Coordination of Competitive Advertising via Investing in Transportation Lead Time Reduction

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Abstract

In this study, a contract for vertical and horizontal coordination is developed in which transportation mode and carbon emissions tax play a key role in determining the values of the contract parameters. The contract is designed for simultaneous coordination of cooperative advertising and periodic review replenishment decisions of a supplier and two competitive retailers. To obtain the optimal decisions, firstly, the traditional decision-making structure is modeled. After that, the centralized structure is modeled to obtain decisions that are profitable for the whole supply chain. Finally, for convincing the competitive retailers to accept the centralized decisions, the supplier applies a lead time crashing contract in which two transportation modes, i.e. fast and slow, can be used. Considering the carbon emissions tax imposed by the government, the coordination contract is designed in such a way that the supplier considers the trade-off between reducing lead time and paying tax on carbon emissions while providing enough incentives for the competitive retailers. Results of the sensitivity analyses showed that the proposed model is profitable from economic and environmental viewpoints. From environmental viewpoint, considering the carbon tax leads to a decrease in the carbon emissions that will be released by the transportation modes. From economic viewpoint, coordinating coop (cooperative) advertising and replenishment decisions of the SC members, enhances demand and provides a higher service level, which increases the SC profit. The contract is conditionally applicable under situations where the carbon emissions tax or lead time reduction costs become high.

Keywords: Transportation lead time, competition, cooperative advertising, transportation mode, carbon emissions tax, periodic review replenishment system

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1. Introduction

Transportation lead time is the interval between the leaving time of shipper from depot and its arriving time to the customer. Since the interval may become long, transportation lead time may affect the service level of downstream members of supply chain, which is the fraction of demand that is met in a particular time period [Johari et al. 2017]. Consequently, it affects downstream members profit and their ordering decision. One way to reduce the transportation lead time, is using a fast transportation mode instead of a slow mode since there are different transportation modes that can be applied for transporting products between supply chain echelons. Fast transportation modes usually lead to higher carbon emissions produced through consuming more amount of fuel by the transportation vehicle [Corbett et al. 2009]. On the other hand, because of the impacts of industries' operations on environmental pollution, different laws are established to prevent from these effects. One is the tax that the government imposes on high emission levels. Accordingly, for reducing the transportation lead time, it is valuable to consider the trade-off between using a fast transportation mode and paying tax on emissions. Moreover, because of the dependency of lead time reduction by upstream and ordering decisions of downstream members, managing these decisions in logistics and transportation through a coordination model is of high importance.

In traditional supply chains, decisions are made individually by each member (i.e. under decentralized model) which may lead to double marginalization effect and inefficiency of the supply chain [Zhang and Chen, 2013; Nematollahi et al. 2017a]. Accordingly, various schemes are applied in the supply chain literature to coordinate SC members' decisions and improve the efficiency of the SC performance. One of the schemes in management and coordination of supply chain, which has attracted researchers' attention is lead time reduction (crashing) scheme. Chaharsooghi and Heydari showed that lead time mean and variance are two factors that highly affect SC performance, especially its ordering system [Chaharsooghi and Heydari, 2010]. Li et al. studied coordination in a vendor-buyer SC by considering the possibility of reducing lead time with a crashing cost [Li et al. 2011]. In another study, Li et al. showed that lead time reduction decreases inventory costs and the coordination models will be more effective [Li et al. 2012]. Heydari proposed an incentive scheme based on reducing lead time fluctuations in a two-echelon supply chain [Heydari, 2014]. Heydari et al. applied crashing lead time as a scheme for coordinating ordering decisions in a seller-buyer SC by considering different transportation modes [Heydari et al. 2016]. Lin investigated the effect of reducing lead time fluctuation in minimizing the total cost in an integrated two-echelon system [Lin, 2016]. Johari et al. proposed a lead-time crashing scheme based on different transportation modes for achieving coordination in a two-echelon supply chain [Johari et al. 2017]. Similarly, in the current study, a coordination scheme based on crashing lead time will be developed with the difference that the contract is designed to coordinate decisions in a competitive situation, carbon emissions tax is considered in this contract and there is a trade-off between reducing lead time and paying carbon tax.

Different laws are established by the government to regulate firms' carbon emissions. One is carbon tax policy which charges the company for its emissions by taxes [Toptal et al. 2014]. On the other hand, one of the main contributors to carbon emissions is transportation [Hoen et al. 2013]. Changing logistics and transportation decisions is one of the ways that companies can apply to reduce carbon emissions [Toptal et al. 2014]. Fast transportation modes usually lead to more carbon emissions since they use more fuel in comparison with slow ones [Corbett et al. 2009]. Thus, when a company needs to switch from a slow mode to a fast mode, it may face with taxes on additional emissions. Accordingly, in this paper, the lead time crashing contract is developed to consider carbon emission tax. To be more precise, under lead time crashing contract, the supplier can reduce the lead time with a slow transportation mode up to a specified level and for reductions more than this level, he may change the slow mode into a fast one. As mentioned above, fast modes lead to higher carbon emissions and more tax payment. Thus, the supplier reduces the lead time in such a way that not only enough incentives are provided for the retailers, but also the transportation and emissions costs under the coordination contract does not reduce the supplier profit in comparison with his profit under the decentralized model. To model these effects, both the transportation cost and carbon emissions tax are considered under the coordination contract.

Because of the importance of transportation issues and its effects on supply chain performance, some studies have addressed these issues through scheduling and various optimization models. Beheshtinia and Ghasemi developed a metaheuristic algorithm for solving a mutli-objective model with the aims of minimizing the delivery time of the orders and the total distance that the vehicle travels [Beheshtinia and Ghasemi, 2017]. In another study, Beheshtinia et al. proposed a shared transportation system for reducing transportation expenditure as well as production expenditure and developed a new genetic algorithm to solve the objectives of the problem [Beheshtinia et al. 2017]. Borumand and Beheshtinia considered not only the objectives of minimizing delivery time and production expenditures, but also considered the objective of minimizing the emissions by the supplier and vehicles and maximizing the product quality [Borumand and Beheshtinia, 2017]. They stated that scheduling plays an important role in supply chain coordination.

In addition to the transportation lead time, managing inventory decisions is closely related to the system that is used by the company for replenishing the inventory. There are two inventory systems which are used for reviewing the inventory level and replenishing the items. One is continuous review and another is periodic review model [Eynan and Kropp, 2007]. In both systems, due to the effects of downstream member's replenishment decisions on the level of stock-outs or overstocking and consequently its effect on the other SC members' profitability, coordinating replenishment decisions is of high importance [Nouri, Hosseini-Motlagh and Nematollahi, 2018]. Managing ordering and replenishment decisions through coordination models is much studied under continuous review systems as in [Chaharsooghi, Heydari and Kamalabadi, 2011; Heydari, 2013; Heydari and Norouzinasab, 2016; Cobb, 2016]. Recently, there are few studies which have considered coordination of replenishment decisions under periodic review inventory systems. Nematollahi, et al. studied coordination of visit interval and service level decisions under periodic review setting by proposing a novel collaboration model [Nematollahi, et al. 2017b and 2018]. After that, Johari et al. proposed a quantity discount contract for coordinating replenishment decisions under periodic review inventory model [Johari et al. 2017]. Ebrahimi et al. proposed a delay in payments contract for coordinating replenishment and promotional efforts decisions under a periodic review inventory setting [Ebrahimi et al. 2017]. Recently, Johari et al. developed a bi-level credit period model for coordinating credit period, pricing and periodic review inventory decisions [Johari et al. 2018]. Although these studies have investigated SC decisions under periodic review inventory model, they did not consider coordination of cooperative and competitive advertising as well as replenishment decisions under periodic review model, which the current study aims to do.

In today's competitive market, several efforts are done by companies in order to increase their market share and make brand image. Cooperative advertising is an example of these effort by supply chain members. Bergen and John define cooperative advertising as an agreement between the upstream and downstream members of a supply chain, in which the upstream member shares a fraction of the downstream members' investment in advertising [Bergen and John, 1997]. For achieving an effective agreement, many researches in supply chain literature have studied cooperative advertising to find the optimal values of the fraction to be shared and the level of advertising investment. For instance, Xie and Neyretstudied cooperative advertising policy under Nash and Stackelberg games and under cooperation model to derive the optimal solutions of the policy [Xie and Neyretstudied, 2009]. Wang et al. investigated cooperative advertising in a competitive supply chain under different non-cooperative game structures and also proposed a cost-sharing contract to obtain the agreement parameters under coordination model [Wang et al. 2011]. Yang et al. calculated the agreement parameters in a manufacturer-retailer channel by considering the retailer's fairness concerns by which they dislike unfair achievements in comparison to their rivals [Yang et al. 2013]. Karray and Amin studied coop advertising under different game models and under coordination and stated that under some situations coop advertising may not be beneficial [Karray and Amin, 2014]. Karray and Surti evaluated the effect of coop advertising and quantity discount contract on supply chain and its members' performance and showed that the profitability of each mechanism is related to the other mechanism [Karray and Surti, 2016]. Johari and Hosseini-Motlagh coordinated cooperative promotional efforts via promotion cost-sharing contract under different game structures [Johari and Hosseini-Motlagh, 2018]. Most of these papers have calculated the optimal values of the coop advertising model under non-cooperative game structures and a few of them have considered coordination model for obtaining these variables. To the best of our knowledge, there is no previous study that obtains these variables with a coordination model under a competitive situation, except Wang et al. and Johari and Hosseini-Motlagh, who proposed a cost-sharing contract for achieving this aim [Wang et al. 2011; Johari and Hosseini-Motlagh, 2018].

According to the literature surveyed above, the contributions of this paper are: (1) Coordination of periodic review replenishment decisions and cooperative advertising in a supply chain with a supplier and two competitive retailers. (2) Proposing a novel coordination scheme based on lead time reduction and carbon emission tax for achieving coordination. The contract is a development of the contract proposed by Heydari [Heydari, 2014]. To be more precise, in his contract, lead time is reduced in two transportation modes. We developed this model and considered the effects of carbon emissions tax on the contract, i.e. the supplier considers the trade-off between lead time reduction and carbon tax payment. Moreover, the contract is applied for coordinating a supply chain under a competitive situation, which was not considered by Heydari [Heydari, 2014]. (3) Vertical and horizontal coordination by applying a lead time reduction contract which is restricted by carbon emission tax.

The paper is structured as follows. In the next section, the main problem is described. In Section 3, different structures are modeled and solved. In Section 4, the model is analyzed based on numerical experiments and sensitivity analyses. In section 5, the conclusion of the paper is provided.

2. Problem Definition

In this paper, a supply chain consisting of a monopolistic supplier and duopolistic retailers is investigated. Each retailer uses a periodic review replenishment system (R, T) for making orders and decides on the safety stock level that he holds to prevent from shortages. If the shortages occur in the retailer side, they are partially backordered. The orders are received by the retailers after a constant lead-time. Moreover, there is an advertising competition between the duopolistic retailers for enhancing their stochastic demand and they decide on adverting investment level. Similar to many real-world cases, we have assumed that the retailers are in the same level and not a retailer's power dominates another retailer. Thus, we have analyzed two different games that the duopolistic retailers may follow when they are in the same level: (1) Cournot and (2) Collusion game. Under the Cournot behavior, each retailer individually set the safety factor and advertising decisions. Cournot model is followed by the retailers in the competitive markets. For instance, GOME and Suning, Wal-Mart and Tesco, Carrefour and Auchan are the retailers who follow Cournot structure [Modak et al. 2015]. Another behavior of the duopolistic retailers is Collusion. Under the Collusion behavior, the retailers cooperate with each other to make their decisions. The collusion behavior is followed by the retailers in order to improve their additional profit [Johari and Hosseini-Motlagh, 2018]. The collusion behavior is common in the Chinese market [Modak et al. 2015]. On the other hand, the supplier applies cooperative advertising to affect the competitive retailers' decisions on advertising. Thus, he shares a fraction of the retailers' investment in advertising, which is one of his decision variables. For replenishing the inventory, the supplier follows a lot-for-lot mechanism. In fact, if the retailers make orders per period with length T, the supplier will replenish its inventory per nT times and decides on the multiplier n. In order to obtain the optimal values of the supply chain members' decisions, three structures are investigated. (1) Decentralized structure in which each SC member tries to maximize its profit and does not consider others' profit. Under this model, two game structures, i.e. Cournot and Collusion are investigated. (2) Centralized structure in which decisions are determined from the whole SC perspective, which may not be profitable for all SC members, especially the retailers. (3) Coordination structure in which the supplier applies lead-time crashing to encourage the retailers to accept centralized decisions. Through this mechanism, the supplier can choose to reduce lead-time in two transportation modes, i.e. fast mode or slow mode. For example, the supplier could reduce the lead time through train or truck. The train is the slow mode and truck is the fast mode. The train can reduce the lead time to a certain level. If the supplier wants to reduce the lead time beyond that level, he has to choose the truck instead of the train in order to send the products to the retailers [Heydari et al. 2016]. Reducing lead-time may lead to more amounts of carbon emissions produced by consuming fuels. On the other hand, the government imposes tax on emissions that are higher than a determined level. Thus, under the coordination model, the supplier considers the trade-off between reducing lead-time and paying tax on carbon emissions. The notations used for the parameters and decision variables are represented in Table 1.

3. Model Formulation and Solution Procedures

In this section, the profit functions of SC members are formulated. The market demand of each retailer is dependent on his/her advertising and their rival's advertising [Johari and Hosseini-Motlagh, 2018]. The market demand follows a normal distribution (D_{r_i}, σ^2). D_{r_i} is formulated as follows:

$D_{r_i} = d_0 + \mu a_i - \gamma a_j; \;\; i = 1,2, i \neq j$ (1)

Retailer i applies a periodic review replenishment system and decides on the order-up-to-level and the value of advertising investment. According to Montgomery et al. under the periodic review replenishment system, the ordering cost is calculated as $\frac{A_r}{T}$, and the holding cost is computed as $h_r\left(R - D_{r_i}L - \frac{D_{r_i}T}{2} + \alpha E(x-R)^+\right)$, [Montgomery et al. 1973]. Moreover, the shortage cost is $\frac{\pi+\alpha(p-w)}{T}E(X-R)^+$. Furthermore, the retailer invests in advertising with an amount of $\frac{\eta {a_i}^2}{T}$ and the supplier shares a fraction of advertising investment. Thus, the profit function of retailer i is calculated as follows:

$$\Pi_{r_i}(R_i, a_i) = (p-w)(d_0 + \mu a_i - \gamma a_j) - \frac{A_r}{T} - h_r\left(R_i - (d_0 + \mu a_i - \gamma a_j)L - \frac{(d_0+\mu a_i - \gamma a_j)T}{2} + \alpha E(x-R_i)^+\right) - \frac{\pi+\alpha(p-w)}{T}E(X-R_i)^+ - \frac{1}{2}(1-\varphi)\frac{\eta {a_i}^2}{T} \quad (2)$$

where the first term is the sales revenue of the retailer. The second term shows the ordering cost. The third and fourth terms denote the holding cost and shortage cost, respectively. The last term is the advertising cost. The order-up-to-level is determined as $R_i = D_{r_i}(T+L) + k\sigma\sqrt{T+L}$. Thus, the expected shortage at the end of each interval can be calculated as follows:

$$E(X-R)^+ = \int_R^\infty (x-R) f_x(x)dx = \int_k^\infty \sigma\sqrt{T+L}\,(Z-K) f_z(z)dz = \sigma\sqrt{T+L}G(K) \quad (3)$$

where $G(k)$ is:

$$G(K) = \int_k^\infty (Z-K) f_z(z)dz = \varphi(K) - K[1 - \Phi(K)] \quad (4)$$

By substituting $E(X-R)^+$ and order-up-to-level in Eq. (2), the profit function of retailer can be transformed to:

$$\Pi_{r_i}(k_i, a_i) = (p-w)(d_0 + \mu a_i - \gamma a_j) - \frac{A_r}{T} - h_r\left(\frac{T(d_0+\mu a_i - \gamma a_j)}{2} + k_i\sigma\sqrt{T+L} + \alpha\sigma\sqrt{T+L}G(k_i)\right) - \frac{\pi+\alpha(p-w)}{T}\sigma\sqrt{T+L}G(k_i) - \frac{1}{2}(1-\varphi)\frac{\eta {a_i}^2}{T} \quad (5)$$

The total demand received by the supplier is the sum of the retailers' demand minus lost sale that can be calculated as follows:

$$D_T = D_{r_1} + D_{r_2} - \frac{\alpha}{T}\sigma\sqrt{T+L}\big(G(k_1) + G(k_2)\big) = 2d_0 + (\mu-\gamma)(a_1+a_2) - \frac{\alpha}{T}\sigma\sqrt{T+L}\big(G(k_1)+G(k_2)\big) \quad (6)$$

The supplier's profit function is formulated as follows:

$$\Pi_s(n, \varphi) = (w-m)\left(2d_0 + (\mu-\gamma)(a_1 + a_2) - \frac{\alpha}{T}\sigma\sqrt{T+L}\big(G(k_1)+G(k_2)\big)\right) - \frac{A_s}{nT} - h_s\left[\frac{(n-1)\Big([2d_0+(\mu-\gamma)(a_1+a_2)]T - \alpha\sigma\sqrt{T+L}\big(G(k_1)+G(k_2)\big)\Big)}{2}\right] - \frac{\varphi\eta}{2T}({a_1}^2 + {a_2}^2) \quad (7)$$

where the first term shows the revenue. The second and third terms are ordering cost and holding cost, respectively. The last term illustrates the cost of sharing the retailers' advertising investment.

Table 1. The notations used for the parameters and decision variables

Parameters and Variables	
Decisions variable	
R	Order-up-to-level
a_i	Retailer i's advertising level
n	Supplier's replenishment cycle multiplier
φ	The fraction of advertising cost that is shared by the supplier
Parameters	
k	Retailer's safety factor
d_0	Initial demand
μ	Coefficient of the retailer's advertising on increasing the demand
γ	The rival's sensitively coefficient of advertising on demand
P	Selling price
T	Length of the review period
L	Length of the lead time
σ	Standard deviation of the demand
A_r	Retailer's ordering cost per order
h_r	Retailer's inventory holding cost per item
α	Fraction of the demand during the stock-out period that will be lost
π	Shortage cost per item
w	Wholesale price
m	Supplier's purchase cost per item
A_s	Supplier's fixed ordering cost per order
h_s	Supplier's inventory holding cost per item
θ	Amount of CO2 emissions from fuel per gallon consumed (ton/gallon)
C_t	Carbon emissions tax
C_s	Cost of lead time reduction in the slow mode
C_f	Cost of lead time reduction in the fast mode
g_s	fuel volume need per trip (gallons) in the slow mode
g_f	fuel volume need per trip (gallons) in the fast mode
A	Point at which more lead time reduction requires shifting to the fast mode
M	Maximum possible lead time crashing
LTR	Percentage of lead time crashing
R_1	Bargaining power of the retailer 1
R_2	Bargaining power of the retailer 2
R_3	Bargaining power of the supplier

Note: the superscripts Ct, Cn, Cen and Co mark the Cournot behavior, Collusion behavior, centralized model and coordinated model, respectively.

3.1 Decentralized Model

Under the decentralized model, each SC member makes decisions individually in order to maximize its own profitability. In this section, a Supplier-Stackelberg game is used in which the supplier as the leader firstly decides on his/her replenishment cycle multiplier (n) and the fraction of sharing the advertising cost. Then, the retailers as the followers decide on the safety factor (k) and advertising (a_i). In the following, two different behaviors of the retailers (Cournot and Collusion) are investigated in this paper.

3.1.1 The retailer's Cournot behavior

Under the Cournot behavior, both retailers simultaneously determine the value of investment in the advertising. Backward induction is used in order to solve the SC members' problem.

Theorem 1. Under the Cournot behavior, the profit function of retailer i is concave with respect to k_i and a_i.

Proof. See Appendix A.

By setting $\frac{\partial \Pi_{r_i}}{\partial a_i} = 0$ and $\frac{\partial \Pi_{r_i}}{\partial k_i} = 0$, the optimal values of the retailers' decisions are obtained as follows:

$$a_i^{Ct} = \frac{T\left(\mu(p-w)-h_r\frac{T\mu}{2}\right)}{(1-\varphi)\eta} \quad (8)$$

$$1-\phi\left(K_i^{Ct}\right) = \frac{h_rT}{h_r\alpha T+\pi+\alpha(p-w)} \quad (9)$$

By substituting Eq. (8-9) into the profit function of the supplier, under the Cournot behavior, the supplier's problem is modeled as follows:

$$Max\ \Pi_s\,(n,\varphi) = (w-m)\left(2d_0+(\mu-\gamma)(a_1+a_2)-\frac{\alpha}{T}\sigma\sqrt{T+L}\left(G(k_1)+G(k_2)\right)\right)-\frac{A_s}{nT}$$
$$-h_s\left[\frac{(n-1)\left([2d_0+(\mu-\gamma)(a_1+a_2)]T-\alpha\sigma\sqrt{T+L}\left(G(k_1)+G(k_2)\right)\right)}{2}\right]-\frac{\varphi\eta}{2T}\left({a_1}^2+{a_2}^2\right) \quad (10)$$

Subject to $\quad a_1, K_1 \in argmax\ \Pi_{r_1} \quad$ (10a)

$\quad a_2, K_2 \in argmax\ \Pi_{r_2} \quad$ (10b)

Theorem 2. Under the Cournot behavior, the profit function of the supplier is concave with respect to n and φ.

Proof. See Appendix B.

By solving $\frac{\partial \Pi_s}{\partial n} = 0$ and $\frac{\partial \Pi_s}{\partial \varphi} = 0$, the optimal values of supplier's decisions are determined as follows:

$$n^{Ct} = \sqrt{\frac{2A_s}{Th_s\left(\left(2d_0+2(\mu-\gamma)\left(\frac{\theta_1 T}{(1-\varphi)\eta}\right)\right)T-2\alpha\sigma\sqrt{T+L}G(\theta_2)\right)}} \quad (11)$$

$$\varphi^{Ct} = \frac{2T\theta_1(w-m)(\mu-\gamma)-T^2\theta_1h_s(n-1)(\mu-\gamma)-T{\theta_1}^2}{2T\theta_1(w-m)(\mu-\gamma)-T^2\theta_1h_s(n-1)(\mu-\gamma)+T{\theta_1}^2} \quad (12)$$

where $\theta_1 = \mu(p-w)-h_r\frac{T\mu}{2}$, $\theta_2 = \phi^{-1}(1-\frac{h_rT}{h_r\alpha T+\pi+\alpha(p-w)})$.

3.1.2 The Retailer's Collusion Behavior

Under the Collusion behavior, the duopolistic retailers jointly determine the value of investment in advertising. Therefore, the retailers' problem under the Collusion behavior is modeled as follows:

$$\Pi_r(k_1,a_1,k_2,a_2) = \sum_{i=1}^{2}\Pi_{r_i}(k_i,a_i\,) \quad (13)$$

Theorem 3. Under the Collusion behavior, the profit function of retailers is concave with respect to k_1, k_2, a_1 and a_2.

Proof. See Appendix C.

By solving the first derivatives equal to zero, under the Collusion behavior, the optimal values of decisions are obtained as follows:

$$a_1^{Cn} = \frac{T\left((\mu-\gamma)(p-w)-h_r\frac{T}{2}(\mu-\gamma)\right)}{(1-\varphi)\eta} \quad (14)$$

$$1-\phi\left(K_1^{Cn}\right) = \frac{h_rT}{h_r\alpha T+\pi+\alpha(p-w)} \quad (15)$$

$$a_2^{Cn} = \frac{T\left((\mu-\gamma)(p-w)-h_r\frac{T}{2}(\mu-\gamma)\right)}{(1-\varphi)\eta} \quad (16)$$

$$1-\phi\left(K_2^{Cn}\right) = \frac{h_rT}{h_r\alpha T+\pi+\alpha(p-w)} \quad (17)$$

By substituting Eq. (14-17) into the profit function of the supplier, under the Collusion behavior, the supplier's problem is formulated as follows:

$$Max\ \Pi_s\,(n,\varphi) = (w-m)\left(2d_0+(\mu-\gamma)(a_1+a_2)-\frac{\alpha}{T}\sigma\sqrt{T+L}\left(G(k_1)+G(k_2)\right)\right)-\frac{A_s}{nT}$$
$$-h_s\left[\frac{(n-1)([2d_0+(\mu-\gamma)(a_1+a_2)]T)}{2}\right]+h_s[\alpha\frac{\sigma\sqrt{T+L}\left(G(k_1)+G(k_2)\right)}{2}]-\frac{\varphi\eta}{2T}\left({a_1}^2+{a_2}^2\right) \quad (18)$$

Subject to $\quad a_1, K_1, a_2, K_2 \in argmax\ \Pi_r \quad$ (19)

Theorem 4. Under the Collusion behavior, the profit function of the supplier is concave with respect to n and φ.

Proof. See Appendix D.

By setting $\frac{\partial \Pi_s}{\partial n} = 0$ and $\frac{\partial \Pi_s}{\partial \varphi} = 0$, the optimal values of supplier's decisions are determined as follows:

$$n^{Cn} = \sqrt{\frac{2A_s}{Th_s\left(\left(2d_0+2(\mu-\gamma)\left(\frac{\theta_3 T}{(1-\varphi)\eta}\right)\right)T-2\alpha\sigma\sqrt{T+L}G(\theta_2)\right)}} \quad (20)$$

$$\varphi^{Cn} = \frac{2T\theta_3(w-m)(\mu-\gamma)-T^2\theta_3 h_s(n-1)(\mu-\gamma)-T{\theta_3}^2}{2T\theta_3(w-m)(\mu-\gamma)-T^2\theta_3 h_s(n-1)(\mu-\gamma)+T{\theta_3}^2} \quad (21)$$

where $\theta_1 = (\mu-\gamma)(p-w) - h_r \frac{T}{2}(\mu-\gamma)$.

3.2 Centralized Model

Under the centralized model, a decision maker determines all SC decisions (replenishment cycle multiplier, fraction of sharing advertising cost, safety factor and advertising level) to optimize the whole SC profitability. In this model, the profit function of the whole SC is the sum of retailers' and supplier's profit functions and can be formulated as follows:

$$\begin{aligned}\Pi_{sc}(K_1,a_1,K_2,a_2,n) &= \Pi_s(n) + \Pi_{r_1}(K_1,a_1) + \Pi_{r_2}(K_2,a_2)\\ &= (p-m)\big(2d_0+(\mu-\gamma)(a_1+a_2)\big) - \frac{1}{T}\left(\frac{A_s}{n}+2A_r\right)\\ &-\frac{1}{T}\left[\pi+\alpha(p-m)-\frac{h_s\alpha(n-1)T}{2}\right]\sigma\sqrt{T+L}\big(G(K_1)+G(K_2)\big)\\ &-\frac{\big(2d_0+(\mu-\gamma)(a_1+a_2)\big)T}{2}\left[(n-1)h_s+h_r\right]-\frac{1}{2}\frac{\eta({a_1}^2+{a_2}^2)}{T}\\ &-h_r\left[K_1\sigma\sqrt{T+L}+\alpha\sigma\sqrt{T+L}G(K_1)\right]\\ &-h_r\left[K_2\sigma\sqrt{T+L}+\alpha\sigma\sqrt{T+L}G(K_2)\right] \end{aligned} \quad (22)$$

Theorem 5. Under the centralized model, the profit function of the whole SC is concave with respect to K_1, a_1, K_2, a_2, n.

Proof. See Appendix E.

The optimal values of decisions are obtained as follows:

$${a_1}^{Cen} = \frac{T(2(\mu-\gamma)(p-w)-h_rT(\mu-\gamma)-h_s(n-1)(\mu-\gamma)T)}{2\eta} \quad (23)$$

$$1-\phi\left({K_1}^{Cen}\right) = \frac{2h_rT}{2h_r\alpha T+2\pi+2\alpha(p-m)-Th_s\alpha(n-1)} \quad (24)$$

$${a_2}^{Cen} = \frac{T(2(\mu-\gamma)(p-w)-h_rT(\mu-\gamma)-h_s(n-1)(\mu-\gamma)T)}{2\eta} \quad (25)$$

$$1-\phi\left({K_2}^{Cen}\right) = \frac{2h_rT}{2h_r\alpha T+2\pi+2\alpha(p-m)-Th_s\alpha(n-1)} \quad (26)$$

$$n^{Cen} = \sqrt{\frac{2A_s}{Th_s\big((2d_0+(\mu-\gamma)(a_1+a_2))T-2\alpha\sigma\sqrt{T+L}G(\theta_2)\big)}} \quad (27)$$

Although the centralized structure improves the entire SC profitability (since the decisions are determined by optimizing the entire SC profit function) in comparison with the decentralized model, the profitability of all SC members does not necessarily increase in the centralized model compared to the decentralized model (Nematollahi et al., 2017b). Thus, the centralized model may not be accepted by the member who incurs losses. To overcome this drawback, an incentive contract is used in order to improve not only the entire SC profitability, but also each member profitability.

3.3 Coordinated Model

As mentioned above, in order to enhance the profitability of the entire SC and all SC members, an incentive contract is used. In this paper, a lead time crashing contract is applied to entice the SC members to participate in the joint decision-making. As the lead time gets longer, the order-up-to-level must be increased for more service level which imposes more inventory cost on the retailer [Johari et al. 2017]. Therefore, the lead time impacts on the profitability of the retailer. Under the lead time crashing contract, the supplier decreases the lead time via two transportation modes (slow or fast) in order to motivate the retailers to take part in the joint decision-making structure. The supplier can reduce the lead time with a slow transportation mode up to a specified level and for reductions more than this level, he may change the slow transportation mode into a fast one. Reducing the lead time by two transportation modes is costly for the supplier. Moreover, the carbon tax which is imposed by the government affects the supplier's decisions since reducing the lead time leads to an increase in fuel consumption and consequently increases the carbon emissions. Thus, the supplier has to pay carbon tax to the government for more carbon emissions. Therefore, in order to reduce the lead time, the supplier not only has to pay the lead time reduction costs but also has to pay the carbon emissions tax. Therefore, there is a trade-off between reducing lead time and its incurred costs while providing enough incentives for the retailers. Under the lead time crashing contract, LRT is a factor which is in the interval $[0,1]$. The supplier reduces the lead time from L to L_{new}, which can be computed as follows:

$$L_{new} = (1 - LTR)L \quad (28)$$

In the above equation, if LRT is near zero, it means that reducing the lead time by the supplier is not considerable while, if LRT is close to one, it means that the supplier significantly reduces the lead time. Under the coordinated model, all decisions of SC members (i.e. Safety factor, retailers' advertising, and supplier's replenishment cycle multiplier) are equal to that of the centralized model while the fraction of advertising cost that is shared by the supplier (φ) is an internal decision within the SC; Therefore, that will be omitted in the centralized model. In order to calculate φ in the coordinated model, firstly, the optimal value of advertising is calculated and is set equal to that of the centralized model. This results in an equation from which the optimal value of φ can be calculated. Under the lead time crashing contract, the retailer's profit function is formulated as follows:

$$\Pi_{r_i}{}^{Co}\left(k_i{}^{Cen}, a_i{}^{Co}\right) = (p-w)\left(d_0 + \mu a_i{}^{Co} - \gamma a_j{}^{Cen}\right) - \frac{A_r}{T} - h_r\left(\frac{T\left(d_0+\mu a_i{}^{Co}-\gamma a_j{}^{Cen}\right)}{2} + k_i{}^{Cen}\sigma\sqrt{T+L_{new}} + \alpha\sigma\sqrt{T+L_{new}}G\left(k_i{}^{Cen}\right)\right) - \frac{\pi+\alpha(p-w)}{T}\sigma\sqrt{T+L_{new}}G\left(k_i{}^{Cen}\right) - \frac{1}{2}\left(1-\varphi^{Co}\right)\frac{\eta a_i{}^{Co^2}}{T} \quad (29)$$

Taking the first partial derivative of $\Pi_{r_i}{}^{Co}$ with respect to a_i and setting it equal to zero, we obtain:

$$a_i{}^{Co} = \frac{T\left(\mu(p-w) - h_r\frac{T\mu}{2}\right)}{(1-\varphi)\eta} \quad (30)$$

Under the lead time crashing contract, the retailer sets its advertising equal to that of the centralized model (i.e., $a_i{}^{Co} = a_i{}^{Cen}$). Under such a case, the optimal value of φ will be calculated as follows:

$$\varphi^{Co} = 1 - \frac{\mu(p-w) - h_r\frac{T\mu}{2}}{2(\mu-\gamma)(p-w) - h_r T(\mu-\gamma) - h_s\left(n^{Cen}-1\right)(\mu-\gamma)T} \quad (31)$$

The retailer motivates to participate in the coordinated model if his/her profitability is improved in comparison with the decentralized model. Assume X denotes the type of retailers' behavior, Cournot or Collusion. Thus, the retailer accepts the coordinated model, if the following condition is satisfied:

$$\pi_{r_i}\left(k_i{}^{Cen}, a_i{}^{Cen}, L_{new}\right) > \pi_r\left(k_i{}^X, a_i{}^X\right) \quad (32)$$

The minimum value of LTR which is acceptable for the retailer is determined as follows:

$$LTR_{min} = 1 - \left(\left(\frac{A+B}{-C}\right)^2 - T\right)\frac{1}{L} \quad (33)$$

where

$$A = (p-w)\left(d_0 + \mu a_i{}^X - \gamma a_j{}^X\right) - \frac{A_r}{T} - h_r\left(\frac{T\left(d_0+\mu a_i{}^X-\gamma a_j{}^X\right)}{2} + k_i{}^X\sigma\sqrt{T+L_{new}} + \alpha\sigma\sqrt{T+L_{new}}G\left(k_i{}^X\right)\right) - \frac{\pi+\alpha(p-w)}{T}\sigma\sqrt{T+L_{new}}G\left(k_i{}^X\right) - \frac{1}{2}\left(1-\varphi^X\right)\frac{\eta a_i{}^{X^2}}{T}, \quad (34)$$

$$B = -(p-w)\left(d_0 + \mu a_i{}^{Cen} - \gamma a_j{}^{Cen}\right) - \frac{A_r}{T} + h_r\frac{T\left(d_0+\mu a_i{}^{Cen}-\gamma a_j{}^{Cen}\right)}{2} + \frac{1}{2T}\left(1-\varphi^{Co}\right)\eta a_i{}^{Cen^2} \quad (35)$$

$$C = h_r {k_i}^{Cen}\sigma + h_r\alpha\sigma G\left({k_i}^{Cen}\right) + \frac{\pi+\alpha(p-w)}{T}\sigma G\left({k_i}^{Cen}\right) \qquad (36)$$

On the other hand, the supplier decreases the lead time by a faster transportation mode. A faster transportation mode needs more fuel and consequently emits more carbon into the environment. Moreover, the supplier has to pay a tax per additional carbon emissions. Therefore, the carbon emissions tax and lead time reduction cost are incurred to the supplier. Under the proposed coordinated model, the supplier's replenishment cycle multiplier is equal to that of the centralized model and the fraction of advertising cost that is shared by the supplier (φ) is calculated in the coordinated model. Therefore, the supplier's profit function is calculated as follows:

$$\Pi_s{}^{Co}\left(n^{Cen},\varphi^{Co},L_{new}\right) = (w - m)\left(2d_0+(\mu-\gamma)\left({a_1}^{Cen}+{a_2}^{Cen}\right) - \frac{\alpha}{T}\sigma\sqrt{T+L_{new}}\left(G\left({k_1}^{Cen}\right)+G\left({k_2}^{Cen}\right)\right)\right) - \frac{A_s}{n^{Cen}T} - h_s\left[\frac{\left(n^{Cen}-1\right)\left(\left[2d_0+(\mu-\gamma)\left({a_1}^{Cen}+{a_2}^{Cen}\right)\right]T\right)}{2}\right] - h_s\left[\frac{-\alpha\sigma\sqrt{T+L_{new}}\left(G\left({k_1}^{Cen}\right)+G\left({k_2}^{Cen}\right)\right)}{2}\right] - \frac{\varphi^{Co}\eta}{2T}\left({a_1}^{Cen^2}+{a_2}^{Cen^2}\right) - \frac{LTRC}{T} \qquad (37)$$

The last term in Eq. (37) shows the lead time crashing cost in each interval. The lead time can be reduced by two transportation modes, slow or fast. The cost of lead time reduction in each transportation mode is different and can be calculated as Eq. (38). According to Eq. (38), if the LRT is in the interval $(0, A)$, the slow mode is used for shipment in which the lead time reduction cost is determined from the first criterion. According to Heydari, Zaabi-Ahmadi, and Choi, transportation cost is considered as C_sLTR [Heydari, Zaabi-Ahmadi, and Choi, 2016]. According to Bazan, Jaber, and Zanoni, carbon emissions tax is considered as $(g_s\theta)C_t$ [Bazan, Jaber, and Zanoni, 2015]. Moreover, if the LRT is in the interval (A, M), it means that the fast mode is chosen for shipment and the lead time reduction cost is calculated from the second criterion.

$$LTRC = \begin{cases} C_sLTR + (g_s\theta)LTRC_t & 0 < LTR \le A \\ C_fLTR + (g_f\theta)LTRC_t & A < LTR \le M \end{cases} \qquad (38)$$

The supplier participates in the coordinated model, if the following condition is satisfied:

$$\pi_s\left({k_1}^{Cen},{a_1}^{Cen},{k_2}^{Cen},{a_2}^{Cen},n^{Cen},\varphi^{Co},L_{new}\right) > \pi_s\left({k_1}^{X},{a_1}^{X},{k_2}^{X},{a_2}^{X},n^{X},\varphi^{X}\right) \qquad (39)$$

By substituting Eq. (38) into the supplier's profit function, the supplier's profit function becomes complicated. Therefore, a close-form formula for LTR_{max} cannot be calculated. In order to determine LTR_{max}, we develop an algorithm. According to this algorithm, we firstly suppose that the supplier determines LTR equal to the end of the interval (M) and the supplier's profit function is calculated according to $LRT = M$. Then the supplier's profit is compared in the decentralized and coordinated models. If the profit of supplier in the coordinated model is less than that under the decentralized model, it means that the coordination contract is not acceptable for the supplier. Therefore, a small amount is reduced from LRT and the process is repeated until the difference between the supplier's profit under the coordinated and decentralized models becomes positive. The first LRT that causes the difference between the supplier's profit in the coordinated and decentralized models become positive, is chosen as LTR_{max}. The algorithm is as follows:

Determining LR_{max} algorithm

Step1: Set $LTR = M$.

Step2: Calculate the profit of supplier using Eq. (37).

Step3: Calculate $\pi_s\left({k_1}^{Cen},{a_1}^{Cen},{k_2}^{Cen},{a_2}^{Cen},n^{Cen},\varphi^{Co},L_{new}\right) - \pi_s\left({k_1}^{X},{a_1}^{X},{k_2}^{X},{a_2}^{X},n^{X},\varphi^{X}\right)$.

Step4: If the obtained value in Step 3 is greater than zero, LTR_{max} is equal to LTR;

otherwise let $LTR = LTR - \varepsilon$, where ε is a small positive quantity, and go to step 2.

According to the above algorithm, LTR is accepted as LTR_{max} when the supplier's profit in the coordination model is more than the decentralized model. It means that, for this value of LTR_{max}, the supplier participates in the coordination model. Therefore, Eq. (39) is satisfied and LTR_{max} is calculated by the proposed algorithm. If LTR is in the interval $[LTR_{min}, LTR_{max}]$, the incentive contract can coordinate the proposed SC. The retailer achieves the whole surplus profit of the coordinated model if $LTR = LTR_{max}$. On the other hand, if $LTR = LTR_{min}$ the supplier gains the whole surplus profit of the coordinated model. Under the coordinated model, the surplus profit of each member depends on the value of lead time. Therefore, in this study, a profit-sharing mechanism based on the bargaining power of SC members is used to determine the exact value of LTR. Each SC member whose bargaining power is more than the others, expects to earn more surplus profit. Assume R_1, R_2, R_3 are the bargaining powers of retailer 1, retailer 2 and supplier, respectively, where $R_1 + R_2 + R_3 = 1$. Thus, LTR^{Co} can be determined as follows:

$$LTR^{Co} = R_1 LTR_{max} + R_2 LTR_{max} + R_3\, LTR_{min} \quad (40)$$

The retailers prefer high reductions in lead time and they like it to be near LTR_{max}. On the other hand, the supplier prefers the lead time to be near LTR_{min}. Thus, LTR^{Co} is obtained from Eq. (40) based on the SC members' bargaining power.

4. Numerical Experiments

In this section, the proposed model is analyzed based on three numerical test problems, which the values of parameters are represented in Table 2 and the results of running the model based on these values are represented in Table 3.

As can be seen in Table 3, under the decentralized model, for instance, the Stackelberg-Cournot structure in test problem 1, the retailer invests in advertising with a level of $a_i = 1.87$, which leads to a cost equal to 1/2*0.3* 2*1.87*1.87=1.05$ for the retailer in which the supplier has shared %71 of the retailer's investment in advertising. Moreover, the safety stock factor is 0.44 unit. Under the centralized model, the retailer must invest more in advertising, i.e., from 1.87 under the decentralized model to 2.02 under the centralized model. The safety stock factor is increased from 0.44 to 1.10 unit. Investing more in advertising and holding more amounts of safety stock, leads to more costs for the retailer and consequently its profit is decreased. Thus, he may refuse to participate in the centralized decision-making. On the other hand, the centralized model is profitable for the supply chain since its profit is increased from $330251 (Stackelberg-Cournot) to $353870 (Centralized model), which is a %7 increase in the SC profit. Thus, the supplier as the leader, decides to provide an incentive scheme for the retailers and reduces lead time with %79 which is possible by switching to a fast transportation mode. In test problem 2, the supplier reduces the lead time with %44 in a slow transportation mode since the carbon emission tax is higher than that under test problem 1. Thus, the supplier chooses the slow transportation mode which requires 120 gallons fuel per trip and is much less than the fuel volume of the fast transportation mode, i.e. 470 gallons per trip. By applying this incentive, the retailers accept the centralized decisions on advertising and replenishment and not only the whole SC profit increases, but also the retailers' and the supplier's profits increase in comparison to their profits under the decentralized model. Moreover, the environmental performance of the SC is enhanced since transportation lead time reduction and carbon emission tax payment are determined rationally.

Table 2. Values of the model parameters in three test problems

Parameter	Test Problem 1	Test Problem 2	Test Problem 3
P	100	120	150
w	90	110	138
m	60	90	120
d_0	6000	3000	5000
μ	50	65	75
γ	11	20	22
$T\ (Day)$	50	35	40
A_r	500	450	350
A_s	550	600	450
σ	2000	1000	1700
h_r	30	12	19
h_s	22	10	15
π	4	3	5
α	0.6	0.8	0.7
$L\ (Day)$	35	25	33
A	0.3	0.6	0.4
M	0.85	0.9	0.9
C_s	240	180	380
C_f	380	250	450
g_s	80	120	160
g_f	260	470	670
c_t	60	75	85
R_1	0.3	0.2	0.25
R_2	0.3	0.2	0.25
R_3	0.4	0.6	0.5

Table 3. Results of running the model under three test problems

	Test Problem 1	Test Problem 2	Test Problem 3
Stackelberg-Cournot			
k_i	0.44	1.30	1.07
a_i	1.87	0.94	0.93
n	1	2	1
φ	0.71	0.48	0.39
Π_{r_i}	12435.62	15073.03	28801.98
π_s	305379.89	109660.79	165185.19
π_{sc}	330251.14	139806.87	222789.16
Stackelberg-			
k_i	0.44	1.30	1.07
a_i	1.81	0.87	0.85
n	1	2	1
φ	0.76	0.61	0.53
Π_{r_i}	12508.27	15146.29	28890.71
π_s	305239.41	109528.12	165026.95
π_{sc}	330255.97	139820.71	222808.37
Centralized structure			
k_i	1.10	1.73	1.43
a_i	2.02	1.04	1.05
n	1	2	1
π_{sc}	353870.26	141721.79	226324.71
Coordinated-			
k_i	1.10	1.73	1.43
a_i	2.02	1.04	1.05
n	1	2	1
φ	0.86	0.76	0.72
Π_{r_i}	13279.88	15428.98	30824.70
π_s	343385.53	111145.65	167801.13
π_{sc}	369945.29	142003.62	229450.53
LR_{min}	70%	24%	18%
LR_{max}	85%	67%	86%
LR_{co}	79%	41%	52%
Coordinated-			
k_i	1.10	1.73	1.43
a_i	2.02	1.04	1.05
n	1	2	1
φ	0.86	0.76	0.72
Π_{r_i}	13309.70	15494.40	30939.28
π_s	343386.82	110967.86	167658.75
π_{sc}	369945.29	142003.62	229450.53
LR_{min}	71%	28%	19%
LR_{max}	85%	69%	88%
LR_{co}	79%	44%	54%
Transportation mode	Fast	Slow	Fast

In the following, the model is analyzed according to the sensitivity analyses with respect to some important parameters of the model.

Figure. 1 illustrates the effect of increasing carbon emissions tax on the maximum amount of lead time reduction. As can be seen, as carbon emission tax increases, the maximum amount that is possible for the supplier to reduce lead time, decreases. One reason for this observation is that, when the supplier reduces lead time, more carbon emissions will be produced by the vehicles since they consume more amounts of fuel for a fast transportation. Thus, the supplier should pay more carbon tax. Accordingly, in situations when the government imposes a high tax on carbon emissions, considering the trade-off between lead time reduction and carbon tax payment, the maximum amount of lead time reduction will be decreased. This decision by the supplier, may not cause the retailers to refuse the coordination model until the interval between LTR_{min} and LTR_{max} has not become empty.

Figure. 2 shows the changes in the maximum amount of lead time reduction with respect to increase in lead time reduction cost using a fast transportation mode. From Figure. 2, the increase in lead time reduction cost, restricts the maximum amount that is possible for the supplier to reduce lead time up to that. In fact, by considering the trade-off between reducing lead time for the retailers and encountering high cost levels of lead time reduction, the supplier provides a less decrease in lead time because if he reduces lead time more than a specified level, the costs of lead time reduction may not be compensated by the lead time crashing contract. Another important observation is that the distance between LTR_{min} and LTR_{max} decreases, which are the amount of lead time reduction that are appropriate for the supplier and the retailers, respectively. This indicates that when the lead time reduction cost increases, LTR_{min} and LTR_{max} may have close values and the supplier reduces the lead time with values near the minimum possible amounts of reduction. In fact, he reduces the lead time in such a way that, not only the retailers accept the coordination model, but also the incentive does not incur a loss for himself.

Figure. 3 shows the changes in safety stock factor with respect to the increase in lost sale rate. According to this figure, under this situation, the retailer increases the amount of safety factor. Under the proposed lead time crashing contract, the retailer holds more amount of safety stock in comparison with the decentralized model. Thus, under the coordination scheme, less amount of demand will be lost and the retailer may provide a higher service level.

Holding more amount of safety stock costs more for the retailer but as can be seen in Figure. 4, under lead time crashing contract, the retailer profit is still greater than its profit under the decentralized model. Thus, by using the lead time crashing contract, the supply chain not only is more responsive to customer demand when the lost sale rate increases, but also can make more profit rather than the decentralized model.

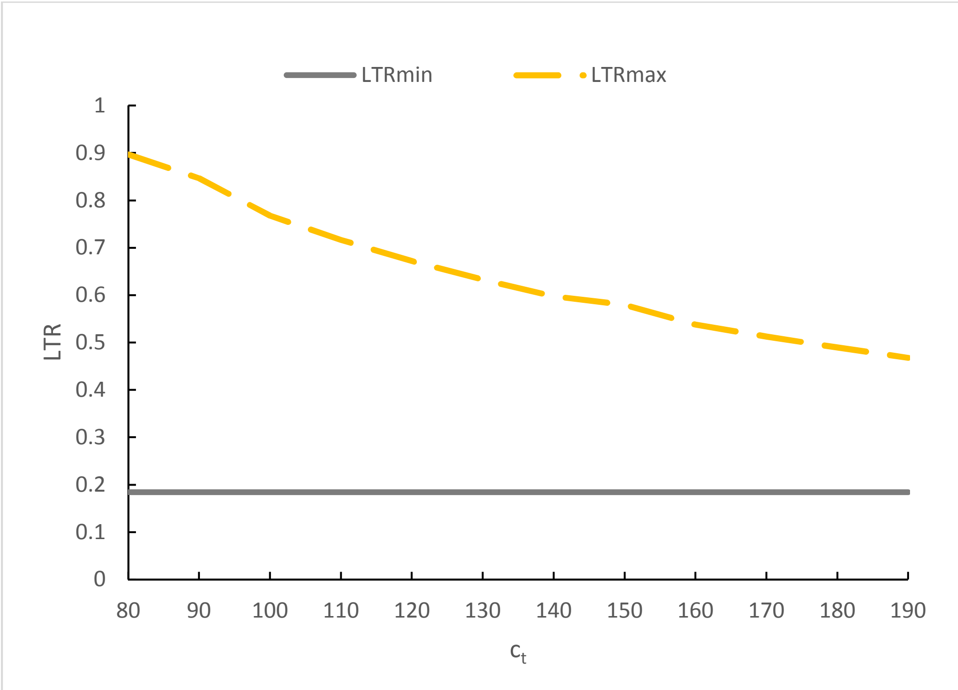


Figure. 1. Changes of LTR_{min} and LTR_{max} w.r.t. increase in carbon emission tax

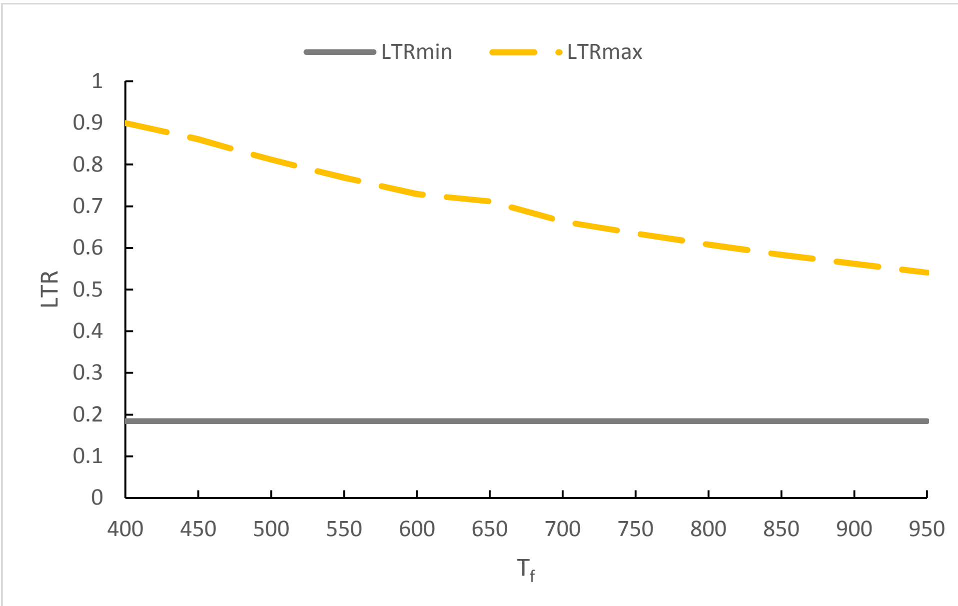


Figure. 2. Changes of LTR_{min} and LTR_{max} w.r.t. increase in lead time reduction cost in fast transportation mode

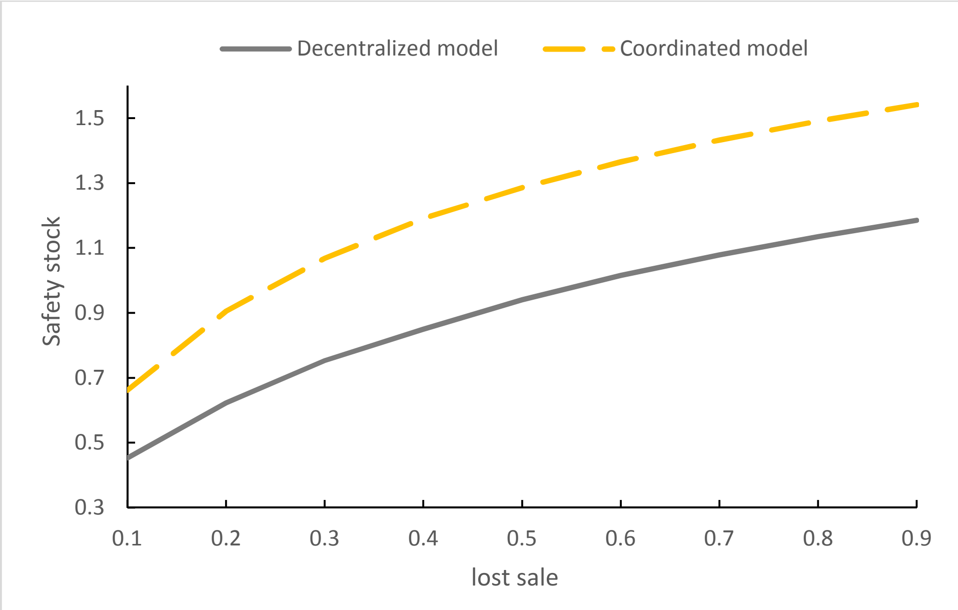


Figure. 3. Effects of increasing lost sale rate on the safety stock factor

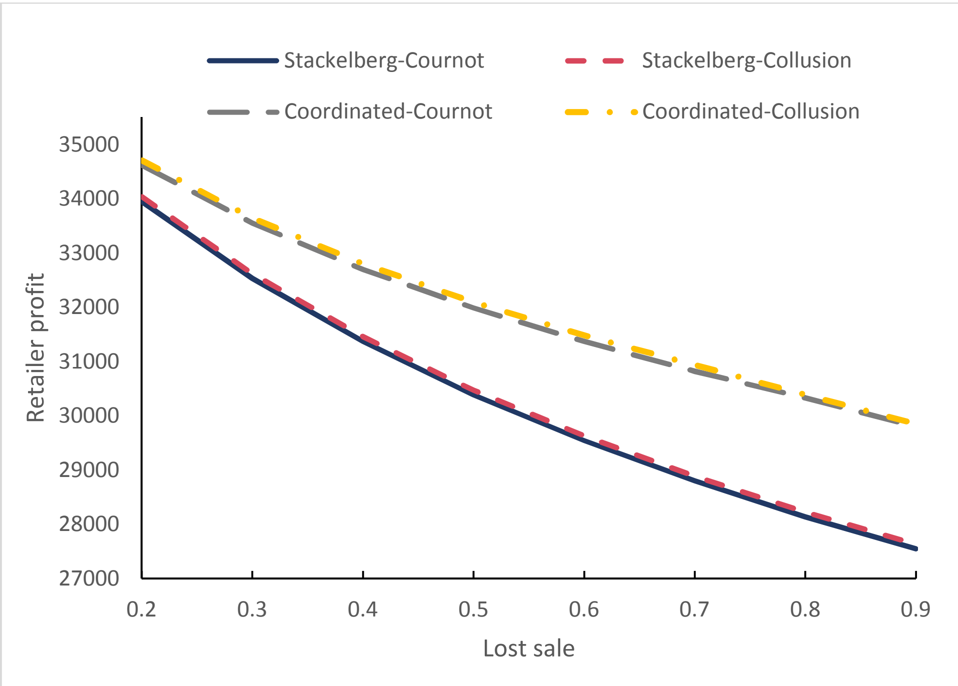


Figure. 4. Changes in the retailer profit according to the increase in lost sale rate

The effect of increasing demand elasticity coefficient of advertising on the supply chain profit is illustrated in Figure. 5. According to this figure, when advertising elasticity coefficient of demand increases, which means that customers are more sensitive to advertising efforts, the supply chain profit increases under all decision-making models. Under lead time crashing contract, the increase in supply chain profit is the most in comparison with the other models. Thus, when customers are more sensitive to advertising effort, the manager of the supply chain can apply the coordination model to convince the retailers to invest more in advertising efforts which lead to enhancing demand and consequently make more profit for the supply chain.

Figure. 6 shows the trends of supplier profit with respect to the advertising competition factor. According to this figure, as the competition degree between the retailers increases, the supplier profit decreases, under both the decentralized structures and the coordinated models. Under the coordination model of both Cournot and Collusion structures, the graph of the supplier profit is above its graph under both the decentralized structures. Thus, in a highly competitive situation, if the supplier applies the lead time reduction contract, he will make more profits rather than the decentralized model. One reason is that, by applying the coordination contract, the supplier encourages the retailers to invest more in advertising efforts and shares a greater part of their advertising investment.

Figure. 7 examines the effects of demand uncertainty on the supply chain profit. From Figure. 7, under all decision-making structures, when the demand uncertainty increases, the supply chain profit decreases. Under the coordination structure, the SC profit is greater than its profit under other decision-making structures. In fact, by applying the lead time crashing contract, the orders will be sooner received by the retailers and they may be more responsive to the customers' demand. Thus, the lead time crashing contract is profitable for the supply chain from economic perspective and also is profitable for the retailers for the reason that it enhances their responsiveness in the market.

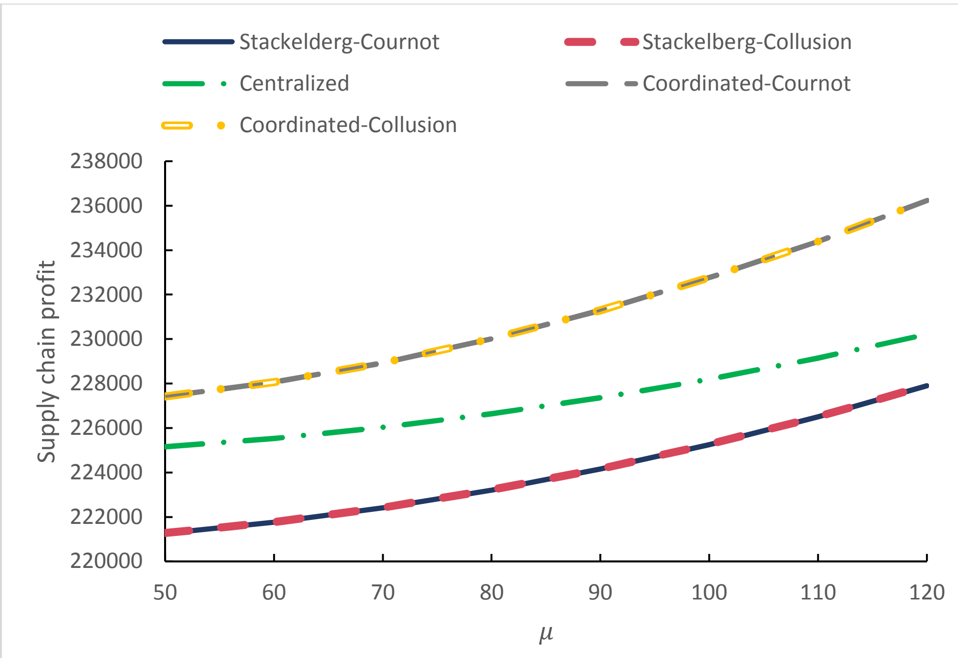


Figure. 5. Effects of changes in advertising elasticity coefficient of demand on the supply chain profit

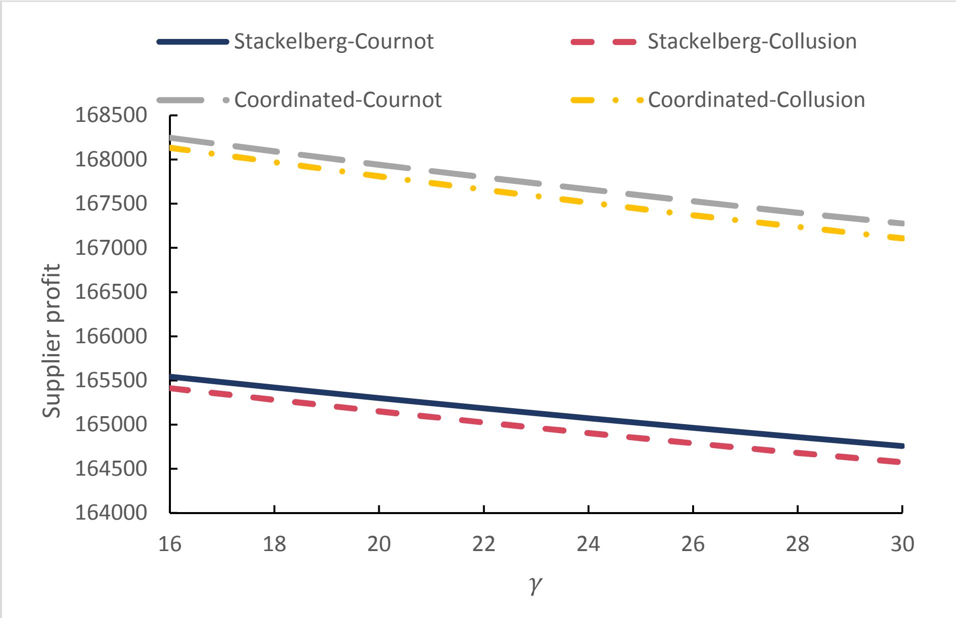


Figure. 6. The effects of increasing competition degree on the supplier profit

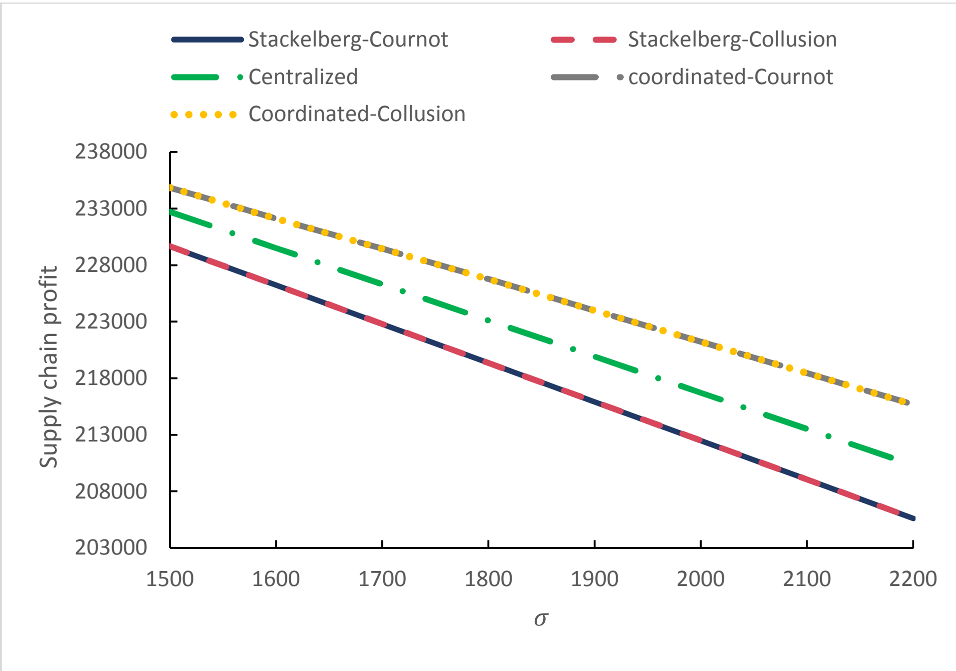
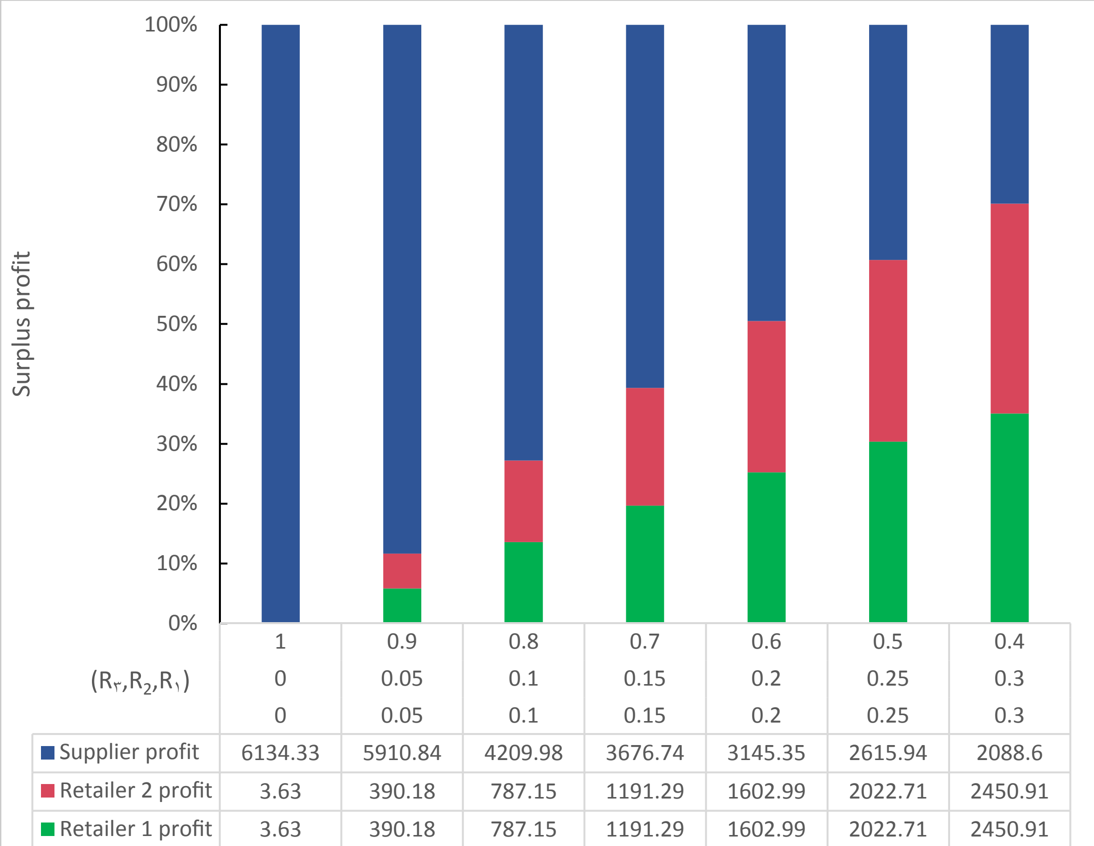


Figure. 7. Changes of the supply chain profit with respect to demand uncertainty

Figure. 8 illustrates different combinations of profit allocation between the supply chain members under the lead time reduction contract based on their bargaining powers. As can be seen in Figure. 8, when the bargaining power of the supplier is $R_3 = \%100$, i.e. he is the most powerful member of the supply chain in the market, all of the surplus profit that is made by the coordination model, is gained by the supplier. As the retailers' powers become greater, they will receive a greater profit. Thus, under the lead time reduction contract, the surplus profit can be allocated between the SC members based on their bargaining power.



(R_3,R_2,R_1)	1, 0, 0	0.9, 0.05, 0.05	0.8, 0.1, 0.1	0.7, 0.15, 0.15	0.6, 0.2, 0.2	0.5, 0.25, 0.25	0.4, 0.3, 0.3
Supplier profit	6134.33	5910.84	4209.98	3676.74	3145.35	2615.94	2088.6
Retailer 2 profit	3.63	390.18	787.15	1191.29	1602.99	2022.71	2450.91
Retailer 1 profit	3.63	390.18	787.15	1191.29	1602.99	2022.71	2450.91

Figure. 8. Surplus profit allocation between SC members under the coordination model

5. Conclusion

The paper developed a coordination model based on transportation lead time crashing in a two-echelon competitive supply chain consisting of a monopolistic supplier and two duopolistic retailers. Demand of the items were stochastic depending on the retailers' competitive advertising efforts. The retailers used a periodic review inventory policy for making orders. The supplier used a lot-for-lot policy and decided on the number of shipments to the retailers. The orders were received by the retailers after a constant lead time with a certain transportation mode. Moreover, it was possible for the supplier to invest in cooperative advertising efforts. Since the retailers' and supplier's decisions were related to each other and each member's profit was affected by the others' decisions, the supplier designed a transportation lead time crashing contract to change the retailers' decisions on coop advertising and replenishment. First, the common structure of decision making was modeled, i.e. the decentralized model, which showed the members' minimum satisfying profit. After that, the centralized structure was modeled to obtain the decisions which were profitable for the whole supply chain. Finally, the lead time crashing contract was developed to achieve coordination in the investigated supply chain. The lead time crashing contract was a development of the works by Heydari et al. and Johari et al. in which, besides their assumptions, the contract was regulated by carbon emission tax imposed by the government. In fact, the contract used in this study considered two aspects, one was the economic and strategic viewpoint on transportation lead time and another was the environmental viewpoint which restricted reducing lead time. Moreover, the contract was developed to be applied in a competitive situation. The model was analyzed based on numerical examples and sensitivity analyses and some managerial insights were derived based on observations, as follows.

- One way to reduce transportation lead time can be switching into a fast transportation mode. This may result in more consumption of fuel by the vehicles and release of more carbon emissions, which is not profitable for the environmental performance of the supply chain. Accordingly, the manager of the supply chain can apply the proposed transportation lead time crashing contract which considers the trade-off between reducing lead time and paying carbon emission tax imposed by the government.
- The proposed transportation lead time crashing contract can be an efficient strategy to be applied for improving the supply chain economic performance, especially, when demand faces uncertainty. It has been observed that the contract is applicable even under high levels of demand uncertainty. By applying this contract, the orders are sooner received by the retailer since the supplier will reduce the transportation lead time. Moreover, under this contract, the retailer can hold more safety stock because the supplier shares the other costs of the retailer, i.e. advertising costs, thus, the holding cost of more safety stock may not incur a loss for the retailer [Heydari et al. 2014; Johari et al. 2017]. Accordingly, by reducing the transportation lead time and holding more safety stock, less amount of demand will be lost under high levels of uncertainty and the retailer provides a satisfying service level.
- The minimum and maximum amount of reduction in transportation lead time can be calculated based on the supplier's and retailers' viewpoints, respectively. This gives a benchmark for the interval of the coordination contract parameters, whereby the coordination is achievable and provides enough incentives for all the SC members. The carbon emissions tax and cost of lead time reduction are two factors which affect the proposed coordination contract and under these situations if the supplier considers the value of maximum amount of lead time reduction, near the values of minimum amount of lead time reduction, the coordination is achievable.
- The results showed that the proposed coordination contract enhances both the environmental and economic performance of the supply chain. From environmental viewpoint, considering the carbon tax decreases the carbon emissions that will be released by the transportation modes. From economic viewpoint, coordinating coop advertising and replenishment decisions of the SC members, enhances demand and provides a higher service level, which increases the SC profit.

This paper can be developed from some aspects. First, there are always more than one supplier in the market and the model can be developed by considering the competition between the suppliers in reducing the transportation lead time. Moreover, we have considered a linear demand function that depends on the competitive advertising by the retailers; In another study, a more complicated demand function which is more similar to the real-world cases, can be considered. In addition, considering the effects of scheduling and number of available vehicles on reducing the transportation lead time, can be interesting.

6. References

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Appendix

Appendix A.

To prove concavity of the profit function of retailer i, the Hessian matrix is computed as follows:

$$H(\Pi_{r_i}) = \begin{bmatrix} \frac{\partial^2 \Pi_{r_i}}{\partial {a_i}^2} & \frac{\partial^2 \Pi_{r_i}}{\partial a_i \partial k_i} \\ \frac{\partial^2 \Pi_{r_i}}{\partial k_i \partial a_i} & \frac{\partial^2 \Pi_{r_i}}{\partial {k_i}^2} \end{bmatrix} \quad \text{(A-1)}$$

$$\frac{\partial \Pi_{r_i}}{\partial a_i} = \mu(p-w) - h_r \frac{T\mu}{2} - (1-\varphi)\frac{\eta a_i}{T} \quad \text{(A-2)}$$

$$H_{11} = \frac{\partial^2 \Pi_{r_i}}{\partial {a_i}^2} = -(1-\varphi)\frac{\eta}{T} < 0 \quad \text{(A-3)}$$

The first minor of the Hessian matrix is always negative.

$$\frac{\partial \Pi_{r_i}}{\partial k_i} = -h_r\sigma\sqrt{T+L} - h_r\alpha\sigma\sqrt{T+L}(\Phi(K_i)-1) - \frac{\pi+\alpha(p-w)}{T}\sigma\sqrt{T+L}(\Phi(K_i)-1) \quad \text{(A-4)}$$

$$\frac{\partial^2 \Pi_{r_i}}{\partial {k_i}^2} = -\sigma\sqrt{T+L}\left(h_r\alpha + \frac{\pi+\alpha(p-w)}{T}\right)\varphi(k_i) < 0 \quad \text{(A-5)}$$

$$\frac{\partial^2 \Pi_{r_i}}{\partial a_i \partial k_i} = \frac{\partial^2 \Pi_{r_i}}{\partial k_i \partial a_i} = 0 \quad \text{(A-6)}$$

$$H_{22} = \frac{\partial^2 \Pi_{r_i}}{\partial {a_i}^2} * \frac{\partial^2 \Pi_{r_i}}{\partial {k_i}^2} = (-(1-\varphi)\frac{\eta}{T}) * \left(-\sigma\sqrt{T+L}\left(h_r\alpha_i + \frac{\pi+\alpha_i(p-w)}{T}\right)\varphi(k)\right) > 0 \quad \text{(A-7)}$$

The second minor of the Hessian matrix is always positive. Thus, the profit function of the retailer is concave.

Appendix B.

To prove concavity of the supplier's profit function, the Hessian matrix is computed as follows:

$$H(\Pi_s) = \begin{bmatrix} \frac{\partial^2 \Pi_s}{\partial n^2} & \frac{\partial^2 \Pi_s}{\partial n \partial \varphi} \\ \frac{\partial^2 \Pi_s}{\partial \varphi \partial n} & \frac{\partial^2 \Pi_s}{\partial \varphi^2} \end{bmatrix} \quad \text{(B-1)}$$

$$\frac{\partial \Pi_s}{\partial n} = \frac{A_s}{n^2 T} - \frac{h_s}{2}\left(\left(2d_0 + 2(\mu - \gamma)\left(\frac{\theta_1 T}{(1-\varphi)\eta}\right)\right)T - 2\alpha\sigma\sqrt{T+L}G(\theta_2)\right) \quad \text{(B-2)}$$

$$H_{11} = \frac{\partial^2 \Pi_s}{\partial n^2} = -\frac{2A_s}{n^3 T} < 0 \quad \text{(B-3)}$$

The first minor of Hessian matrix is always negative.

$$\frac{\partial \Pi_s}{\partial \varphi} = \frac{2T\theta_1(w-m)(\mu-\gamma)}{(1-\varphi)^2\eta} - \frac{T^2\theta_1 h_s(n-1)(\mu-\gamma)}{(1-\varphi)^2\eta} - \frac{T{\theta_1}^2}{(1-\varphi)^2\eta} - \frac{2\varphi T{\theta_1}^2}{(1-\varphi)^3\eta} \quad \text{(B-4)}$$

$$\frac{\partial^2 \Pi_s}{\partial \varphi^2} = \frac{4T\theta_1(w-m)(\mu-\gamma)}{(1-\varphi)^3\eta} - \frac{2T^2\theta_1 h_s(n-1)(\mu-\gamma)}{(1-\varphi)^3\eta} - \frac{2T{\theta_1}^2}{(1-\varphi)^3\eta} - \frac{6\varphi T{\theta_1}^2}{(1-\varphi)^4\eta} \quad \text{(B-5)}$$

$$\frac{\partial^2 \Pi_s}{\partial \varphi \partial n} = \frac{\partial^2 \Pi_s}{\partial n \partial \varphi} = \frac{-h_s(\mu-\gamma)T^2\theta_1}{(1-\varphi)^2\eta} \quad \text{(B-6)}$$

$$H_{22} = \left(-\frac{2A_s}{n^3 T}\right)\left(\frac{4T\theta_1(w-m)(\mu-\gamma)}{(1-\varphi)^3\eta} - \frac{2T^2\theta_1 h_s(n-1)(\mu-\gamma)}{(1-\varphi)^3\eta} - \frac{2T{\theta_1}^2}{(1-\varphi)^3\eta} - \frac{6\varphi T{\theta_1}^2}{(1-\varphi)^4\eta}\right) - \left(\frac{-h_s(\mu-\gamma)T^2\theta_1}{(1-\varphi)^2\eta}\right)^2 \quad \text{(B-7)}$$

The second minor of Hessian matrix is positive under the following condition.

$$-2A_s\big((\mu-\gamma)(1-\varphi)\eta(4(w-m) - 2h_s(n-1)T) - 2\theta_1(1-\varphi)\eta - 6\varphi\theta_1\eta\big) > n^3\theta_1(h_s(\mu-\gamma)T^2)^2 \quad \text{(B-8)}$$

Appendix C.

To prove concavity of the profit function of retailers, the Hessian matrix is computed as follows:

$$H(\Pi_r) = \begin{bmatrix} \frac{\partial^2\Pi_r}{\partial {a_1}^2} & \frac{\partial^2\Pi_r}{\partial a_1 \partial a_2} & \frac{\partial^2\Pi_r}{\partial a_1 \partial k_1} & \frac{\partial^2\Pi_r}{\partial a_1 \partial k_2} \\ \frac{\partial^2\Pi_r}{\partial a_2 \partial a_1} & \frac{\partial^2\Pi_r}{\partial {a_2}^2} & \frac{\partial^2\Pi_r}{\partial a_2 \partial k_1} & \frac{\partial^2\Pi_r}{\partial a_2 \partial k_2} \\ \frac{\partial^2\Pi_r}{\partial k_1 \partial a_1} & \frac{\partial^2\Pi_r}{\partial k_1 \partial a_2} & \frac{\partial^2\Pi_r}{\partial {k_1}^2} & \frac{\partial^2\Pi_r}{\partial k_1 \partial k_2} \\ \frac{\partial^2\Pi_r}{\partial k_2 \partial a_1} & \frac{\partial^2\Pi_r}{\partial k_2 \partial a_2} & \frac{\partial^2\Pi_r}{\partial k_2 \partial k_1} & \frac{\partial^2\Pi_r}{\partial {k_2}^2} \end{bmatrix} \quad \text{(C-1)}$$

$$\frac{\partial \Pi_r}{\partial a_1} = (\mu - \gamma)(p - w) - h_r \frac{T}{2}(\mu - \gamma) - (1 - \varphi)\frac{\eta a_1}{T} \quad \text{(C-2)}$$

$$H_{11} = \frac{\partial^2\Pi_r}{\partial {a_1}^2} = -(1 - \varphi)\frac{\eta}{T} < 0 \quad \text{(C-3)}$$

The first minor of the Hessian matrix is always negative.

$$\frac{\partial \Pi_r}{\partial a_2} = (\mu - \gamma)(p - w) - h_r \frac{T}{2}(\mu - \gamma) - (1 - \varphi)\frac{\eta a_2}{T} \quad \text{(C-4)}$$

$$\frac{\partial^2\Pi_r}{\partial {a_2}^2} = -(1 - \varphi)\frac{\eta}{T} < 0 \quad \text{(C-5)}$$

$$\frac{\partial^2\Pi_r}{\partial a_1 \partial a_2} = \frac{\partial^2\Pi_r}{\partial a_2 \partial a_1} = 0 \quad \text{(C-6)}$$

$$H_{22} = \left(-(1 - \varphi)\frac{\eta}{T}\right)^2 > 0 \quad \text{(C-7)}$$

The second minor of the Hessian matrix is always positive.

$$\frac{\partial^2\Pi_r}{\partial k_1} = -h_r\sigma\sqrt{T + L} - h_r\alpha\sigma\sqrt{T + L}(\Phi(K_1) - 1) - \frac{\pi + \alpha(p - w)}{T}\sigma\sqrt{T + L}(\Phi(K_1) - 1) \quad \text{(C-8)}$$

$$\frac{\partial^2\Pi_r}{\partial {k_1}^2} = -\sigma\sqrt{T + L}\left(h_r\alpha + \frac{\pi + \alpha(p - w)}{T}\right)\varphi(k_1) < 0 \quad \text{(C-9)}$$

$$\frac{\partial^2\Pi_r}{\partial k_1 \partial a_1} = \frac{\partial^2\Pi_r}{\partial k_1 \partial a_2} = \frac{\partial^2\Pi_r}{\partial a_1 \partial k_1} = \frac{\partial^2\Pi_r}{\partial a_2 \partial k_1} = 0 \quad \text{(C-10)}$$

$$H_{33} = \frac{\partial^2\Pi_r}{\partial {a_1}^2} * \frac{\partial^2\Pi_r}{\partial {a_2}^2} * \frac{\partial^2\Pi_r}{\partial {k_1}^2} = \left(-(1 - \varphi)\frac{\eta}{T}\right)^2 * \left(-\sigma\sqrt{T + L}\left(h_r\alpha + \frac{\pi + \alpha(p - w)}{T}\right)\varphi(k_1)\right) < 0 \quad \text{(C-11)}$$

The third minor of the Hessian matrix is always negative.

$$\frac{\partial^2\Pi_r}{\partial k_2} = -h_r\sigma\sqrt{T + L} - h_r\alpha\sigma\sqrt{T + L}(\Phi(K_2) - 1) - \frac{\pi + \alpha(p - w)}{T}\sigma\sqrt{T + L}(\Phi(K_2) - 1) \quad \text{(C-12)}$$

$$\frac{\partial^2\Pi_r}{\partial {k_2}^2} = -\sigma\sqrt{T+L}\left(h_r\alpha + \frac{\pi+\alpha(p-w)}{T}\right)\varphi(k_2) < 0 \tag{C-13}$$

$$\frac{\partial^2\Pi_r}{\partial k_2\partial a_1} = \frac{\partial^2\Pi_r}{\partial k_2\partial a_2} = \frac{\partial^2\Pi_r}{\partial a_1\partial k_2} = \frac{\partial^2\Pi_r}{\partial a_2\partial k_2} = \frac{\partial^2\Pi_r}{\partial k_1\partial k_2} = \frac{\partial^2\Pi_r}{\partial k_2\partial k_1} = 0 \tag{C-14}$$

$$H_{44} = \frac{\partial^2\Pi_r}{\partial {a_1}^2} * \frac{\partial^2\Pi_r}{\partial {a_2}^2} * \frac{\partial^2\Pi_r}{\partial {k_1}^2} * \frac{\partial^2\Pi_r}{\partial k_2} = \left(-(1-\varphi)\frac{\eta}{T}\right)^2 * \left(-\sigma\sqrt{T+L}\left(h_r\alpha + \frac{\pi+\alpha(p-w)}{T}\right)\varphi(k_1)\right) * \left(-\sigma\sqrt{T+L}\left(h_r\alpha + \frac{\pi+\alpha(p-w)}{T}\right)\varphi(k_2)\right) > 0 \tag{C-15}$$

The fourth minor of the Hessian matrix is always positive. Thus, the profit function of the retailers is concave.

Appendix D.

To prove concavity of the supplier's profit function, the Hessian matrix is computed as follows:

$$H(\Pi_s) = \begin{bmatrix} \frac{\partial^2\Pi_s}{\partial n^2} & \frac{\partial^2\Pi_s}{\partial n\partial\varphi} \\ \frac{\partial^2\Pi_s}{\partial\varphi\partial n} & \frac{\partial^2\Pi_s}{\partial\varphi^2} \end{bmatrix} \tag{D-1}$$

$$\frac{\partial\Pi_s}{\partial n} = \frac{A_s}{n^2T} - \frac{h_s}{2}\left(\left(2d_0 + 2(\mu-\gamma)\left(\frac{\theta_3 T}{(1-\varphi)\eta}\right)\right)T - 2\alpha\sigma\sqrt{T+L}G(\theta_2)\right) \tag{D-2}$$

$$H_{11} = \frac{\partial^2\Pi_s}{\partial n^2} = -\frac{2A_s}{n^3T} < 0 \tag{D-3}$$

The first minor of Hessian matrix is always negative.

$$\frac{\partial\Pi_s}{\partial\varphi} = \frac{2T\theta_3(w-m)(\mu-\gamma)}{(1-\varphi)^2\eta} - \frac{T^2\theta_4 h_s(n-1)(\mu-\gamma)}{(1-\varphi)^2\eta} - \frac{T{\theta_4}^2}{(1-\varphi)^2\eta} - \frac{2\varphi T{\theta_4}^2}{(1-\varphi)^3\eta} \tag{D-4}$$

$$\frac{\partial^2\Pi_s}{\partial\varphi^2} = \frac{4T\theta_3(w-m)(\mu-\gamma)}{(1-\varphi)^3\eta} - \frac{2T^2\theta_3 h_s(n-1)(\mu-\gamma)}{(1-\varphi)^3\eta} - \frac{2T{\theta_3}^2}{(1-\varphi)^3\eta} - \frac{6\varphi T{\theta_3}^2}{(1-\varphi)^4\eta} \tag{D-5}$$

$$\frac{\partial^2\Pi_s}{\partial\varphi\partial n} = \frac{\partial^2\Pi_s}{\partial n\partial\varphi} = \frac{-h_s(\mu-\gamma)T^2\theta_3}{(1-\varphi)^2\eta} \tag{D-6}$$

$$H_{22} = \left(-\frac{2A_s}{n^3T}\right)\left(\frac{4T\theta_3(w-m)(\mu-\gamma)}{(1-\varphi)^3\eta} - \frac{2T^2\theta_3 h_s(n-1)(\mu-\gamma)}{(1-\varphi)^3\eta} - \frac{2T{\theta_3}^2}{(1-\varphi)^3\eta} - \frac{6\varphi T{\theta_3}^2}{(1-\varphi)^4\eta}\right) - \left(\frac{-h_s(\mu-\gamma)T^2\theta_3}{(1-\varphi)^2\eta}\right)^2 \tag{D-7}$$

The second minor of Hessian matrix is positive under the following condition.

$$-2A_s\big((\mu-\gamma)(1-\varphi)\eta(4(w-m)-2h_s(n-1)T)-2\theta_3(1-\varphi)\eta-6\varphi\theta_1\eta\big)> n^3\theta_3(h_s(\mu-\gamma)T^2)^2 \quad \text{(D-8)}$$

Appendix E.

To prove concavity of the whole SC profit function, the Hessian matrix is calculated as follows:

$$H(\Pi_{sc})=\begin{bmatrix} \frac{\partial^2\Pi_{sc}}{\partial a_1{}^2} & \frac{\partial^2\Pi_r}{\partial a_1\partial a_2} & \frac{\partial^2\Pi_r}{\partial a_1\partial k_1} & \frac{\partial^2\Pi_r}{\partial a_1\partial k_2} & \frac{\partial^2\Pi_r}{\partial a_1\partial n} \\ \frac{\partial^2\Pi_r}{\partial a_2\partial a_1} & \frac{\partial^2\Pi_{sc}}{\partial a_2{}^2} & \frac{\partial^2\Pi_r}{\partial a_2\partial k_1} & \frac{\partial^2\Pi_r}{\partial a_2\partial k_2} & \frac{\partial^2\Pi_r}{\partial a_2\partial n} \\ \frac{\partial^2\Pi_{sc}}{\partial k_1\partial a_1} & \frac{\partial^2\Pi_{sc}}{\partial k_1\partial a_2} & \frac{\partial^2\Pi_{sc}}{\partial k_1{}^2} & \frac{\partial^2\Pi_{sc}}{\partial k_1\partial k_2} & \frac{\partial^2\Pi_{sc}}{\partial k_1\partial n} \\ \frac{\partial^2\Pi_{sc}}{\partial k_2\partial a_1} & \frac{\partial^2\Pi_{sc}}{\partial k_2\partial a_2} & \frac{\partial^2\Pi_{sc}}{\partial k_2\partial k_1} & \frac{\partial^2\Pi_{sc}}{\partial k_2{}^2} & \frac{\partial^2\Pi_{sc}}{\partial k_2\partial n} \\ \frac{\partial^2\Pi_{sc}}{\partial n\partial a_1} & \frac{\partial^2\Pi_{sc}}{\partial n\partial a_2} & \frac{\partial^2\Pi_{sc}}{\partial n\partial k_1} & \frac{\partial^2\Pi_{sc}}{\partial n\partial k_2} & \frac{\partial^2\Pi_{sc}}{\partial n^2} \end{bmatrix} \quad \text{(E-1)}$$

$$\frac{\partial\Pi_{sc}}{\partial a_1}=(p-m)(\mu-\gamma)-\frac{T(\mu-\gamma)}{2}\big((n-1)h_s+h_r\big)-\frac{\eta a_1}{T} \quad \text{(E-2)}$$

$$H_{11}=\frac{\partial^2\Pi_{sc}}{\partial a_1{}^2}=-\frac{\eta}{T}<0 \quad \text{(E-3)}$$

The first minor is always negative.

$$\frac{\partial\Pi_{sc}}{\partial a_2}=(p-m)(\mu-\gamma)-\frac{T(\mu-\gamma)}{2}\big((n-1)h_s+h_r\big)-\frac{\eta a_2}{T} \quad \text{(E-4)}$$

$$\frac{\partial^2\Pi_{sc}}{\partial a_2{}^2}=-\frac{\eta}{T} \quad \text{(E-5)}$$

$$\frac{\partial^2\Pi_{sc}}{\partial a_2\partial a_1}=\frac{\partial^2\Pi_{sc}}{\partial a_1\partial a_2}=0 \quad \text{(E-6)}$$

$$H_{22}=(-\tfrac{\eta}{T})*(-\tfrac{\eta}{T})>0 \quad \text{(E-7)}$$

The second minor is always positive.

$$\frac{\partial\Pi_{sc}}{\partial k_1}=-\frac{1}{T}\Big(\pi+\alpha(p-m)-\frac{h_s\alpha(n-1)}{2}T\Big)\sigma\sqrt{T+L}[\Phi(K_1)-1]$$
$$-h_r[\sigma\sqrt{T+L}+\alpha\sigma\sqrt{T+L}(\Phi(k_1)-1)] \quad \text{(E-8)}$$

$$\frac{\partial^2\Pi_{sc}}{\partial K_1{}^2}=-\frac{1}{T}\Big[\pi+\alpha(p-m)+\alpha_1Th_r-\frac{h_s\alpha(n-1)T}{2}\Big]\sigma\sqrt{T+L}\varphi(K_1) \quad \text{(E-9)}$$

$$\frac{\partial^2\Pi_{sc}}{\partial a_1\partial k_1}=\frac{\partial^2\Pi_{sc}}{\partial k_1\partial a_1}=\frac{\partial^2\Pi_{sc}}{\partial a_2\partial k_1}=\frac{\partial^2\Pi_{sc}}{\partial k_1\partial a_2}=0 \tag{E-10}$$

$$H_{33}=-\frac{\eta}{T}\left[-\frac{\eta}{T}\left(-\frac{1}{T}\left[\pi+\alpha(p-m)+\alpha_1 Th_r-\frac{h_s\alpha(n-1)T}{2}\right]\sigma\sqrt{T+L}\varphi(K_1)\right)\right] \tag{E-11}$$

The third minor is negative under the following condition:

$$\pi+\alpha(p-m)+\alpha Th_r>\frac{h_s\alpha(n-1)T}{2} \tag{E-12}$$

$$\frac{\partial\Pi_{sc}}{\partial k_2}=-\frac{1}{T}\left(\pi+\alpha(p-m)-\frac{h_s\alpha(n-1)}{2}T\right)\sigma\sqrt{T+L}[\Phi(K_2)-1]$$
$$-h_r[\sigma\sqrt{T+L}+\alpha\sigma\sqrt{T+L}(\Phi(k_2)-1)] \tag{E-13}$$

$$\frac{\partial^2\Pi_{sc}}{\partial {K_2}^2}=-\frac{1}{T}\left[\pi+\alpha(p-m)+\alpha Th_r-\frac{h_s\alpha(n-1)T}{2}\right]\sigma\sqrt{T+L}\varphi(K_2) \tag{E-14}$$

$$\frac{\partial^2\Pi_{sc}}{\partial a_1\partial k_2}=\frac{\partial^2\Pi_{sc}}{\partial k_2\partial a_1}=\frac{\partial^2\Pi_{sc}}{\partial a_2\partial k_2}=\frac{\partial^2\Pi_{sc}}{\partial k_2\partial a_2}=\frac{\partial^2\Pi_{sc}}{\partial k_1\partial k_2}=\frac{\partial^2\Pi_{sc}}{\partial k_2\partial k_2}=0 \tag{E-15}$$

$$H_{44}=\left(-\frac{\eta}{T}\right)^2\left(-\frac{1}{T}\left[\pi+\alpha(p-m)+\alpha Th_r-\frac{h_s\alpha(n-1)T}{2}\right]\sigma\sqrt{T+L}\big(\varphi(K_1)+\varphi(K_2)\big)\right)>0 \tag{E-16}$$

The forth minor is positive by considering the condition (E-12).

$$\frac{\partial\Pi_{sc}}{\partial n}=\frac{1}{T}\left(\frac{A_s}{n^2}\right)-\frac{h_s}{2}\Big(\big(2d_0+(\mu-\gamma)(a_1+a_2)\big)T-\alpha\sigma\sqrt{T+L}\big(G(k_1)+G(k_2)\big)\Big) \tag{E-17}$$

$$\frac{\partial^2\Pi_{sc}}{\partial n^2}=-\frac{1}{T}\left(\frac{2A_s}{n^3}\right) \tag{E-18}$$

$$\frac{\partial^2\Pi_{sc}}{\partial n\partial a_1}=\frac{\partial^2\Pi_{sc}}{\partial n\partial a_2}=-\frac{h_s}{2}T(\mu-\gamma) \tag{E-19}$$

$$\frac{\partial^2\Pi_{sc}}{\partial n\partial k_1}=\frac{\partial^2\Pi_{sc}}{\partial k_1\partial n}=\frac{h_s}{2}\alpha\sigma\sqrt{T+L}(\Phi(k_1)-1) \tag{E-20}$$

$$\frac{\partial^2\Pi_{sc}}{\partial n\partial k_2}=\frac{\partial^2\Pi_{sc}}{\partial k_2\partial n}=\frac{h_s}{2}\alpha\sigma\sqrt{T+L}(\Phi(k_2)-1) \tag{E-21}$$

The fifth minor is negative under the following condition:

$$\frac{1}{T}\left(\frac{2A_s}{n^3}\right)\varphi(K_1)\varphi(K_2)\left[\pi+\alpha(p-m)+\alpha Th_r-\frac{h_s\alpha(n-1)T}{2}\right]>\frac{{h_s}^2}{4}\alpha^2\sigma\sqrt{T+L}[\varphi(K_1)(\Phi(k_2)-1)^2$$

$$+\varphi(K_2)(\Phi(k_1)-1)^2] \quad \text{(E-22)}$$