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Research Paper

# Evaluating the Effectiveness of Burr Mixtures for Travel Time Reliability Analysis

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## Abstract

Travel time reliability plays a key role in travelers' decision-making processes and serves as a significant indicator for evaluating road performance. The initial stage of enhancing travel time reliability involves measuring it, which guarantees on-time arrivals and minimizes travel costs. A thorough understanding of travel time distribution is required to accurately measure travel time reliability. Traditionally, travel time has been represented using unimodal distributions. However, recent studies have indicated that travel time distribution is often multimodal. Failing to accurately model travel time distribution can lead to overestimation or underestimation of travel time reliability, resulting in suboptimal solutions in practical applications. This study mainly focuses on examining the effectiveness of six different mixture distributions, namely Burr mixture, Gamma mixture, Inverse Gaussian mixture, Log-normal mixture, Normal mixture, and Weibull mixture, in modeling travel time reliability. The Bayesian information criterion is utilized to compare and select the best model for representing travel time distribution. The results of this study indicate that the Burr mixture model can characterize the travel time distribution more accurately compared to its alternatives. Furthermore, the results show that using burr mixture distributions leads to more accurate estimations of travel time reliability measures compared to single distributions.

**Keywords:** Travel Time Reliability, Probabilistic Modeling, Mixture Models

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# 1. Introduction

It is widely acknowledged that travel times on road networks are subject to variations due to uncertainties in travel demand and transportation supply disruptions, such as road closures, accidents, and adverse weather conditions [Ghavidel et al., 2022]. This variability in travel time makes travel time reliability a primary concern for travelers and transportation professionals.

Travel time reliability affects travelers' decision-making as they choose alternatives that reduce the risk of being late. Additionally, travelers consider travel time reliability when planning their trips by allocating extra time to ensure that they arrive on time. Improving travel time reliability can reduce this buffer time, providing a significant advantage to travelers [de Jong & Bliemer, 2015; Taylor, 2017]. The pivotal role of travel time reliability in travelers' choices has been substantiated by empirical research. For example, Bhat and Sardesai (2006) examined the effects of midday and commute stop-making and travel time reliability on mode choice. They utilized revealed and stated preference data and estimated mixed logit models using a mean-variance approach. The results of their models indicate that midday stops and travel time reliability substantially affect mode choice. Additionally, it was observed that commuters with fixed work schedules place more value on reliability compared to those with flexible schedules. Small et al. (2005) employed revealed preference data collected on California State Route 91 to estimate the value of travel time (VOT) and the value of travel time reliability (VOR). According to their estimation, the value of travel time is approximately 93% of the average wage, whereas the value of reliability is approximately 85% of the usual wage rate. This indicates that travelers highly value travel time and its reliability. Mishra et al. (2018) developed a framework to estimate, forecast, and integrate travel time reliability into the transportation planning process. In their study, the value of travel time reliability was estimated to be \$56.31/h, which is nearly four times the value of the travel time. In a recent study by Ho et al. (2020), VOT and VOR were estimated to be \$23.13/h ah \$51.65/h, respectively, for car commuters and \$12.83/h and \$65.39/h for public transport commuters. In another study, Alonso-González et al. (2020) addressed the VOT and VOR for pooled on-demand services. They found that the values of reliability in the waiting and in-vehicle stages of trips were approximately half of their respective values of time. For further details on the value of travel time reliability, interested readers can refer to Li et al. (2010), Carrion and Levinson (2012), and Shams et al. (2017).

Transportation professionals are also concerned with travel time reliability. They aim to measure travel time reliability as a performance indicator and integrate it into a wide range of transportation planning and traffic management applications. To measure travel time reliability, several indexes have been proposed. In their seminal work, Lomax et al. (2003) introduced multiple indexes to measure travel time reliability. The authors examined these indexes and suggested employing percent variation, the misery index, and the buffer time index. Van Lint et al. (2008) proposed three new indexes to capture both the skew and width of the travel time distribution. Rajabi-Bahaabadi et al. (2019) introduced two travel time reliability indexes that can account for the scheduling preferences of travelers and differentiate between early and late arrivals. Li et al. (2019) treated travel time as a discrete time series and presented a distribution-free index to estimate the predictability of travel time. They employed the Lempel–Ziv algorithm adopted from the field of information theory to establish an upper bound on travel time predictability using historical data. Chen and Liu (2021) devised an alternative index based on a statistical distance to measure travel time reliability and applied it to detect freeway bottlenecks. Fredriksson et al. (2023) proposed a median-based index for measuring travel time reliability. The index calculates the relative difference between the slow travel speeds and the free-flow speed and can be employed to identify road segments that might adversely affect travel times. Recently, Saw et al. (2024) modeled travel time variability by a triangular membership function and introduced a travel time reliability index using the concept of possibility theory when limited or ambiguous information is available. This index quantifies the degree of confidence associated with completing a trip within a predetermined travel time. It is also necessary for transportation professionals to consider travel time reliability in the routing and scheduling of public transport services and delivery systems to ensure on-time operations. As an illustration, Yao et al. (2014) introduced a method to design a transit network by taking into account the reliability of travel time. To achieve this, they developed a robust optimization model to maximize the efficiency and reliability of the transit network. A tabu search algorithm was used to solve the model. The study showed that the method effectively improved the reliability of the transit network and reduced travel time for passengers. Similarly, Tong et al. (2021) addressed the electric transit route network design, assuming that the locations of charging depots are predetermined. Their model aimed to maximize travel time reliability while maintaining total costs within a specified range. A genetic algorithm was employed to solve the model. The results of the study demonstrated the effectiveness of the model in enhancing travel reliability with a minimal cost increase. Li et al. (2016) studied two variants of the Share-a-Ride problem. The first variant addressed stochastic travel times, while the second involved stochastic delivery. The study found that stochastic information can significantly enhance the performance of a taxi-sharing system compared to deterministic approaches. Ricard et al. (2024) addressed the multiple depot vehicle scheduling problem with stochastic travel times to decrease the propagation of delays in schedules and increase reliability. A heuristic branch-and-price algorithm hybridized with a labeling algorithm, was employed to solve the problem. They showed that their method could improve reliability with a slight increase in operational costs. Another potential application of travel time reliability is embedding it into the road network design problem. For example, Chootinan et al. (2005) developed a bi-level model to address the reliability-based network design problem. The lower level employed a probit-based stochastic user equilibrium to capture travel time reliability, and the upper level maximized a new capacity reliability index. Owing to the importance of travel time reliability, it is also considered in the road pricing problem and cost-benefit analysis of projects. Owing to its importance, travel time reliability has also been factored into road pricing and project cost-benefit analysis. For example, Tirachini et al. (2014) proposed a new approach to optimal pricing in a multimodal setting that accounts for travel time variability as a source of disutility for car and bus users. The study revealed that by incorporating travel time variability, optimal car tolls increased significantly, resulting in substantial increases in toll revenue. de Jong and Bliemer (2015) proposed a framework for considering travel time reliability in project appraisals. They recommended implementing travel time reliability into existing transport models in the short, medium, and long terms. Since this study’s primary focus is not on the practical applications of travel time reliability, readers interested in gaining further insights into the topic can refer to the study conducted by Zang et al. (2022).

The travel time distribution describes the nature of travel time reliability. It is vital to accurately model this distribution to assess travel time reliability and to provide essential inputs for various applications. Inaccurate modeling of the travel time distribution can give rise to erroneous estimates of the benefits of enhancing travel time reliability. This, in turn, may lead to inefficient solutions, incorrect decisions, and erroneous conclusions.

The literature on modeling travel time reliability focused on unimodal parametric distributions. For example, Taylor (1982) proposed the normal distribution to represent travel time. However, empirical research has shown that the travel time distribution is usually skewed and asymmetric [Van Lint et al., 2008]. Therefore, some studies recommend using the lognormal [Lu & Dong, 2018], Weibull [Al-Deek & Emam, 2006], Burr [Ganjkhanloo et al., 2020], and gamma [Polus, 1979] distributions to account for skewness and asymmetry in real travel time data. In recent studies, Ma et al. (2016), Yang and Wu (2016), and Ganjkhanloo et al. (2020) provided compelling evidence regarding the multimodality of the travel time distribution. Ma et al. (2016) compared the performance of various distributions to model the travel time distribution and found that the normal mixture distribution outperformed its unimodal alternatives in terms of fitting accuracy. Similarly, Ganjkhanloo et al. (2020) showed that the normal mixture distribution is superior to single distributions in the characterization of travel time distribution. Furthermore, Yang and Wu (2016) recommended using three mixture distributions, normal, gamma, and lognormal, to better capture the variability in travel time.

As mentioned earlier, previous studies have focused on evaluating the appropriateness of unimodal distributions and three specific cases of mixture distributions for representing travel time reliability. This study seeks to broaden the scope of mixture modeling beyond existing models. The main goal is to explore the effectiveness of six finite mixture models in capturing travel time variability. These models are based on parametric distribution families, including Normal, Burr, Weibull, Inverse Gaussian, Log-normal, and Gamma distributions. In summary, the main contributions of this paper are as follows:

- Assessing the appropriateness of a wide range of mixture distributions for modeling travel time reliability.
- In contrast to previous studies that typically relied on travel time data from only one or two roads, our research analyzed real travel time data collected from eight freeways in Iran over a period of one year. By examining a more extensive dataset, we hope to identify better the most appropriate model for characterizing travel time variability and draw more comprehensive conclusions about the travel time distribution.
- We also aim to investigate how using single distributions instead of mixture ones can impact the accuracy of travel time reliability measures.

The remainder of this paper is organized as follows. The next section briefly explains mixture distributions, outlines the algorithm used to estimate the parameters of mixture models, and presents criteria for model selection. The third section provides an overview of the study area and the travel time dataset used in this study. Subsequently, the fourth section presents the results of fitting different distributions to the travel time data. Finally, the last section concludes the paper and suggests new directions for future studies.

## 2. Methodology

This section presents a methodology for estimating link travel time distributions. It commences with an overview of mixture distributions, followed by an explanation of the estimation algorithm. Finally, we present criteria for selecting the most appropriate model.

### 2.1. Mixture Distributions

Finite mixture distributions offer a flexible and computationally efficient approach to modeling complex probability distributions that cannot be accurately represented using standard parametric distributions. The probability density function of a mixture distribution can be expressed as the convex combination of the density functions of its constituent components. In particular, a finite mixture density with *K* components is given by [Miljkovic & Grün, 2016]:

$$g\left(x\,|\,\mathbf{\Theta}\right)=\sum_{k=1}^{K}w_k f_k\left(x\,|\,\mathbf{\theta}_k\right), \qquad (1)$$

Where $\mathbf{\Theta}=\left\{w_k,\mathbf{\theta}_k;k=1,..,K\right\}$ is the set of parameters of a mixture distribution, $f_k$ denotes the probability density function of component *k* parameterized by $\mathbf{\theta}_k$, and $w_k$ represents the mixing weight associated with the *k*-th component.

It is important to note that the mixing weights are non-negative and sum up to one. Moreover, in Equation 1, component densities are presumed to belong to the same parametric family and differ solely in component parameters $\mathbf{\theta}_k$. To model travel time variability, we explore six types of density functions: Normal, Log-normal, Burr, Gamma, Inverse Gaussian, and Weibull. These parametric distributions have been widely employed in the literature for modeling travel time variability. Therefore, we adopt them in this research as basic building blocks with the aim of creating more flexible distributions by integrating them into the finite mixture framework.

### 2.2. Parameter Estimation

In the present study, we employ the expectation-maximization (EM) algorithm to estimate the parameters of mixture distributions. The algorithm is a well-known iterative approach for finding maximum likelihood estimates of unknown parameters of a given model. It is particularly useful in cases of incomplete data or data with missing values [Miljkovic & Grün, 2016]. The EM algorithm, as its name implies, is a method that involves two sequential steps in each iteration, namely the expectation step and the maximization step. Suppose a random variable denoted by $\mathbf{X}=\left(X_1,X_2,...,X_n\right)$ that corresponds to the observed data. Additionally, Let $\mathbf{Z}=\left(z_{ik}\in\left\{0,1\right\},i=1,...,n\right)$ represents a set of latent random variables that indicate the component from which each observation is derived. The logarithm of the complete data likelihood can be defined as

$$L\left(\mathbf{\Theta}\right)=\sum_{i=1}^{n}\sum_{k=1}^{K}z_{ik}\left(\log\left(w_k\right)+\log\left(f_k\left(x_i\,|\,\mathbf{\theta}_k\right)\right)\right), \qquad (2)$$

Where *n* represents the number of observations, *K* stands for the number of components, and $z_{ik}$ is equal to 1 if $x_i$ is derived from component *k*; otherwise, it is equal to 0.

The E-step of the *s*-th iteration entails the computation of the conditional expectation of the complete data log-likelihood (*L*) given the observed data and the parameter estimates obtained at the previous iteration *s*-1. Due to the linearity of *L* in the latent variables $z_{ik}$, the E-step necessitates solely the computation of the expectation of $z_{ik}$ conditional on the observed data and the parameter estimates obtained in the previous iteration. The expected values can be readily obtained as follows:

$$w_{ik}^{s}=E\left(z_{ik}\,|\,x_i,\mathbf{\Theta}^{s-1}\right)=\frac{w_k^{s-1}f_k\left(x\,|\,\mathbf{\theta}_k^{s-1}\right)}{\sum_{j=1}^{K}w_j^{s-1}f_j\left(x\,|\,\mathbf{\theta}_j^{s-1}\right)}. \qquad (3)$$

In Equation 3, $w_{ik}^{s}$ represents the probability that $x_i$ belongs to the *k*-th component of the mixture distribution, calculated at the *s*-th iteration of the EM algorithm. By employing Equation 3, the expectation of *L* (Q-function) can be derived as

$$Q\left(\mathbf{\Theta}\,|\,\mathbf{\Theta}^{s-1}\right)=\sum_{i=1}^{n}\sum_{k=1}^{K}w_{ik}^{s}\left(\log\left(w_k\right)+\log\left(f_k\left(x_i\,|\,\mathbf{\theta}_k\right)\right)\right) \qquad (4)$$

The Q-function is maximized during the M-step to obtain updated estimates for unknown parameters $\mathbf{\Theta}=\left\{w_k,\mathbf{\theta}_k\right\}$. To obtain updated estimates for $\mathbf{\theta}_k$, a weighted ML estimation problem is solved for each component distribution. This can be accomplished analytically or through numerical optimization methods. The mixing weights in the *s*-th iteration can be updated using the following equation:

$$w_k^s = \frac{1}{n} \sum_{i=1}^{n} w_{ik}^s . \tag{5}$$

### 2.3. Model Selection Criteria

Increasing the number of components in the mixture model tends to improve the log-likelihood value. Therefore, it is essential to consider an additional penalty term while comparing models with different numbers of parameters to make a trade-off between model fit and complexity. The present study employs the Bayesian Information Criterion (BIC) to evaluate and compare various mixture models. The criterion considers both model fit and complexity to prevent overfitting and underfitting and provides a fair assessment. *BIC* is defined as

$$BIC = -2\ln\left(\hat{L}\right) + p\ln\left(n\right), \tag{6}$$

where $\hat{L}$ represents the maximized value of the likelihood function of the model and $p$ denotes the number of estimated parameters.

It is worth noting that a lower value of *BIC* indicates a better model. We also use *BIC* to determine the optimal number of components of a mixture distribution.

## 3. Data Description

This study uses travel time data from eight freeway routes located in Iran. These freeways are equipped with Bluetooth sensors that enable the recording of time stamps and media access control (MAC) addresses of Bluetooth-enabled devices. Mapping the MAC addresses makes it possible to estimate vehicle travel time. Vehicle travel times were aggregated every five minutes to mitigate short-duration travel time fluctuations. The travel time data used in this study were collected from March 21, 2019, to March 21, 2020. Table 1 presents a list of all the routes, along with their primary characteristics. To ensure data quality, a screening process was implemented. To this end, the mean absolute deviation (MAD) method, specifically the MAD-3delta criterion, as described by Pearson (2002), was used to identify and exclude outliers. Data points exceeding the calculated MAD-3delta bounds were classified as outliers and were excluded from the dataset.

## 4. Results

One of the primary benefits of mixture distributions, as opposed to unimodal parametric distributions, is their ability to effectively capture the complex multimodal nature of a given dataset. The Hartigan dip test is employed in this study to assess the existence of multimodality within the travel time distribution [Hartigan & Hartigan, 1985]. The null hypothesis of the dip test ($H_0$) is that the travel time distribution is unimodal. If the p-value exceeds 0.05, then the zero hypothesis cannot be rejected, indicating a unimodal distribution. In this study, we divided the day into 24 one-hour intervals and tested the multimodality of the travel time distribution for each hour. The test results revealed that all *p*-values were below 0.05, indicating that all hourly travel time distributions were multimodal for all routes. As an example, Figure 1 depicts the travel time distribution of Route 3 (Karaj-Qazvin) for the 20:00-21:00 p.m. interval. As shown in Figure 1, the travel time distribution is not unimodal but exhibits several modes. This suggests the potential advantage of using mixture distributions to model travel time reliability accurately. Our findings here are consistent with the results reported by Ma et al. (2016).

**Table 1. Main Characteristics of Routes**

| Name | Free-flow travel time (min) | Mean travel time (min) |
|---|---|---|
| Route 1: Bumehen-Tehran | 15 | 25 |
| Route 2: Qazvin-Karaj | 61 | 70 |
| Route 3: Karaj-Qazvin | 62 | 72 |
| Route 4: Karaj-Tehran | 15 | 18 |
| Route 5: Tehran-Karaj | 15 | 18 |
| Route 6: Qazvin-Rasht | 110 | 120 |
| Route 7: Rasht-Gazvin | 115 | 126 |
| Route 8: Saveh-Tehran | 68 | 77 |

In order to identify the most suitable distribution for modeling travel time variability and to explore the potential application of mixture models, six single distributions, including Burr, Gamma, Normal, Inverse Gaussian, Log-normal, and Weibull distributions, as well as their respective mixture models were considered. Travel time observations for each route were then divided into 24 hourly data sets. Subsequently, single distributions and their corresponding mixture models were fitted to each travel time data set.

The R package flexmix [Grün & Leisch, 2008] was employed to fit the models to the data. In this study, we tested mixture models with varying numbers of distributions (*K*) and selected the best distribution based on the BIC criterion. The number of distributions in the mixture models ranged from one (i.e., corresponding to a single distribution) to ten, allowing for the contribution of up to ten individual distributions in the mixture models for fitting the travel time observations.

We found that mixture distributions were the best model for representing travel time variability in all 192 cases (24 hours × 8 routes). This highlights the advantages of mixture distributions over single distributions when modeling variability in travel time. For instance, Figure 2 presents the single distributions fitted to the travel time observations for Route 3 during the 20:00-21:00 pm interval. As shown in the figure, none of the unimodal distributions can effectively capture the multimodal characteristics of the travel time data. Figure 3 compares the cumulative distribution of the best-fitting mixture distribution to its single distribution counterpart. In this comparison, the cumulative distribution function (CDF) of the mixture distribution closely approximates the empirical distribution, whereas the single distribution fails to accurately represent the travel time distribution. Finally, Figure 4 illustrates the probability density function of each component of the best-fitting mixture distribution. As depicted in Figure 4, the mixture model consists of three components. Additionally, it demonstrates how the integration of these components effectively represents the distribution of travel times and captures the multimodal characteristics inherent in travel time data.

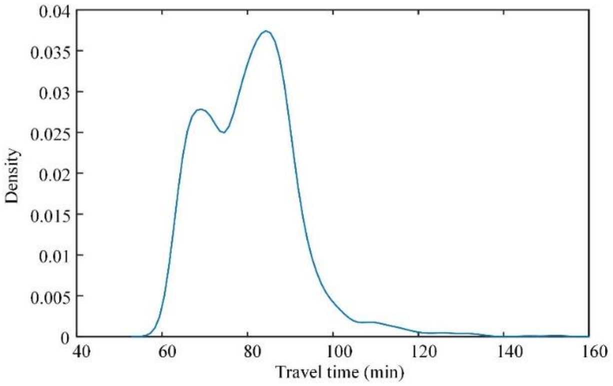


Figure 1. Travel Time Distribution with Several Modes

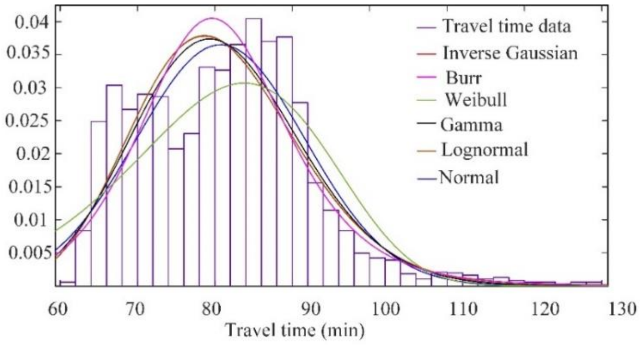


Figure 2. Fitting Several Unimodal Distributions

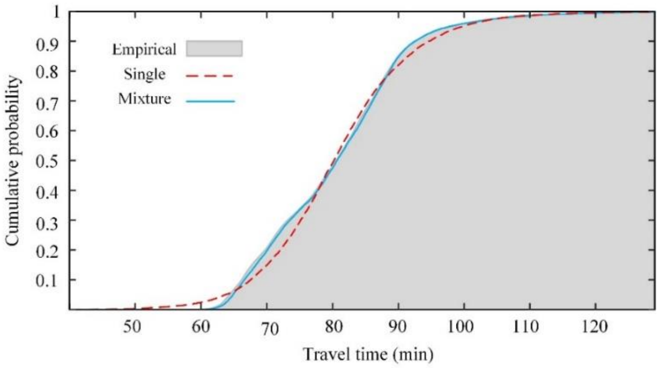


Figure 3. Fitting Results (Case: Route 3 for the 20:00-21:00 p.m. Interval)

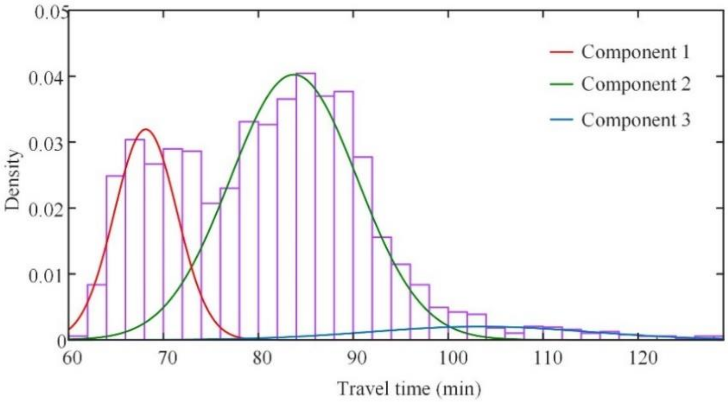


Figure 4. Fitting a Mixture Model with Three Components

Figure 5 depicts the number of routes in which each candidate mixture distribution is identified as the best-fitting distribution for each hour over the whole day. It is apparent from Figure 5 that the Burr mixture distribution has the highest proportion of the best-fitting distributions (51%) compared to its alternatives. Following the Burr mixture distribution, the Gamma mixture has the next highest proportion (25%) of the best-fitting distributions. Empirical research indicates that the travel time distribution exhibits a strong positive skew and a long upper tail [Ganjkhanloo et al., 2020]. Therefore, the appropriateness of the Burr distribution in modeling travel time distributions can be attributed to its ability to accommodate distributions with a strong positive skew and long upper tails.

Figure 6 demonstrates the frequency of each candidate distribution identified as the best-fitting hourly distribution for each route. As shown in Figure 6, the Burr distribution provides the best fit to the travel time data for most hours of the day. For example, it is identified as the best-fitting distribution for 18 out of 24 hours of the day for Route 8.

To examine the impact of using single distributions (Burr, Gamma, Lognormal, Weibull, and Inverse Gaussian distributions) instead of mixture distributions on the accuracy of travel time reliability measurements, we considered five commonly used reliability measures, including the buffer time index (*BI*), planning time index (*PI*), $\lambda^{\mathrm{var}}$ and $\lambda^{skew}$. These measures are defined as follows [Pu, 2011; Van Lint et al., 2008]:

$$BI = \frac{T_{95} - \mu}{\mu}, \tag{7}$$

$$PI = \frac{T_{95}}{T_{15}}, \tag{8}$$

$$\lambda^{skew} = \frac{T_{90} - T_{50}}{T_{50} - T_{10}}, \tag{9}$$

$$\lambda^{\mathrm{var}} = \frac{T_{90} - T_{10}}{T_{50}}, \tag{10}$$

where $T_i$ denotes the *i*-th percentile of the travel time distribution, *S* is the standard deviation of travel time, and $\mu$ represents the mean travel time.

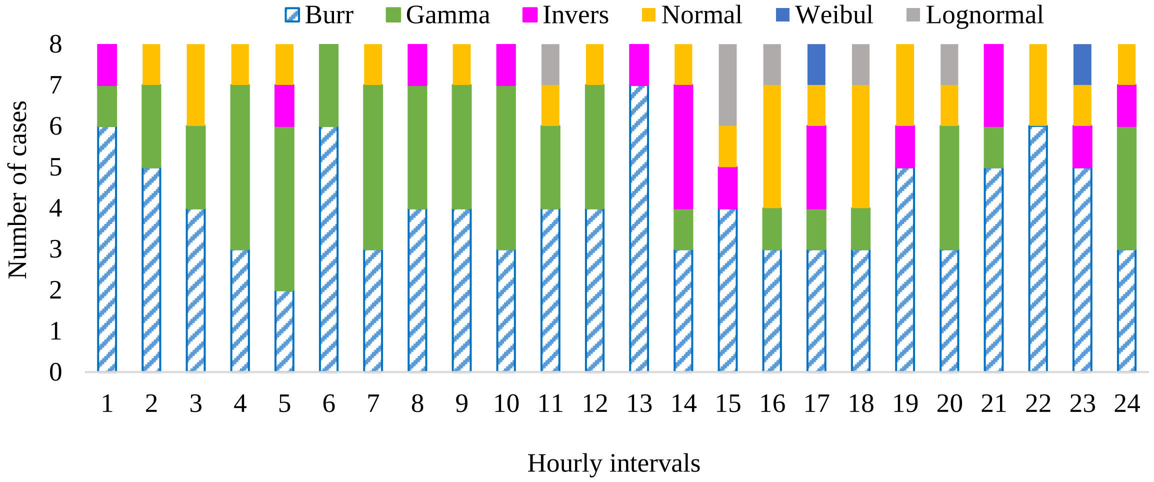


**Figure 5. Summary of Best-Fitting Distributions for Each Hour**

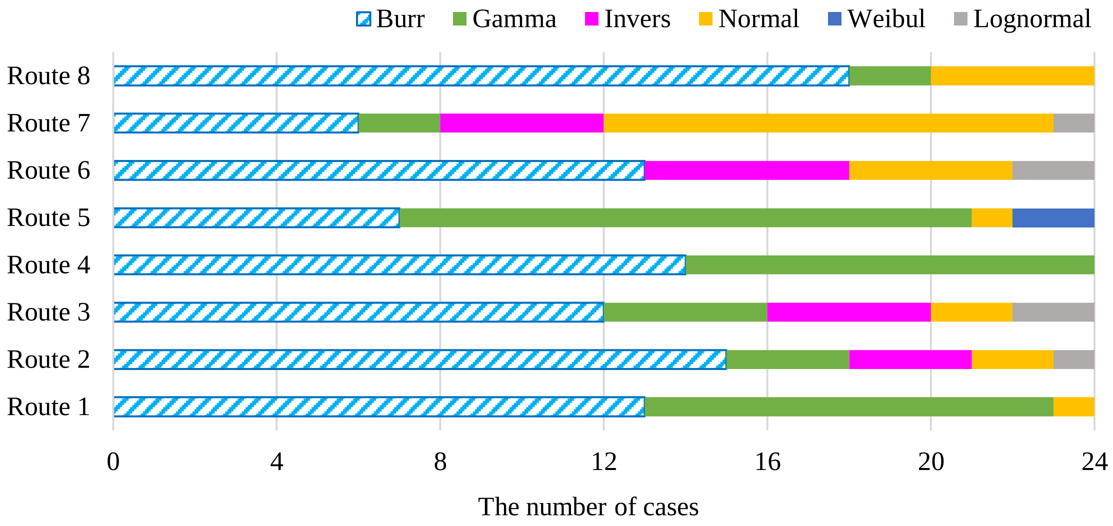


**Figure 6. Summary of Best-Fitting Distributions for Each Route**

The value of each measure was computed using three different methods for every hour and route. First, the best-fitting single distribution was identified for each hour and route to derive the value for each measure (*Method 1*). Second, the best-fitting mixture method was employed to derive the value of each measure from the mixture distribution (*Method 2*). Lastly, each measure was obtained numerically from the empirical distribution (*Method 3*). The measures derived directly from the empirical distribution are considered true (benchmark) values. The mean absolute error (*MAPE*) was used to compare the estimated values obtained by methods 1 and 2 with the true values. The mean absolute percentage error is computed as:

$$MAPE = \frac{100}{n}\sum_{i=1}^{n}\left|\frac{y_i - y_i^*}{y_i^*}\right|, \tag{11}$$

where $y_i$ is the estimate obtained by *Method 1* or *Method 2*, $y_i^*$ is the true value for the *i*-th estimation, and *n* represents the number of estimations. It should be noted that lower *MAPE* values indicate more precise estimates of reliability measures.

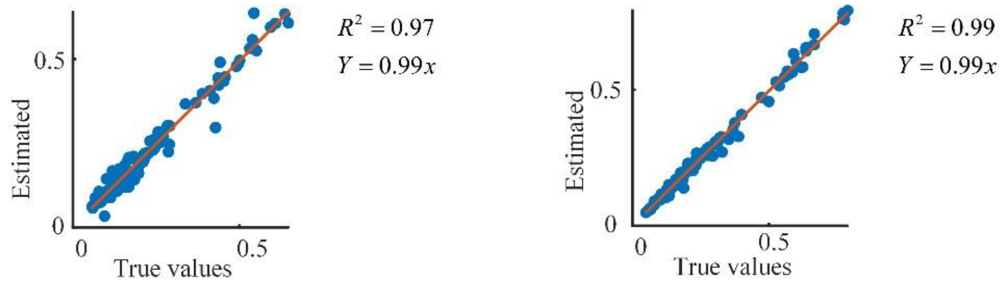
Table 2 presents the *MAPE* values for the reliability measures obtained once when the travel time distribution is modeled by a single distribution (*Method 1*) and once when it is modeled by a mixture distribution (*Method 2*). As indicated in Table 2, the results demonstrate consistently lower *MAPE* values for all measures when the estimation is based on mixture distributions. For example, the value of *MAPE* for the buffer time measure decreases by 133% when a mixture distribution is used for modeling travel time reliability. This indicates that using mixture distributions can lead to more accurate estimations of reliability measures than using single distributions. It is also evident from Table 2 that significant errors are associated with the estimation of the $\lambda^{skew}$ measure when a single distribution is employed to represent the travel time distribution. Consequently, it can be inferred that single distributions fail to capture the skewness of the travel time distribution.

**Table 2. MAPE for Reliability Measures**

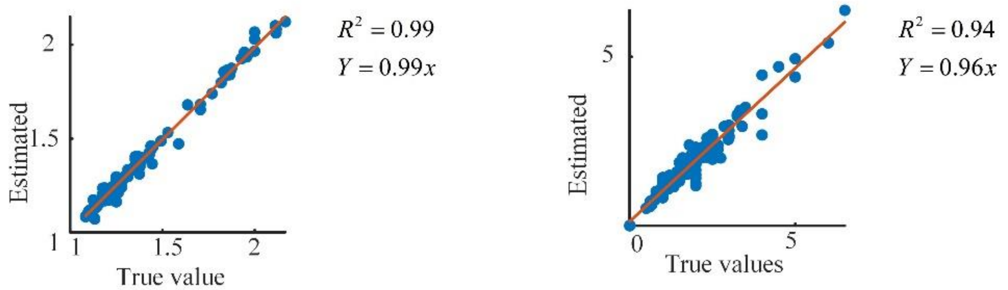
| Measure | Mixture | Single |
|---|---|---|
| *BI* | 6.44 | 13.52 |
| *PI* | 1.06 | 2.16 |
| $\lambda^{var}$ | 3.42 | 14.51 |
| $\lambda^{skew}$ | 7.30 | 27.48 |

Figure 7 shows a scatterplot for each travel time reliability measure that compares the true values of the measures with their corresponding estimated values derived from the mixture distributions. As shown in Figure 7, the travel time reliability estimates are close to the corresponding true values. This conclusion can also be derived by analyzing the fitted line equations and $R^2$ values since both the slope of each regression line and its $R^2$ value are very close to 1. Accordingly, it can be concluded that travel time reliability measures can be estimated accurately when mixture distributions are employed for modeling travel time distributions.

In theoretical applications, it is often assumed that travel time follows a normal distribution. This assumption simplifies the study of various aspects of travel time reliability and allows researchers to investigate how travel time reliability impacts applications such as travel behavior, reliable route finding, network design, and network equilibrium. To assess this assumption and explore its effects on travel time reliability measures, a comparative analysis was conducted. First, for each route and hour, travel time reliability measures were calculated using three distinct approaches, each corresponding to a different distributional assumption: 1) travel time reliability measures were calculated assuming a normal distribution fitted to the observed travel time data, 2) the best-fitting mixture distribution was used to calculate the measures, and 3) reliability measures were calculated directly from the observed travel time data (benchmark). Subsequently, the mean absolute percentage error was used to quantify the accuracy of estimates obtained from approaches 1 and 2 by comparing them to the benchmark values obtained by approach 3. Figure 8 presents the MAPE values for both approaches. As shown in Figure 8, the MAPE values are consistently lower when using mixture distributions. Therefore, although the normality assumption offers some simplifications, it can lead to inaccurate results.



(a) $BI$ $(b)$ $\lambda^{\text{var}}$ (mixture)



(c) $PI$ $(d)$ $\lambda^{skew}$ (mixture)

**Figure 7. Scatterplots of True Value versus Estimated Value**

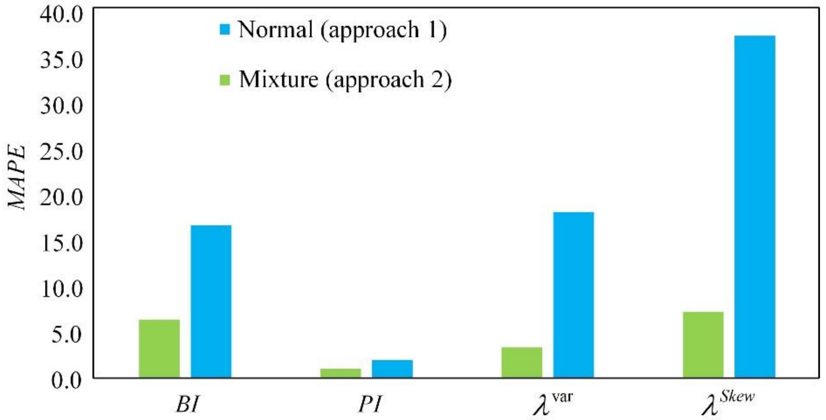


**Figure 8. Mean Absolute Percentage Errors for Reliability Measures**

The findings of this research indicate that the burr mixture is more effective than its alternatives for modeling travel time distribution. Furthermore, the study demonstrates that utilizing unimodal distributions for modeling travel time distribution may result in erroneous estimations of travel time reliability. Such inaccuracies in estimating travel time reliability can have considerable practical consequences in various areas of transportation engineering. For example, Srinivasan et al. (2014) found that assuming normal or lognormal distributions for travel times resulted in suboptimal solutions for finding the most reliable paths in 14% and 12% of cases, respectively. Antit et al. (2022) highlighted the impact of inaccurate travel time distribution modeling on routing solutions within the context of the capacitated vehicle routing problem with time windows. They showed inaccurate modeling of travel time distribution can change considerably the routing solutions. As another example, Shen et al. (2019) examined the effects of assuming incorrect travel time distribution in the context of the vehicle scheduling problem. They found that the cost and on-time performance of vehicle schedules can be improved by using the mixture distributions. Furthermore, inaccurate modeling of travel time distribution can lead to erroneous reliability assessment of transit services, which finally leads to suboptimal transit strategies, increased operational costs, and reduced service reliability [Ma et al., 2016]. Accordingly, accurate modeling of travel time distribution is crucial and offers a significant advantage over traditional unimodal distributions by capturing the complexities of real-world travel time data.

## 5. Conclusion

Travel time reliability is a crucial factor in the assessment of transportation systems. To effectively incorporate travel time reliability into transportation-related applications, it is necessary to model the travel time distribution. In this study, we focused on modeling travel time distributions. Our findings indicate that travel time distribution is typically multimodal; therefore, a single unimodal distribution fails to model travel time distribution accurately. We evaluated the performance of several mixture models using real travel time data collected from eight rural routes. The results of this study demonstrated that the burr mixture distribution is the most appropriate model, followed by the gamma mixture distribution. Furthermore, we examined the effects of employing single distributions instead of mixture distributions on the precision of travel time reliability assessments. Our findings showed that utilizing mixture distributions results in more precise estimations of reliability measures when compared to single distributions.

Mixture distributions have a major limitation in their lack of robustness to outliers. The presence of outliers often results in an increased number of components being utilized in the model. Additionally, mixture distributions exhibit instability in each algorithm run due to the random initialization of parameters. Therefore, it is essential to carefully clean the data and determine the optimal number of components for practical applications.

Several issues are still challenging and deserve to be addressed in future studies. In our research, we analyzed travel time data collected on rural freeways. However, it would be valuable to examine travel time data from urban roads to see how well mixture distributions can represent travel time variability in urban areas. It would also be interesting to establish the relationship between the parameters of mixture distributions and traffic variables. We also plan to incorporate the Bayesian model averaging [Hoeting et al., 1999] approach into the mixture modeling framework.

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