Solving a bi-objective multi-product vehicle routing problem with heterogeneous fleets under an uncertainty condition

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Abstract:

This paper presents a novel bi-objective multi-product capacitated vehicle routing problem with uncertainty in demand of retailers and volume of products (UCVRP) and heterogeneous vehicle fleets. The first of two conflict fuzzy objective functions is to minimize the cost of the used vehicles, fuel consumption for full loaded vehicles and shortage of products. The second objective is to minimize the shortage of products for all retailers. In order to get closer to a real-world situation, the uncertainty in the demand of retailers is applied using fuzzy numbers. Additionally, the volume of products is applied using robust parameters, because the possible value of this parameter is not distinct and belongs to a bounded uncertainty set. The fuzzy-robust counterpart model may be larger than the deterministic form or the uncertain model with one approach and it has with further complexity; however, it provides a better efficient solution for this problem. The proposed fuzzy approach is used to solve the bi-objective mixed-integer linear problem to find the most preferred solution. Moreover, it is impossible to improve one of the objective functions without considering deterioration in the other objective functions. In order to show the conflict between two objective functions in an excellent fashion, a Pareto-optimal solution with the ε-constraint method is obtained. Some numerical test problems are used to demonstrate the efficiency and validity of the presented model.

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1. Introduction

A vehicle routing problem (VRP) was one of the most crucial and essential issues in recent years. The objective of this problem is obtained through optimal route and distribution of vehicles in different nodes in a network. Vehicles start their travel from the depot and after serving all vertices (nodes), return to the depot (except in an open VRP). There are different types of a VRP according to different types of real-world applications.

There are several assumptions in formulating VRP. Some considered assumptions in this study are include: a capacitated VRP considering the load on each route does not exceed the capacity of a vehicle. A multi-commodity VRP that is considering different types of products transported on each route. A heterogeneous or homogeneous fleet considers different types or capacity for vehicles. Also, a multi or single-depot VRP is one of the critical assumption.

One of the most regular objective functions is to minimize the cost. For instance, the cost of fuel consumption with respect to the distance between each node is considered in a few papers. It is one of the most important issues in the company to save the fuel cost in transportation. Some papers discussed about the effect of distance travel on the fuel consumption, [Kara, Kara and Yetis, 2007] proposed a capacitated VRP with a new cost function based on the distance and the load of a vehicle. The empirical analysis of [Sahin et al, 2009] discussed for the truck with the capacity of 20 tons, when the fuel cost for 1000 km traveled included 60% of the total cost. Therefore, it is important to reduce the fuel consumption at the operational level. Hence, the travel distance is one of the critical factors effecting on the fuel consumption when it is considered the full loaded vehicle capacity.

Also, one of the problems dealing with the VRP is inventory decision. The inventory-routing problem (IRP) is regarded as a medium-term problem; however, the VRP is a short-term problem [Moin and Salhi, 2007]. The first study on the IRP was introduced by [Golden, Assad and Dahl, 1984] to identify a routing problem take into account the inventory assumptions. One of the critical issue is to integrate inventory management with a routing decision. As recent studies on this problem [Yu, Chen and Chu, 2008] proposed a model with split delivery and homogeneous fleet. They used a Lagrangian relaxation method that is decomposed in two sub-problems; inventory and routing problems in order to solve the large-scale problem. [Raa and Aghezzaf, 2009] proposed an integrated model of inventory management and vehicle route with a fleet size in a practical case in the context of vendor-managed inventory (VMI) with a cyclic approach. The aim is to minimize the average distribution and inventory cost. [Coelho, Cordeau and Laporte 2012] proposed and IRP model with a more practical case in the context of VMI. They assumed that products could be transshipped from a supplier to a customer and a customer to another customer. [Li et al, 2014] investigated a novel model for the IRP for a gasoline distribution industry, which minimizes travel time. They provided tabu search and an adapted US algorithm as a part of a solution procedure to obtain a better solution.

In the last two decades, the applications of this problem under uncertainty have increased in both practitioners and researchers area. This uncertainty existed in the board spectrum of the theories that include stochastic, fuzzy and robust uncertainty.

Fuzzy uncertainty is associated with the ambiguity of linguistic statements. These ambiguous situations may occur in objectives or in the parameters. It is often happening in parameters, such as demand and time windows. [Cheng, Gen and Tozawa 1995] proposed a VRP model with time windows and considered due-time instead of fixed time windows. Their fuzzy membership function shows the degree of customer satisfaction of the service time. [Wang and Wen, 2002] investigated a VRP model for a Chinese postman problem and considered fuzzy time windows. [Tang et al, 2009] proposed bi-objective VRP model with fuzzy soft time windows and considered a linear and concave fuzzy membership. Their objective functions are to minimize the routing cost and maximize the overall customer satisfaction level. [Ghannadpour et al, 2014] proposed a multi-objective dynamic VRP with fuzzy time windows. The objective functions are to minimize the total travel distance and waiting time on vehicles and maximize customer preferences for service. They proposed a genetic algorithm with three basic modules to solve this problem.

Other studies have considered demand and travel time as fuzzy parameters. [Gupta, Singh and Pandey, 2010] solved a multi-objective VRP with fuzzy time windows. The objective functions are to minimize the fleet size, maximize the average

grade of satisfaction, minimize the total travel distance, and minimize the total waiting time for vehicles. [Cao and Lai, 2010] proposed an open vehicle routing problem (OVRP) model. They considered fuzzy demands and used a fuzzy chance-constrained programing model based on a fuzzy credibility theory to solve this model.

Some papers have considered travel time as fuzzy parameters. [Kuo, Chiu and Lin, 2004] proposed a VRP model with time windows and fuzzy travel time. They used ant colony optimization (ACO) to solve this problem. [He and Xu, 2005] considered a single depot VRP with fuzzy demand and probability travel times between customers that minimizes travel time by considering capacity and arrival time constraints on the greatest degree. Because of the complexity of an uncertain VRP, they used a genetic algorithm to solve it. [Zheng and Liu, 2006] proposed a single-depot VRP model with time windows and considered travel time as fuzzy variables. They designed the integrated fuzzy simulation and genetic algorithm to solve this problem. [Xu, Yan and Li, 2011] proposed a bi-objective VRP model with soft time windows and fuzzy random environment. The objective functions are to minimize the total travel time and maximize the average satisfaction level of all customers. In order to obtain the equivalent crisp, the concept of the fuzzy random expected value is used. Some recent studies have considered a dynamic condition. [Ghannadpour, Noori and Tavakkoli-Moghaddam, 2013] considered the customer satisfaction level by using the concept of fuzzy time windows with the previous assumptions. Also, the first objective function is to minimize the total traveling distance and waiting time on vehicles and the second one is to maximize the customer preference for service.

Some recent studies have used the concept of robust optimization in a VRP model. This concept is used to compensate for the limitation of a stochastic approach for this problem, because this approach does not increase the complexity of the problem. [Montemanni et al, 2007] proposed a new extension for solving travel salesman problem by robust optimization. They applied a robust deviation criterion to improve optimization over an interval data problem. [Sungur, Ordónez and Dessouky, 2008] proposed a VRP model with demand uncertainty to introduce robust optimization to solve the problem. The aim is to minimize transportation costs with satisfying all demands. [Sungur et al, 2010] proposed an

uncertain VRP model. They considered scenario-based stochastic programming for uncertainty in customers and robust optimization for uncertainty in service times. [Li, Xu and Zhou, 2012] provided chance-constrained programming for a fuzzy demand in a VRP model. Also, they found the lowest total mileage in the model by using the improved tabu search algorithm. [Gounaris, Wiesemann and Floudas 2013] proposed capacitated VRP model with uncertain demand. They used robust optimization to solve the problem by developed robust rounded-capacity inequalities for two broad classes of the demand capacity. [Agra et al. 2013] proposed a VRP model with time windows and proposed a new formulation of a robust problem. The first formulation extends the well-known resource inequality formulation by considering adjustable robust optimization and the second approach formulation generalizes a path inequalities formulation for the uncertain context. [Solano-Charris, Prins and Santos, 2014] considered a robust VRP with uncertain travel time with discrete scenarios. The objective function is to minimize the worst total cost in each scenario. They used a genetic algorithm to solve this problem for small and medium-sized problems. [Sun and Wang, 2015] proposed a VRP model with uncertainty in the demand and transportation cost. They used robust optimization considered situations possibly related to bidding and capital budgets to solve the problem.

To summarize the contribution of this paper, a bi-objective capacitated VRP with uncertainty (UCVRP) is presented. Also, an assumption of multi-product and heterogeneous vehicles (with different capacity) is considered. The fuzzy objective functions are considered that minimize the traditional cost by considering the fuel consumption cost due to the traveled distance for fully loaded vehicles as the first objective. According to the constraints of inventory management in the model, the second objective is to minimize the shortage of products (i.e., negative inventory). Two types of uncertain parameters (e.g., demand of retailers and volume of products) are considered in the model. The uncertainty in the demand of retailers is due to the linguistic ambiguity of people, so it is expressed as fuzzy parameter. Also, the uncertainty in the volume of products is expressed as robust parameter, because the level of uncertainty of the volume of products belongs to the small bound without knowing

possible value. As a result, a new model is proposed by considering two uncertainty approaches and fuzzy objective functions in this study.

The rest of this paper is organized as follows. Classifying and reviewing the literature in Section 1. In Section 2 introduces the deterministic formulation of the model. Section 3 presents a fuzzy approach used in the objective functions and demand parameter and then rewrites fuzzy formulation of the model. Section 4 presents a robust optimization approach for an uncertain volume of products and rewrites the final form of the model. Numerical results and sensitive analysis are provided in Section 5, and we conclude this study in Section 6.

2. Problem Statement

In this paper, we consider the bi-objective multi-product VRP with heterogeneous fleet and retailer's inventory assumption. We assume that the vehicles are driving the same speed with full load on each tour. Additionally, we consider an uncertain demand for retailers, volume of products and objective functions. Additionally, the aim of this model is to minimize the total cost and shortage of products (i.e., negative inventory) with fuzzy objectives. The total cost consists of used vehicle, fuel consumption and shortage cost as lack of inventory in each demand point. It is notable that the cost of fuel consumption is calculated with respect to the travel distance, because the speed of full loaded vehicles are considered the same on each route.

2.1 Assumptions

The main assumptions of the present model are listed below.

- Considering the capacitated vehicle routing problem (CVRP).
- Considering different types of products.
- Considering a vehicle with the different capacity (i.e., heterogeneous fleet).
- Considering an inventory policy.
- Considering fuel consumption as a part of cost function in the first objective.
- Minimize the shortage of products as the second objective.
- Demands of retailers are assumed to be in a fuzzy nature.
- Volumes of products are assumed to be in a robust nature.
- Considering the fuzzy objective functions.

Figure 1 provides an example of a mathematical model in order to illustrate this problem with the above assumptions. This sample problem is described for nine retailers, three full loaded vehicles with different capacity and same speed and three types of products. According to the described model, the tours constructed from a central depot to visit retailers are shown. Also, the vehicles which choose to carry the amount of three types of selected products are shown.
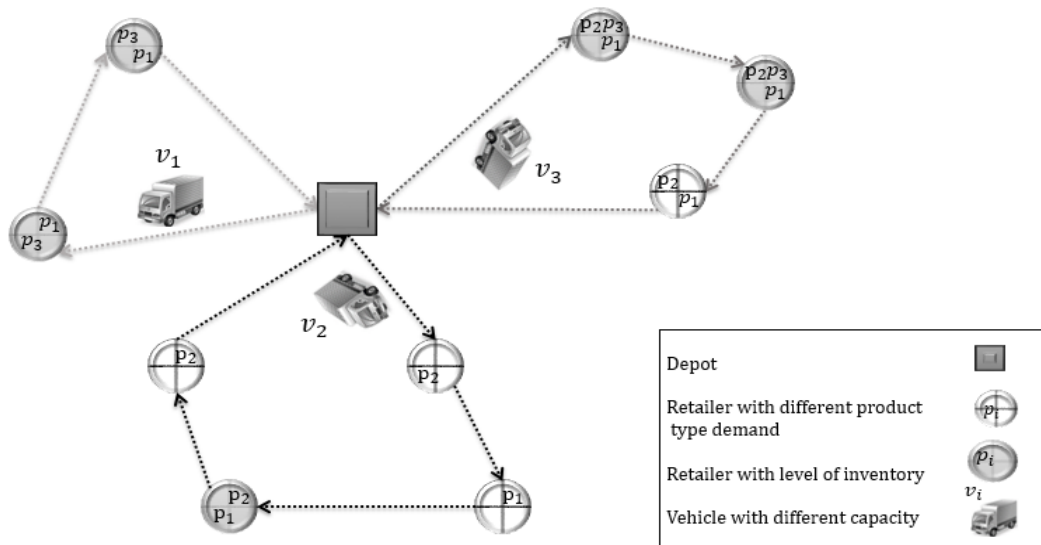

Figure 1. Illustrative example of the given problem

2.2 Model Formulation

Applied sets, indices, parameters and modeling variables are as follows:

Sets

$p \in \{1,...,P\}$ Set of products

$v \in \{1,...,V\}$ Set of vehicles

$c \in \{1,...,C\}$ Set of retailers

Parameters

dem_{pc} The amount of demand of retailer c for product p

cap_v Capacity of vehicle v

$d_{\hat{c}c}$ Distance between retailer c and $\hat{c}$

c_v Cost of vehicle v

cap_p Capacity of product p in central depot

vol_p Volume of product p

f_v Amount of fuel consumption of vehicle v per unit distance and unit vehicle weight

k_p Density per each type of product p

c^{fuel} Price of fuel consumption per unit distance per unit vehicle weight

c_p Cost of shortage (i.e., negative inventory) for product p

zl_1, zu_1
zl_2, zu_2 Fuzzy multi objective parameters

zll, zuu Upper bound and lower bound of right hand side of fuzzy constraints

Γ_v, Γ' Robust parameters

$M \sim \infty$

Decision variables

$x_{pvc} \geq 0$ Amount of products delivered to retailer c by vehicle v

$u_{pvc} \geq 0$ Amount of products when entering in retailer c by vehicle v

inv_{pc} Amount of inventory levels of product p for retailer c

$invp_{pc} \geq 0$
$invn_{pc} \geq 0$ Amount of positive and negative inventories of product p for retailer c

$\lambda_{v\hat{c}c}$ $\begin{cases} 1 \\ \\ 0 \end{cases}$ If vehicle v is assumed to travel from retailer c to $\bar{c}$
Otherwise

z_v^{veh} $\begin{cases} 1 \\ \\ 0 \end{cases}$ If vehicle v is used
Otherwise

y_{pvc}, z_v, p_p Robust variables

$y'_{pv\hat{c}c}, z', p'_{pv\hat{c}c}$

λ Fuzzy variable

$\eta_{pv\hat{c}c}$ The Auxiliary variable for linearization

2.3 Mathematical model

$$z_1 = Min \sum_{v \in V} c_v \times z_v^{veh} + \sum_{v \in V} \sum_{\hat{c} \in C} \sum_{c \in C} c^{fuel} \times f_v \times \lambda_{v\hat{c}c} \times d_{\hat{c}c} + \sum_{p \in P} \sum_{c \in C} c_p \times invn_{pc} \tag{1}$$

$$z_2 = Min \sum_{p \in P} \sum_{c \in C} invn_{pc} \tag{2}$$

s.t.

$$\sum_{v \in V} \sum_{\hat{c} \in C} \lambda_{v\hat{c}c} \leq 1 \qquad \forall c \in C, c > 1 \tag{3}$$

$$\sum_{\hat{c} \in C} \lambda_{v\hat{c}c} = \sum_{\hat{c} \in C} \lambda_{vc\hat{c}} \qquad \forall v \in V, c \in C \tag{4}$$

$$\sum_{v \in V} \sum_{c \in C} x_{pvc} \leq cap_p \qquad \forall p \in P \tag{5}$$

$$\sum_{p \in P} \sum_{c \in C} vol_p \times x_{pvc} \leq cap_v \times z_v^{veh} \qquad \forall v \in V \tag{6}$$

$$u_{pvc} \geq (u_{pv\hat{c}} - x_{pv\hat{c}}) \times \lambda_{v\hat{c}c} \qquad \forall p \in P, v \in V, \hat{c}, c \in C, c > 1 \tag{7}$$

$$u_{pv1} \geq \sum_{c \in C} x_{pv\hat{c}} \qquad \forall p \in P, v \in V \tag{8}$$

$$\sum_{\hat{c} \in C} \lambda_{v\hat{c}c} \leq z_v^{veh} \qquad \forall v \in V, c \in C \tag{9}$$

$$\sum_{p \in P} x_{pvc} \leq M \times \sum_{\hat{c} \in C} dem_{pc} \qquad \forall v \in V, c \in C \tag{10}$$

$$inv_{pc} = \sum_{v \in V} x_{pvc} - dem_{pc} \qquad \forall p \in P, c \in C \tag{11}$$

$$inv_{pc} = invp_{pc} - invn_{pc} \qquad \forall p \in P, c \in C \tag{12}$$

$$\lambda_{v\hat{c}c} \in \{0,1\} \qquad \forall \hat{c}, c \in C \tag{13}$$

$$z_v^{veh} \in \{0,1\} \qquad \forall v \in V \tag{14}$$

$$x_{pvc} \geq 0 \qquad \forall p \in P, v \in V, c \in C \tag{15}$$

$$u_{pvc} \geq 0 \qquad \forall p \in P, v \in V, c \in C \tag{16}$$

$$invp_{pc}, invn_{pc} \geq 0 \qquad \forall p \in P, v \in V \tag{17}$$

$$inv_{pc} \text{ free} \qquad \forall p \in P, c \in C \tag{18}$$

As mentioned before, this paper aims at minimizing the cost of purchasing the vehicle, fuel consumption and shortage as the first objective function and minimizing the shortage of products (i.e., negative inventory) as the second objective function. Both of them are shown as the first and second equations. The first constraint as shown in Eq. (3) ensures that each retailer stands on just one route. Eq. (4) guarantees whether vehicle v enter in node c, it should leave this node. Eq. (5) ensures that the amount of delivering product type p does not exceed its capacity. Eq. (6) is the capacity of a vehicle constraint. Eqs. (7) and (8) are sub-tour elimination constraints based on the amount of products when entering in nodes and ensure that no route is apart from a central depot. Eq. (9) ensures that each vehicle is assigned to one route. Eq. (10) states that product type p is delivered to retailer c by vehicle v, if vehicle v is allocated the route of retailer c. Eqs. (11) and (12) are the inventory constraints stating that the amount of delivered products to a retailer (i.e., inventory level) is the difference between load delivering to retailers and the demand for them. On the other hand, it is the difference between the inventory level of retailer and the shortage of products in one period.

2.4   Linearization Step

Due to the non-linear term appeared in the aforementioned mathematical model, a linearization procedure for Eq. (7) is applied as follows:

$$u_{pvc} \le u_{pv\hat{c}} - x_{pv\hat{c}} + bigm \times (1 - \lambda_{v\hat{c}c}) \qquad (19)$$

$$\forall p, v, \hat{c}, c > 1$$

3. Fuzzy Mathematical Programming

The proposed model for the VRP presented in the preceding section is a deterministic model. The fuzzy multi-objective mixed-integer linear programming (FMOMILP) is provided in this section to overcome the rigidity of the deterministic model. The mathematical model in the fuzzy environment can be included vague goal and parameters.

In the deterministic formulation, the total cost consists of the cost of the vehicle, cost of fuel consumption and penalty cost for shortage of products (i.e., negative inventory). In the real world, most of these costs are not easily measured, because human perception is effected in this evaluation. When we face with this issue, the decision maker wants to reach some aspiration

levels of the objective functions and allow some violations in the parameters of constraints. In addition, the fuzzy objective functions, the demand is considered as fuzzy parameter represented by intervals of tolerance.

This paper uses a fuzzy approach of [Zimmermann, 1978] and other [Chen and Chou, 1996] to deal with the imprecise parameters and fuzzy situation of objective functions. The linear membership functions are employed for constraints and objective functions. This approach is presented below.

$$\text{Max} \quad \lambda \qquad (20)$$

s.t.

$$\lambda \le \mu_{zr}^{\min}(x) \qquad \text{Minimum objective function} \qquad (21)$$

$$\lambda \le \mu_{zs}^{\max}(x) \qquad \text{Maximum objective function} \qquad (22)$$

$$\lambda \le \mu_{gl}(x) \qquad \text{Fuzzy constraint} \qquad (23)$$

$$g_p(x) \le b_p \qquad \text{Deterministic constraint} \qquad (24)$$

In the above model,

$\mu_{zr}^{\min}(x)$, $\mu_{zs}^{\max}(x)$ and $\mu_{gl}(x)$ refer to a minimum objective, a maximum objective and fuzzy constraints, respectively. The membership functions based on preference are linear as shown below:

$$\mu_{z_r^{\min}}(x) = \begin{cases} 1 & z_r > z_r^u \\ \dfrac{z_r^u - z_r(x)}{z_r^u - z_r^l} & z_r^l \le z_r(x) \le z_r^u \\ 0 & z_r < z_r^l \end{cases} \qquad (25)$$

$$\mu_{z_s^{\max}}(x) = \begin{cases} 1 & z_s > z_s^u \\ \dfrac{z_s(x) - z_s^l}{z_s^u - z_s^l} & z_s^l \le z_s(x) \le z_s^u \\ 0 & z_s < z_s^l \end{cases} \qquad (26)$$

With two membership functions for minimization and maximization objective functions, we can obtain an aspiration level for the value of the objective functions.

$$\mu_{g_p}(x) = \begin{cases} 1 & g_p(x) > b_p \\ \dfrac{g(x) - b_p}{d_p} & b_p \le g_p(x) \le b_p + d_p \\ 0 & g_p(x) < b_p + d_p \end{cases} \qquad (27)$$

where the left hand side of the fuzzy constraints is $g_p(x)$, the p-th fuzzy value is b_p and the tolerance level that the decision maker can consider in the p-th constraint of the fuzzy inequality is d_p.

According to the fuzzy objective functions and constraints, the MILP formulation can be expressed as follows:

$$\max \lambda \qquad (28)$$

$$(z_1^u - z_1^l) \times \lambda \leq z_1^u - z_1 \qquad (29)$$

$$(z_2^u - z_2^l) \times \lambda \leq z_2^u - z_2 \qquad (30)$$

$$\lambda \times (dem_{pc}^u - dem_{pc}) \leq dem_{pc}^u - (inv_{pc} + \sum_{v \in V} x_{pvc}) \quad \forall p, c \qquad (31)$$

$$\lambda \times (dem_{pc} - dem_{pc}^l) \leq -dem_{pc} - inv_{pc} + \sum_{v \in V} x_{pvc} \quad \forall p, c \qquad (32)$$

Constraints (3)-(6), (14), (8)-(10), (12) and (13).

$$0 \leq \lambda \leq 1 \qquad (33)$$

In order to obtain upper and lower bounds of two fuzzy objective functions, use the following procedure. In minimization goals, z_i^l can be obtained by solving each of single objective linear programming. Also, in minimization goals, z_i^u is the non-ideal solution (i.e., maximum value). After calculating of upper bound of each objective, if the upper bound is infinite, eliminated the basic constraint (e.g., sub-tour elimination) of the problem and calculated the upper bound of objective functions by minimizing them.

4. Robust Mathematical Programing

Now, the robust counterpart problem for a fuzzy VRP is presented. The volume of each product type is the other parameter of this model, which is faced with uncertainty. It is better to use a bound uncertainty set of data to solve this model efficiently in an uncertain condition. The expectation is that such a solution with a robust parameter will be efficient in this bound for every possible outcome.

According to the robust formulation [Bertsimas and Sim, 2004], the following constraint with uncertainty in parameter vol_p is considered, where J_p is the set of coefficients for this parameter and takes the value according to a symmetric distribution with the mean that is equal to the nominal value vol_p in the interval $\left[\overline{vol}_p - v\hat{o}l_p, \overline{vol}_p + v\hat{o}l_p \right]$.

$$\sum_{p \in P} \sum_{c \in C} vol_p \times x_{pvc} \leq cap_v \times z_v^{veh}$$

Parameter Γ_p is introduced that takes a value in interval $\left[0, |\Gamma_p| \right]$ for each p. The role of Γ_p is adjusted the robustness of the proposed method against the level of protection of the solution. The aim of this type of robustness is protected against that all cases up to $\lfloor \Gamma_p \rfloor$ change, and only one coefficient ($v\hat{o}l_p$) is changed by $(\Gamma_p - \lfloor \Gamma_p \rfloor) v\hat{o}l_p$. The obtained robust solution is feasible deterministically, even if more than one $\lfloor \Gamma_p \rfloor$ change, the robust solution will be feasible with very high probability. The formulation is shown below:

$$\max \ c'x$$

s.t.

$$\sum_{p \in P} \sum_{c \in C} \overline{vol}_p \times x_{pvc} +$$

$$\max_{\left\{ S_p \cup \{t_p\} \mid S_p \subseteq J_p, |S_p| = \lfloor \Gamma_p \rfloor, t_p \in J_p \setminus S_p \right\}}$$

$$\left\{ \sum_{p \in P} \sum_{c \in C} v\hat{o}l_p y_{pvc} + \left(\Gamma_p - \lfloor \Gamma_p \rfloor \right) v\hat{o}l_p y_{pvc} \right\}$$

$$\leq cap_v \times z_v^{veh}$$

$$-y_{pvc} \leq x_{pvc} \leq y_{pvc}$$

$$x \geq 0, y \geq 0$$

The value of the second part of the left hand side of Constraint (36) can be computed for each vehicle v and retailer c. Also, by considering $\Gamma_p \in \left[0, |J_p| \right]$, the robustness of the model against the level of protection is more flexible than $\Gamma_p = |J_p|$.

This model is equal to the objective function of the linear model as shown below:

$$\beta_i(x^*, \Gamma_i) = \max(\sum_{p \in P} \sum_{c \in C} v\hat{o}l_p |x_{pvc}^*| z_v) \qquad (39)$$

$$\sum_{v \in V} z_v \leq \Gamma_p \qquad \forall p \qquad (40)$$

$$0 \leq z_v \leq 1 \qquad (41)$$

With regards to the final form of the model introduced by [Bertsimas and Sim 2004], the new form of model is rewritten, which is established based on the dual of the model.
First, the dual of the model is considered below:

$$\min \quad \sum_{p \in P} p_p + \Gamma_v z_v \tag{42}$$

$$z_v + p_p \geq v\hat{o}l_p \times \sum_{c \in C} y_{pvc} \qquad \forall c, v \tag{43}$$

$$z_v \geq 0 \qquad \forall v \tag{44}$$

$$p_p \geq 0 \qquad \forall p \tag{45}$$

The objective value is obtained from the dual problem is feasible and bounded for all $\Gamma_p \in \left[0, \left| J_p \right| \right]$.

Now, the first model by considering the fuzzy objectives and fuzzy demands as well as a robust volume of products in constraints is rewritten.

$$\max \lambda \tag{46}$$

s.t.

Normal constraints

$$\sum_{v \in V} \sum_{\hat{c} \in C} \lambda_{v\hat{c}c} \leq 1 \qquad \forall c \in C, c>1 \tag{47}$$

$$\sum_{\hat{c} \in C} \lambda_{v\hat{c}c} = \sum_{\hat{c} \in C} \lambda_{vc\hat{c}} \qquad \forall v \in V, c \in C \tag{48}$$

$$\sum_{v \in V} \sum_{c \in C} x_{pvc} \leq cap_p \qquad \forall p \in P \tag{49}$$

$$u_{pvc} \leq u_{pv\hat{c}} - x_{pv\hat{c}} + bigm \times (1 - \lambda_{v\hat{c}c}) \qquad \forall p, v, \hat{c}, c > 1 \tag{50}$$

$$u_{pv1} \geq \sum_{c \in C} x_{pv\hat{c}} \qquad \forall p \in P, v \in V \tag{51}$$

$$\sum_{\hat{c} \in C} \lambda_{v\hat{c}c} \leq z_v^{veh} \qquad \forall v \in V, c \in C \tag{52}$$

$$\sum_{p \in P} x_{pvc} \leq M \times \sum_{\hat{c} \in C} \lambda_{v\hat{c}c} \qquad \forall v \in V, c \in C \tag{53}$$

$$inv_{pc} = invp_{pc} - invn_{pc} \qquad \forall p \in P, c \in C \tag{54}$$

Fuzzy constraints for objective function

$$(z_1^u - z_1^l) \times \lambda \leq z_1^u - z_1 \tag{55}$$

$$(z_2^u - z_2^l) \times \lambda \leq z_2^u - z_2 \tag{56}$$

Fuzzy constraints for demand

$$\lambda \times (dem_{pc}^u - dem_{pc}) \leq dem_{pc}^u - (inv_{pc} + \sum_{v \in V} x_{pvc}) \qquad \forall p \in P, c \in C \tag{57}$$

$$\lambda \times (dem_{pc} - dem_{pc}^l) \leq -dem_{pc} - inv_{pc} + \sum_{v \in V} x_{pvc} \qquad \forall p \in P, c \in C \tag{58}$$

Robust constraints for volume of product

$$\sum_{p \in P} \sum_{c \in C} vol_p \times x_{pvc} + z_v \times \Gamma_v + \sum_{p \in P} p_p \leq cap_v^{veh} \times z_v^{veh} \qquad \forall v \in V \tag{59}$$

$$z_v + p_p \geq v\hat{o}l_p \times \sum_{c \in C} y_{pvc} \qquad \forall p \in P, v \in V \tag{60}$$

$$x_{pvc} \leq y_{pvc} \qquad \forall p \in P, v \in V, c \in C \tag{61}$$

$$x_{pvc} \geq -y_{pvc} \qquad\qquad \forall p \in P, v \in V, c \in C \qquad (62)$$

$$0 \leq \lambda \leq 1 \qquad (63)$$

$$x \geq 0, y \geq 0 \qquad (64)$$

$$z_v \geq 0, p_p \geq 0, x_{pvc} \geq 0, u_{pvc} \geq 0, invp_{pc} \geq 0, invn_{pc} \geq 0 \qquad (65)$$

$$\lambda_{v\hat{c}c} \in \{0,1\}, z_v^{veh} \in \{0,1\} \qquad (66)$$

5. Extension of the Model with Decreasing the Load on Each Route

Changing in the load of the vehicle has a direct impact on fuel consumption. After delivery of products with large sizes (in the weight), the amount of fuel consumption has decreased. However, in the medium and small sizes (in the weight) of the delivery process, the decreasing in the fuel consumption is very low. So, the amount of fuel consumption can be considered fixed.

In this section, the extension of the model is considered for assessing the effect of the load decreasing for delivered products in large sizes on the vehicle fuel consumption. First, the deterministic form of the model is described as follows.

Put $\eta_{pv\hat{c}c} = u_{pvc} \times \lambda_{v\hat{c}c}$ then

$$z_1 = Min \sum_{v \in V} c_v \times z_v^{veh} + \sum_{p \in P} \sum_{v \in V} \sum_{c \in C} \sum_{\hat{c} \in C} c^{fuel} \times f_v \times vol_p \times k_p \times \eta_{pv\hat{c}c} \times d_{\hat{c}c}$$
$$+ \sum_{p \in P} \sum_{c \in C} c_p \times invn_{pc} \qquad (67)$$

s.t.

$$\eta_{pv\hat{c}c} \leq u_{pvc} + M(1 - \lambda v\hat{c}c) \qquad \forall p \in P, v \in V, c, \hat{c} \in C \qquad (68)$$

$$\eta_{pv\hat{c}c} \geq u_{pvc} - M(1 - \lambda v\hat{c}c) \qquad \forall p \in P, v \in V, c, \hat{c} \in C \qquad (69)$$

$$\eta_{pv\hat{c}c} \leq M \times \lambda v\hat{c}c \qquad \forall p \in P, v \in V, c, \hat{c} \in C \qquad (70)$$

$$\eta_{pv\hat{c}c} \geq 0 \qquad (71)$$

Constraints (2)-(18) $\qquad (72)$

In the Eq. (67) minimize the cost objective function includes the cost of purchasing the vehicle, fuel consumption cost with considering the effect decreasing the load on fuel consumption of a vehicle. Eqs. (68) - (71) are the linearization procedure for cost of fuel consumption term in the first objective function.

Because the uncertainty in the volume of products is considered as robust parameter, it is needed to consider the robust formulation for this parameter in the objective function. Since the generating the robust model based on deterministic model is described before, in this section the relations are just represented. The new model with fuzzy and robust parameters is shown bellow.

Put $\overline{A}_{pv\hat{c}c} = c^{fuel} \times f_v \times \overline{vol}_p \times k_p \times d_{\hat{c}c}$ then

$$z_1 = Min \sum_{v \in V} c_v \times z_v^{veh} + \beta + \sum_{p \in P} \sum_{c \in C} c_p \times invn_{pc} \qquad (73)$$

$$\overline{A}_{pv\hat{c}c} \times \eta_{pv\hat{c}c} \leq \beta \qquad \forall p \in P, v \in V, c, \hat{c} \in C \qquad (74)$$

$$\eta_{pv\hat{c}c} \geq 0 \tag{75}$$

Constraints (68)-(72) (76)

As described before, the robust model in Eq. (74) for mentioning robust factor is shown below.

$$Max \quad \sum_{p\in P}\sum_{v\in V}\sum_{\hat{c}\in C}\sum_{c\in C} \hat{a}_{pv\hat{c}c} \times \left|\eta^*_{pv\hat{c}c}\right| \times z'_{pv\hat{c}c} \tag{77}$$

s.t. (78)

$$\sum_{p\in P}\sum_{v\in V}\sum_{\hat{c}\in C}\sum_{c\in C} z'_{pv\hat{c}c} \leq \Gamma' \tag{79}$$

$$0 \leq z'_{pv\hat{c}c} \leq 1 \tag{80}$$

The dual of the above model is considered below:

$$Min \quad \sum_{p\in P}\sum_{v\in V}\sum_{\hat{c}\in C}\sum_{c\in C} p'_{pv\hat{c}c} + \Gamma' \times z' \tag{81}$$

$$z' + p'_{pv\hat{c}c} \geq \hat{a}_{pv\hat{c}c} \times \left|\eta^*_{pv\hat{c}c}\right| \qquad \forall p\in P, v\in V, c,\hat{c}\in C \tag{82}$$

$$p'_{pv\hat{c}c} \geq 0 \qquad \forall p\in P, v\in V, c,\hat{c}\in C \tag{83}$$

$$z' \geq 0 \tag{84}$$

Now, the final form of the new model considering the fuzzy and robust constraints of original model is shown below.

$$\lambda \tag{85}$$

$$\sum_{p\in P}\sum_{v\in V}\sum_{\hat{c}\in C}\sum_{c\in C} a_{pv\hat{c}c} \times \eta_{pv\hat{c}c} + z' \times \Gamma' + \sum_{p\in P}\sum_{v\in V}\sum_{\hat{c}\in C}\sum_{c\in C} p'_{pv\hat{c}c} \leq \beta \tag{86}$$

$$z' + p'_{pv\hat{c}c} \geq a_{pv\hat{c}c} \times y'_{pv\hat{c}c} \qquad \forall p\in P, v\in V, c,\hat{c}\in C \tag{87}$$

$$-y'_{pv\hat{c}c} \leq \eta_{pv\hat{c}c} \leq y'_{pv\hat{c}c} \qquad \forall p\in P, v\in V, c,\hat{c}\in C \tag{88}$$

$$p'_{pv\hat{c}c}, \eta_{pv\hat{c}c}, z' \geq 0 \tag{89}$$

Constraints (47)-(66) (90)

6. Computational Results

In the previous section, the given problem was described in details. In this section, the numerical results of the deterministic, fuzzy, robust and fuzzy-robust model with Equations (1)-(66) are presented. The results is obtained by solving some instances via general algebraic modelling system (GAMS) 24.1.2/CPLEX. It is essential to create random intelligent various sample problems in order to evaluate the mathematical model. For nominal experiments, eight sample problems with small and medium sizes are shown in Tables 2 and 3. The sample data are chosen from a uniform bound as shown in Table 1. It is notable that the cost of the three parameters in first objective function (e.g., c_p, c_v and c^{fuel}) should be in the same scale. The cost scaled of fuel consumption is different from the cost of the vehicle and shortage. It causes to ignore the effect of fuel consumption cost. In order to overcome this issue, each parameter is normalized than the range of its data.

Solving a Bi-Objective Multi-Product Vehicle Routing Problem with Heterogeneous Fleets under

Table 1. Source of random generations for sample problems

Parameters	Corresponding random distribution	Parameters	Corresponding random distribution
dem_{pc}	Uniform (0,31)	cap_p	Uniform (200000,300000)
cap_v	Uniform (200000,300000)	vol_p	Uniform (5,15)
$d_{\hat{c}c}$	Euclidean distance	f_v	Uniform (5,15)
c_v	Uniform (250000,300000)	k_p	Uniform (1,10)
c_p	Uniform (250000,300000)		

According to the discrete and continuous variables, the proposed model is a mixed-integer linear model. In order to validate the model and examined the result of the sample problems for fuzzy, robust, fuzzy-robust and deterministic models, used GAMS software. In Table 2, dimensions and characteristics of small-sized instances with the result of the deterministic and

fuzzy-robust model is shown.

As presented in Table 2, the total cost of the sample problem included, the cost of using vehicles, fuel consumption and shortage of products for each retailer is between 31995000 and 43820000 in deterministic form. Also, shortage of products as the second objective is between 70 and 107.

Table 2. Small-scale instances: Results for the deterministic and fuzzy-robust model

Problem name	Indices	Deterministic		Fuzzy-robust	
		Obj1	**Obj2**	**Obj1**	**Obj2**
S01	(p)4-(c)5-(v)4	31995000	70	17628000	21.59
S02	(p)4-(c)6-(v)4	42666000	107	28821000	58.03
S03	(p)4-(c)7-(v)5	43820000	101	27230000	44.3
S04	(p)4-(c)8-(v)6	38443000	75	21710000	13.11
Min	(p)4-(c)5-(v)4	31995000	70	17628000	13.11
Mean	(p)4-(c)7-(v)5	39231000	88.25	23847250	34.26
Max	(p)4-(c)8-(v)6	43820000	107	28821000	58.03

(i): indices of products, (c): indices of retailers, (v): indices of vehicles

In order to validate the exact solution of GAMS and performance of model by increasing the complexity of the problem, four sample problems

with a medium size are created and optimized. The results of medium-sized problems are shown in Table 3.

Table 3. Medium-sized instances: results for the deterministic and fuzzy-robust model

Problem name	Indices	Deterministic		Robust-fuzzy	
		Obj1	**Obj2**	**Obj1**	**Obj2**
M01	(p)6-(c)8-(v)6	119900000	368	101110000	297.368
M02	(p)6-(c)9-(v)7	138410000	417	112050000	326.375
M03	(p)7-(c)9-(v)7	180100000	580	152670000	484.299
M04	(p)7-(c)10-(v)7	220070000	723	193570000	623.068
Min	(p)6-(c)8-(v)6	119900000	368	101110000	297.368
Mean	(p)6-(c)9-(v)7	164620000	522	139850000	432.7775
Max	(p)7-(c)10-(v)7	220070000	723	193570000	623.068

(i): indices of products, (c): indices of retailers, (v): indices of vehicles

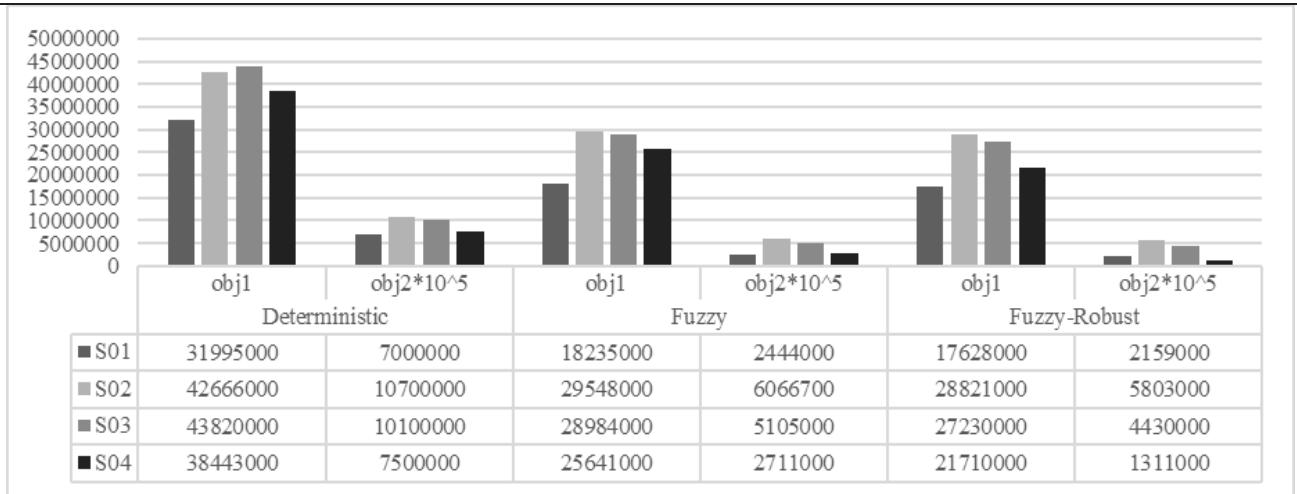|	Deterministic		Fuzzy		Fuzzy-Robust	
	obj1	obj2*10^5	obj1	obj2*10^5	obj1	obj2*10^5
S01	31995000	7000000	18235000	2444000	17628000	2159000
S02	42666000	10700000	29548000	6066700	28821000	5803000
S03	43820000	10100000	28984000	5105000	27230000	4430000
S04	38443000	7500000	25641000	2711000	21710000	1311000

Figure 2. Optimum objective function for small-sized instances in comparison with fuzzy optimum results

The results of the fuzzy and robust models for small instances are shown in Figures 2 and 3, separately. As characteristic of a fuzzy set theory, the solution of the fuzzy model is more stable than the deterministic form and shows the better insight into the model structure. So, the fuzzy results better than deterministic. Also, the result of the robust optimization model is better than the deterministic model, because this robust approach [Bertsimas and Sim 2004] guaranteed that constraints are satisfied by changing data. Also, these results are the same for medium sizes.

The results of comparison fuzzy-robust model with other medium sample problems are shown in Figure 4. It is obvious that the result of both objective functions of the fuzzy-robust model are

the best optimization value than the other ones.

If the uncertainty of parameters is known by the most possible value of them, it is efficient to use a fuzzy optimization approach. If the uncertainty of parameters is unknown and only a small violation of them is known, it is efficient to use a robust optimization approach. The most appropriate solution is achieved with these aggregation of approaches for an uncertain model.

In order to evaluate the flexibility of the presented model, we use a sensitive analysis procedure in different conditions and measure the flexibility of the model. Therefore, various scenarios for robust model and fuzzy model are defined and implementing sensitivity analysis procedures.
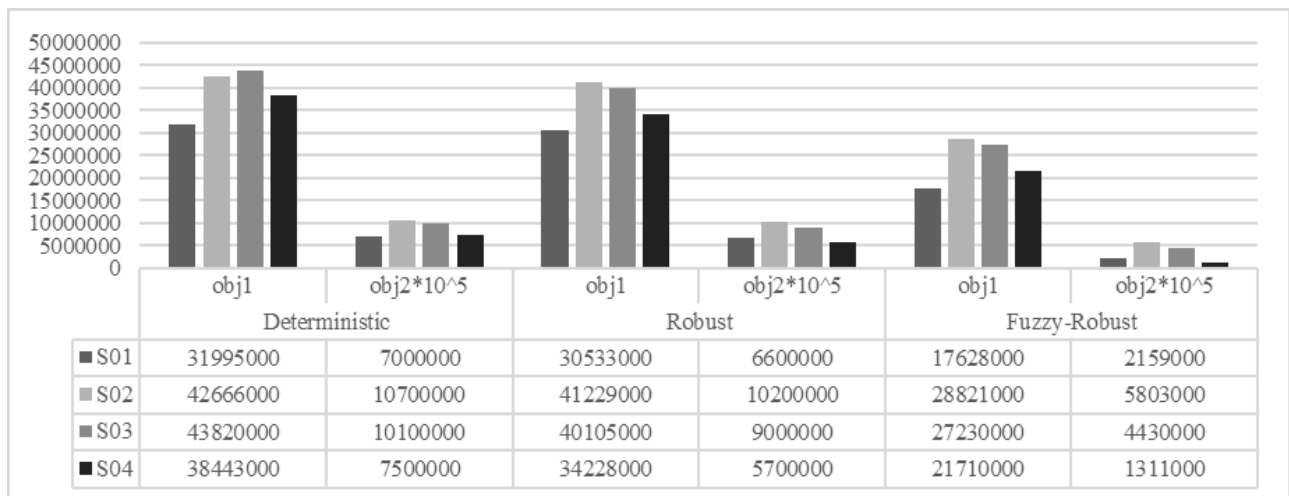

	Deterministic		Robust		Fuzzy-Robust	
	obj1	obj2*10^5	obj1	obj2*10^5	obj1	obj2*10^5
S01	31995000	7000000	30533000	6600000	17628000	2159000
S02	42666000	10700000	41229000	10200000	28821000	5803000
S03	43820000	10100000	40105000	9000000	27230000	4430000
S04	38443000	7500000	34228000	5700000	21710000	1311000

Figure 3. Optimum objective function for small-sized instances in comparison with the robust optimum results

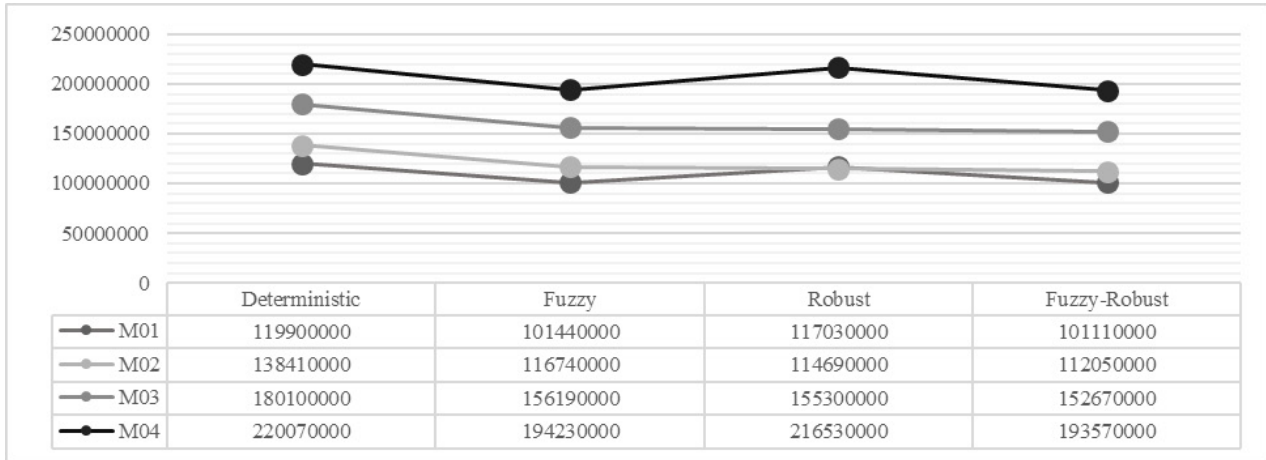|	Deterministic	Fuzzy	Robust	Fuzzy-Robust
M01	119900000	101440000	117030000	101110000
M02	138410000	116740000	114690000	112050000
M03	180100000	156190000	155300000	152670000
M04	220070000	194230000	216530000	193570000

Figure 4. Optimum objective function for medium-sized instances in comparison with fuzzy optimum and robust optimum results

In order to demonstrate the conflict between objective functions, we show the Pareto optimal solution. The augmented ε-constraint method is used to achieve this goal. Table 4 illustrates the ideal (Z^I) and nadir (Z^N) points for objective functions. Also, the diagram of a Pareto front is shown in Figure 5.

Table 4. Upper and lower bound

Ideal and Nadir points	Objective 1 (Cost)	Objective 2 (Shortage)
Upper bound	Z^N= 25200000	Z^N=64
Lower bound	Z^I= 23880000	Z^I=56

According to the non-dominated solution shown in Figure 5, the first two points of the Pareto front have the same value of the cost objective function, while the shortage of products objective function are different. Also, the two other points are the same condition. Conflict among objective functions is not only clear in the Pareto frontier objective, but also is obvious in the model logically. According to the second objective function, if an unlimited amount of negative inventory for the second objective function is allowed, the number of delivered products by each vehicle (Constraints 11 and 12) is reduced. Also, the number of used vehicles and vehicle trips (Constraints 6 and 10) are reduced. So, the value of the cost objective function is reduced.
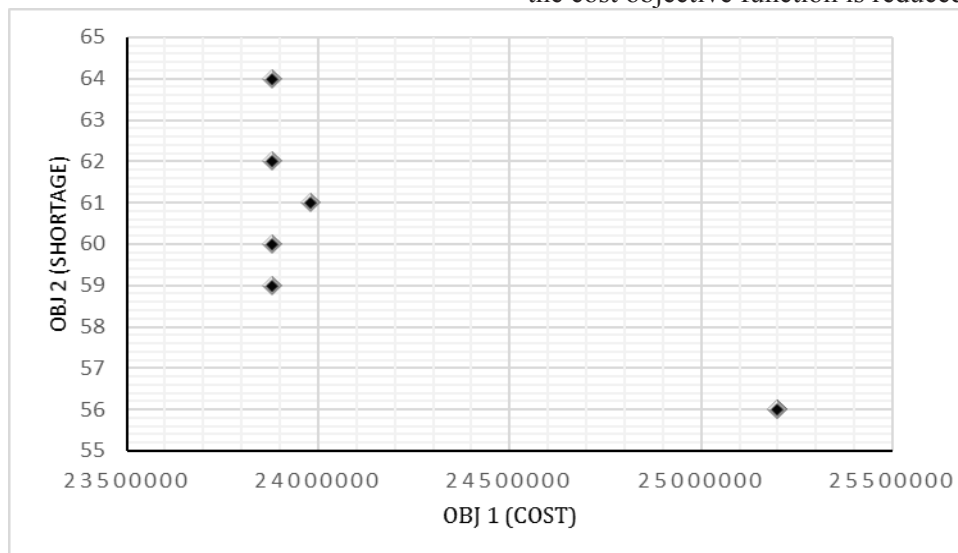

Figure 5. Pareto front

6.1 Analysis Based on the Uncertainty Level of the Demand

The scenarios on reducing demand is considered and chosen sample problem S02 as an example. The result of reducing demand presented in Table 5.

Two analysis is made for reducing the demand. First, the total cost is constant or decreases and the shortage of products decreases too. It is due to the fact that the vehicle number is constant. As a result, the shortage of products decreases means while the cost is constant or decreases. Second, costs decrease a lot and the shortage of products increases, which means that deciding to use fewer vehicles due to the demand reduction. So, the amount of the remained facilities are not capable of supplying more shortage of products.

The results from the fuzzy model and robust-fuzzy model are congruent with first expectation scenarios from the suggested model performance. When the reduction of demands is increasing, the reduction rate of the objective function of the fuzzy-robust model is more than the fuzzy model. It represents the best performance of the fuzzy-robust model. These reduced rates for both models are shown in Figures 6 and 7.

6.2 Analysis Based on a Protection Level for Robust Parameters

It is acceptable to estimate the change in the objective functions with respect to the change in protection level Γ_i of constraint i measure the validation of the robust optimization model. Hence, the price of increasing or decreasing a change is evaluated at the protection level for each constraint by the results of the objective functions in the robust model. Also, the results of the fuzzy-robust optimization model is obtained for each level of protection. Moreover, the probability bounds of constraint violation is obtained [Bertsimas and Sim 2004]. In other words, if more than the $\lfloor \Gamma_i \rfloor$ uncertain parameters of the right hand side of i-th constraint change, the probability of a violation of i-th constraint is at most these bounds. The result of change Γ_i on objective functions and probability bound of constraints for the robust and fuzzy-robust models are shown in Table 6.

Table 5. Scenarios designed for the sensitivity analysis procedure on demand

Scenario	Amount of demand	Fuzzy		Fuzzy-robust	
		Obj1 (Cost)	Obj2 (Shortage)	Obj1 (Cost)	Obj2 (Shortage)
1	0.95×Demand	25401000	48.627	22304000	33.948
2	0.9×Demand	21093000	33.390	20939000	30.345
3	0.85×Demand	17536000	20.133	16443000	17.626
4	0.8×Demand	14919000	9.220	13471000	6.332
5	0.75×Demand	12340000	1.673	12131000	0.789
6	0.7×Demand	11721000	0.087	11767000	0.249
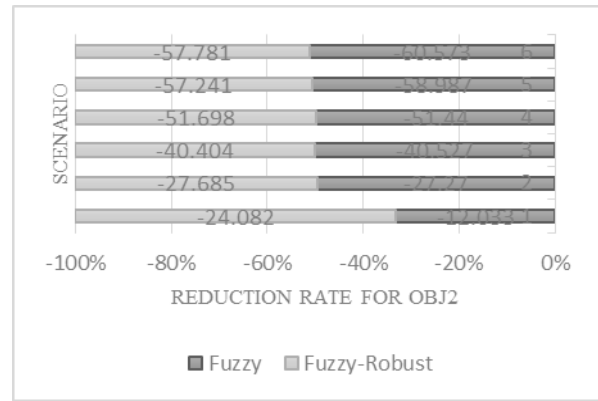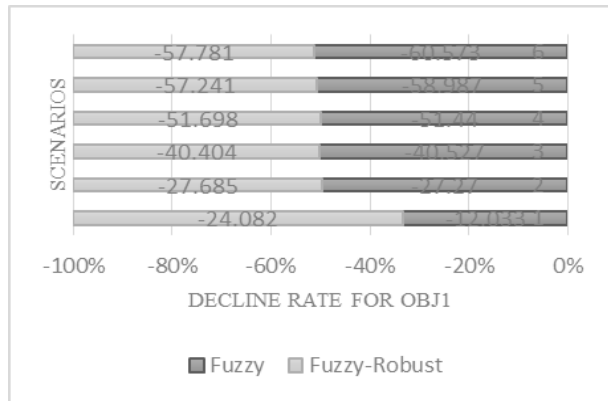
Figure 6. Reduced rate of the second objective function for both models in each scenario

In the robust optimization model, the objective functions are increasing the protection level; however, the trend of the objective functions in the fuzzy-robust model is not distinct. It is obvious that the optimal value of the objective functions in both models is affected when the protection level is increased. For example, in order to have a probability guarantee of a constraint violation at most 15.74%, it needs to reduce the first objective function of the robust model, 69%, while the reduction level for the fuzzy-robust model is 41%.

The effect of the protection level on an optimal value of the first objective function for the robust and fuzzy-robust models are shown in Figure 8. The results illustrate the better performance of the

fuzzy-robust model for each level of protection. Also, Figure 9 illustrates the optimal value of the first objective function with respect to the probability bound of constraint violation.

6.3 Comparison Between Results of Original and Extension Models

In order to show the effect of decreasing of vehicle load (for the model presented in Section 5), the result of the deterministic model described for small-sized instances are shown in Table 7. Also, the result of the fuzzy-robust model is shown in Table 7 to show the effect of the robust parameter in the objective function.

Table 6. Results of the robust and fuzzy-robust solutions for Γ

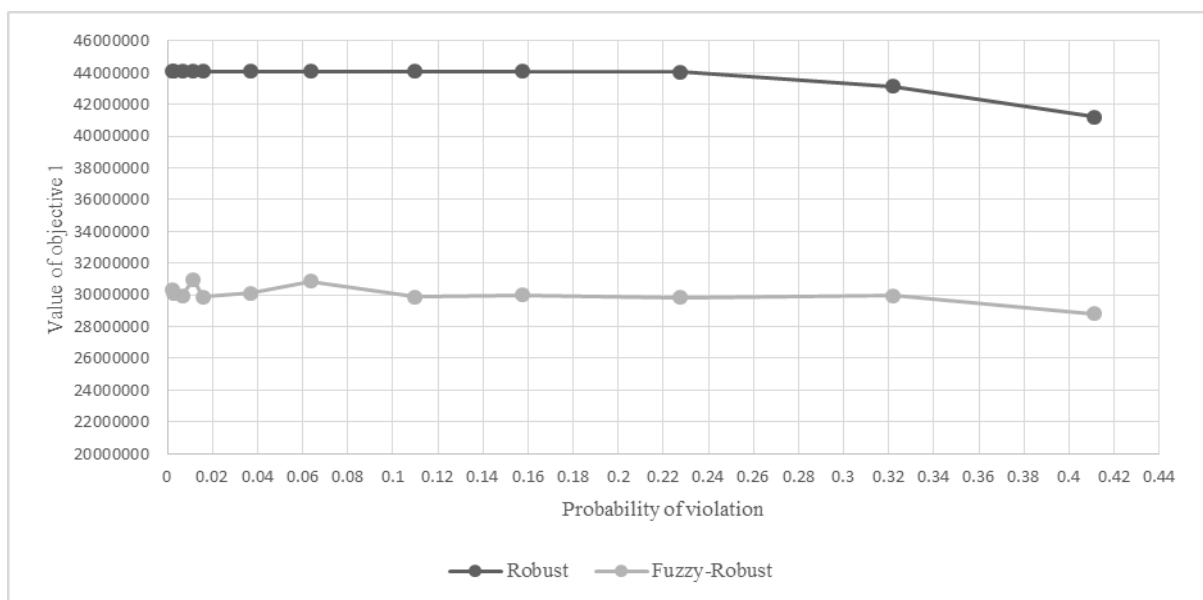| Γ	Probability bound	Robust		Fuzzy-robust	
		Obj1	Obj2	Obj1	Obj2
2	0.4114	41229000	102	28821210	58.030
3.2	0.3221	43132080	110	29953354	63.000
4.6	0.2275	44064589	112	29835145	63.171
5.8	0.1574	44071122	112	29996842	64.423
7	0.1097	44071185	112	29886269	64.765
8.2	0.0634	44071203	112	30862130	65.049
9.4	0.0371	44071236	112	30123720	65.049
10.8	0.0157	44071457	112	29869341	63.797
12	0.0113	44071642	112	30972286	64.655
13.2	0.0065	44079450	112	29953354	65.049
14.4	0.0028	44079592	112	30088715	65.675
15	0.0020	44079754	112	30290140	65.545

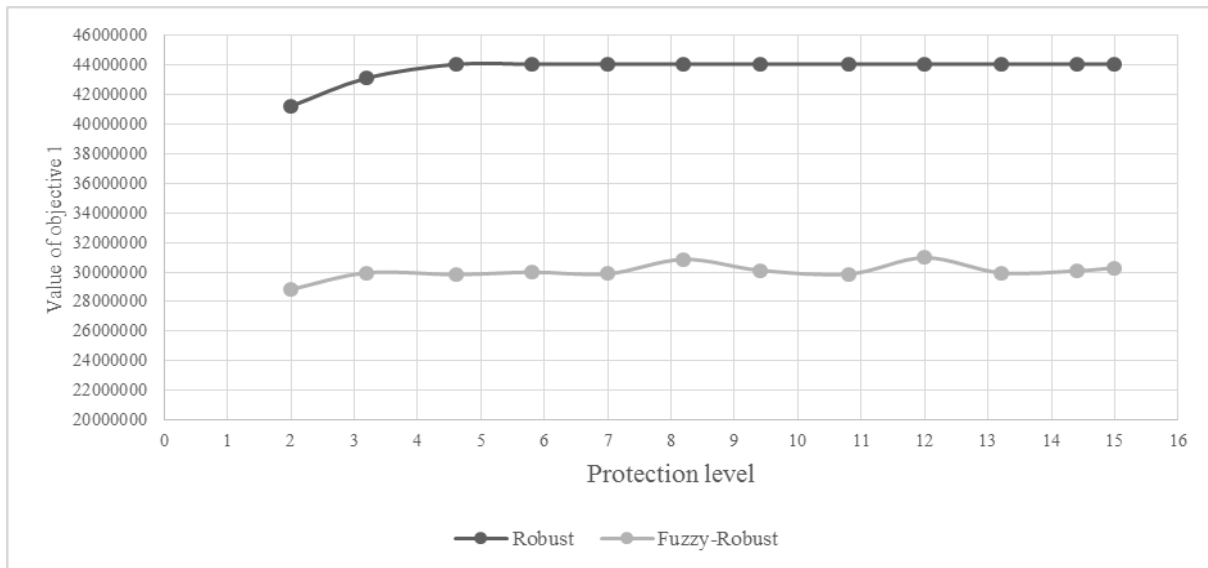Figure 7. Optimal value of the robust and fuzzy-robust models as function of Γ

Figure 8. Optimal value of the robust and fuzzy-robust models as a function of the probability bound of the constraint

Table 7. Comparision between the models for small-sized instances

Problem	Original model				Extension model			
	Deterministic		Fuzzy-robust		Deterministic		Fuzzy-robust	
	Obj1		Obj2		Obj1		Obj2	
S01	31995000	70	17628000	21.59	37847500	246	23942000	34.82
S02	42666000	107	28821000	58.03	47511100	112	31642000	61.94
S03	43820000	101	27230000	44.3	45820800	238	29840000	51.38
S04	38443000	75	21710000	13.11	46465200	201	20432000	45.67

According to the results of Table 7, both objective functions considering load is worse than the original model for a deterministic form. In the fuzzy-robust model, adding the robust parameter to the objective function is changed without any specific behavior in both objective functions.

7. Conclusion

In this study, we proposed a novel bi-objective capacitated vehicle routing problem under uncertainty (UCVRP) and considered the multi-products with heterogeneous fully loaded vehicles (i.e., different capacity) assumptions. The first objective function minimizes the cost of the used vehicles, fuel consumption and shortage of products for each retailer. The second objective minimizes the shortage of products. The inventory constraints are considered in the model to obtain the shortage of products. We considered uncertainty in some parameters included the demand of retailers and volume of products. We utilized fuzzy optimization and robust optimization approaches separately to cope with the uncertain

parameters and objective functions. In order to overcome the ambiguous of linguistic for demand of retailers and most possible amount of known demands in the real world, considered a fuzzy optimization model. Also, the uncertainty in the volume of products belongs to the bounded uncertainty set in the real world. Therefore, we considered a robust optimization model. Finally, an integrated model (i.e., fuzzy-robust model) presented to deal with both uncertainty parameters. Additionally, we coped with fuzzy uncertainty in the bi-objective mixed-integer linear problem. The augmented ε-constraint method is used to demonstrate the conflict between objective functions. Also, an extension of model considering the effect of load of fuel consumption is presented. Some sample problems are generated and solved to demonstrate the efficiency of the fuzzy-robust model. Finally, we proposed a sensitivity analysis based on a reducing demand for the fuzzy and fuzzy-robust model and increasing protection level for the robust and fuzzy-robust model. The results showed the efficiency of the integrated model.

Future studies can be considered a model for a multi-depot VRP or multi-period condition. Also, one can be provided an efficient algorithm to solve given large scale problem.

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