A Monte Carlo Simulation of Chain Reaction Rear End Potential Collisions on Freeways

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Abstract

In recent research on modelling road collisions very little attention has been paid to rear-end chain reaction collisions, which is characterized by more than two vehicles involved in a collision at the same time. The core aim of the present research is to develop a methodology to estimate such potential collision probabilities based on a proactive perspective, where deceleration rate to avoid collision is used as a surrogate safety measure. To consider the uncertainty of variables in estimating the N-vehicle rear-end collision, a methodology based on Monte Carlo simulation is proposed. To show the applicability of the proposed methodology, the NGSIM trajectory database of I-80 interstate freeway is used. The probability density function for drivers' response time is developed through the analysis of 1534 car following situations detected in 45 minutes of movement. The potential risk of two to five vehicle reaction collisions in a five vehicle platoon is estimated by running the simulation through 20 thousand substitutions of randomized generation values drawn from probability density function of response time and maximum available deceleration rate in a following outcome function. Results show that avoiding rear-end collision should be considered as a shared responsibility among the drivers in the platoon. As expected, the methodology considers probability of N vehicles colliding at the same time decreasing as N increases. N-vehicle collision is shown to be directly related to the clearance between the following vehicles within the platoon and the speed of individual vehicles as well as the drivers' reaction time and the maximum deceleration rate available in individual vehicles. The proposed methodology is believed to act more effectively than the ordinary methods, particularly if it is used to alarm drivers of vehicles synchronized based on vehicular ad hoc network (VANET) methodologies.

Keywords: Chain reaction rear end collision, response time, maximum available deceleration rate, Monte Carlo simulation

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1. Introduction

Striking the rear of a leading vehicle by front of a following vehicle is termed as a rear-end collision. As an important aspect of research, a great deal of research has been conducted on rear-end collisions in car following situations. Several rear-end collision prediction models have been developed and validated based on different methodologies so far, ranging from directly using observed crash counts [Wang and Abdel-Aty, 2006] to estimation based on trajectories [Oh and Kim, 2010], real time data [Xu et al. 2013] and combined methods [Yu and Abdel-Aty 2014]. Particularly, the deficient aspects of studies focusing on crash counts as the only data source has been criticized and the necessity for taking all likely interactions between the road users (instead of only interactions leading to a collision) has been well emphasized in proactive approaches [Lord and Mannering, 2010].

In proactive methods of investigating traffic safety during a car following situation, research has been conducted on detecting conflicts (i.e. situations in which two or more vehicles approach each other in time and space to such an extent that there is risk of collision if their movements remain unchanged [Amundsen and Hyden, 1977]) based on different performance indicators (also called surrogate or proximal measures) [Guido et al. 2010]. Despite that the degree of correlation between the surrogates and the real collisions is questionable (e.g. see [Hauer and Garder 1986; Davis et al. 2011]) and only very few indicators have been thoroughly validated to date [Laureshyn 2010], studies of performance in vehicular interactions based on conflicts have been widely used in detection of potential collision situations [Mamdoohi et al. 2014], calibrating in vehicle safety measures and in particular in assessing the safety countermeasures [Cunto and Saccomanno, 2007].

Different aspects of interactions between vehicles have been used by researchers in detection of conflicts in a traffic stream. For instance, Laureshyn et al. [2010]. Categorized indicators to those describing proximity in space, proximity in time and the intensity of necessary evasive action, particularly intensity of necessary evasive action has long been in use by different researchers. Related measures usually focus on braking power as a means to detect the potential collisions in the stream. For example oh et al. (2006) developed the Stopping Distance Index (SDI) based on the above concept to assess the risk of rear-end collision by using the data obtained by inductive loop sensors installed on a freeway segment [Oh, Park, and Ritchie 2006]. Davis (2006) used the same concept to demonstrate that the prevention of rear-end collisions in a platoon of vehicles travelling on the same lane in a freeway is not only the responsibility of the following vehicle, but also a collective responsibility of all drivers in the platoon [Davis and Swenson, 2006]. Present research will also use the deceleration rate to avoid collision as a base to detect the conflicts in car following situations.

While the general body of previous studies have focused on ordinary rear-end collisions (where only one vehicle collides another in a rear ending collision), the literature still extensively lacks research on chain reaction collisions, where more than two vehicles moving in a lane are involved in a rear-end collision. Although such accidents are relatively rare and in despite the lacks of research on this type of collisions, there are media reports elaborating occasions where a large number of vehicles are involved in chain reaction collisions, most often leading to huge burden of vehicular damage, travel delays and sometimes passenger injuries.

In a rear-end chain reaction collision (as will be shown), on a specific deployment of vehicles in a platoon where vehicles separated by specific time headways travelling at specific velocities, driver reaction time and vehicle braking power are crucial parameters in preventing a chain reaction collision. Vehicle braking capacity and driver reaction time are stochastic variables of uncertainty phenomena. The stochastic nature of these variables in estimating the probability of potential rear-ending collisions has rarely been taken into consideration in previous research. Furthermore, prediction models for chain reaction collisions have not been the subject of much research yet. Therefore, considering the stochastic nature of these variables, the present study aims to develop a simulation-based structure to measure the potential collision probability for both a single and a chain reaction rear-end collision.

2. Mechanism of Rear-End Collision

The mechanism of a rear-end collision is usually explained by principles of Newtonian mechanics of movement. To explain the mechanism of chain reaction collision, a single rear-end collision with only two vehicles included is first described.

2.1 Single Collision

Suppose that a platoon of N vehicles numbered by k=1 to n (first to last) is moving on a lane with speeds v1 to vn, respectively. At a particular time, vehicle k brakes to stop with a deceleration rate ak. The following vehicle k+1 (with a space gap xk+1 separating rear bumper of leading vehicle k from the front bumper of following vehicle k+1) brakes to stop after a reaction time rk+1. To avoid a rear-end collision between each two successive vehicles; the distance traversed by the following vehicle before coming to stop should not exceed the distance required by the leading vehicle to stop (Relation 1).

$$x_{k+1} + \frac{v_k^2}{2a_k} \geq r_{k+1} v_{k+1} + \frac{v_{k+1}^2}{2a_{k+1}} \tag{1}$$

Expressing space gap in terms of speed and time headway (h) (Equation 2), driver k+1 will be able to stop before colliding if inequality 3 satisfies.

$$x_{k+1} = v_{k+1} h_{k+1} \tag{2}$$

$$a_{k+1} \geq \frac{v_{k+1}^2}{(v_k^2 / a_k) + 2 \times v_{k+1} \times (h_{k+1} - r_{k+1})} \tag{3}$$

In simple words if available deceleration in the k+1 th vehicle is less than the required deceleration to stop, the rear-end collision will be inevitable.

As an important implication of inequality 3, interactions between different vehicles (in terms of available braking power) and different drivers (in terms of reaction time) as well as the relative position of vehicles should be considered as causes of rear-end collision in a platoon of vehicles. Adopting the causal concept, Brill developed the structure of a causal concept for rear-end collision in a three vehicle platoon as illustrated in Figure 1 [Davis and Swenson, 2006].

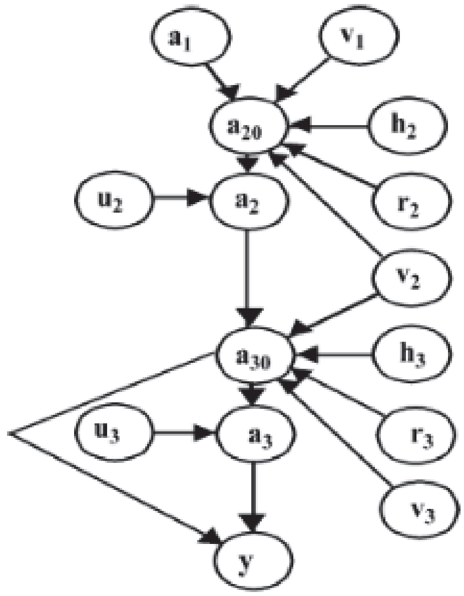

Figure 1. Structure of causal concept for rear-end collision in a platoon of three vehicles [Davis and Swenson 2006]

The scheme in figure 1 is composed of nodes representing model variables and arrows representing the presence and direction of causal dependencies. The brake deceleration, speed and headway for vehicle i in the platoon are represented by a_i, v_i and hi, respectively in this illustration. Driver i reaction time is shown by r_i. a_{i0} is also representing the minimal deceleration rate to avoid colliding with leading vehicle which can be given by the right hand side of relation 3.

The driver may take higher braking deceleration to avoid the collision (i.e. the deceleration rate that driver k needs to avoid collision may exceed a_{k0} by u_k). The deceleration rate, however, is always equal or less than the maximum available deceleration rate (a). The actual deceleration rate for vehicle k can be determined as follows.

$$a_k = \min(a_{k0} + u_k, a) \tag{4}$$

If the deceleration needed by vehicle k to prevent hitting the vehicle in front exceeds the maximum available deceleration rate, a collision may be unavoidable. This is shown by a dichotomous indicator y between each pair of successive vehicles (Equation 5) on a car following situation.

$$y = \begin{cases} 0 & f \quad a_{k0} \le a \\ 1 & f \quad a_{k0} > a \end{cases} \tag{5}$$

As an example suppose that in a three vehicle platoon, v_1=v_2=v_3=44 km/h (12.2 m/s) and the maximum available deceleration rate is 6.1 m/s^2. If the driver 1 brakes to stop with 1.5 m/s^2 and supposing that h_2=2sec and r_2=4sec, equation 3 returns the minimum deceleration rate for driver 2 to avoid colliding with vehicle 1 as 3.05 m/s2. If driver 2 decelerates at 3.2 m/s^2 (corresponding to u_2=3.2-3.05=0.15 m/s^2), assuming that driver 3 with reaction time r_3=2.5 sec and time headway with the leading vehicle h_3=1.5 sec, the minimum deceleration rate for vehicle 3 to avoid colliding to vehicle 2 is a_{30}=6.7 m/s^2 which exceeds the maximum available deceleration rate 6.1 m/s2. It means that rear-end collision between vehicles 3 and 2 is unavoidable.

The question is which driver is causing the collision? Driver 3 could have precluded the collision if he/she maintains a higher amount of headway with vehicle 2. For instance if h3 is counterfactually set as 2.0 sec, the minimum deceleration rate to avoid collision equals to a_{30}=4.3 m/s2 which is still less than maximum available deceleration rate and the collision could have been avoided. But is driver 3 responsible for the collision? The answer is not clear because the collision could also have been prevented if the second driver reacted quicker. For example by setting r_2=2.5 but still keeping u_2=0.15 m/s^2 and a_{30}=2.7 m/s^2 the collision could have been prevented.

This simple example shows that the responsibility of a rear-end collision should be appropriately assigned to a platoon as a whole, rather than to the colliding drivers.

2.2 Chain Reaction Collision

The same causal concept as used to explain a single rear-end collision, is also appropriate in explaining a chain reaction accident. For instance in a three-car platoon, suppose that all vehicles are travelling at the speed of 32 m/s with one second time headway in between. Both following drivers are assumed to have 1.5 second reaction time. Platoon dynamics after the first vehicle (vehicle 1) starts an emergency brake (deceleration at 4 m/s^2) which is illustrated in Figure 2. As the figure depicts, vehicles 2 and 3 both start to brake after a time lag equal to their drivers' reaction time. As a result, vehicle 1 gets hit by vehicle 2 at a distance of 120 m, and subsequently, vehicle 2 is hit by vehicle 3 (i.e. a chain reaction collision happens). The collision could have been prevented if the drivers reacted quickly and exert a higher deceleration rate while braking.

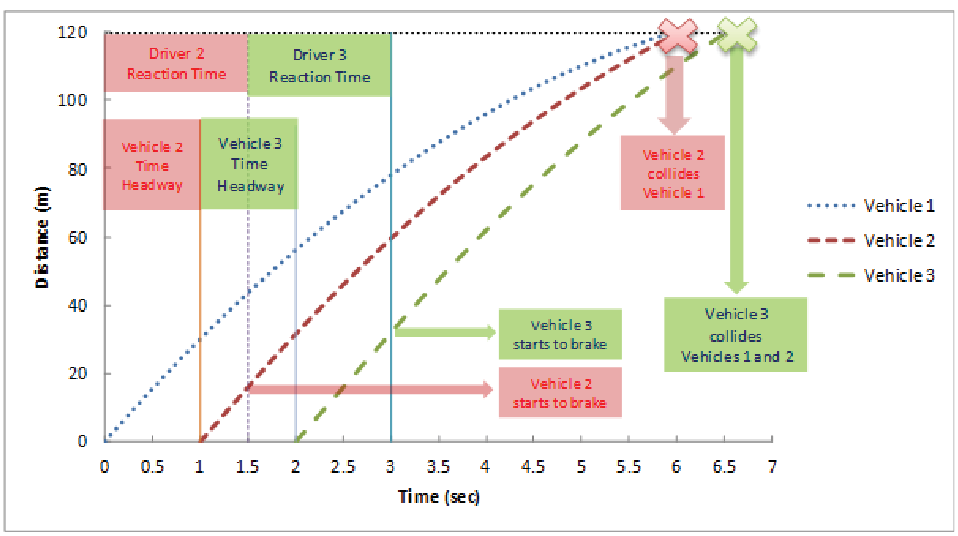

Figure 2. Mechanism of a chain reaction rear-end collision in an example of three vehicles platoon

3. Proposed Methodology

As shown above, safety of a given car-following event in a real world environment is affected by two important variables; the reaction (response) time of the following driver and the available brake power (maximum available deceleration rate) of the following vehicle. Although both variables vary among vehicles and drivers in a platoon, there is no simple way available yet to measure them directly in ordinary observation methods. A better alternative is to consider uncertainty for these variables in calculations.

Based on the suggested causality between the two stochastic variables and the outcome of N-vehicles contribute to a chain reaction collision, we estimate the probability of the potential outcome of a chain reaction collision by comparing the deceleration rate (as a surrogate safety measure) to avoid the collision with the maximum available deceleration rate in the following vehicles. Monte Carlo simulation as an appropriate method for solving problems with uncertainty has been used in estimation. The method represents the solution of a problem as a parameter of a hypothetical population, and using a random sequence of numbers to construct a sample of the population, from which statistical estimates of the parameter can be obtained [James 1980]. Independent observations are generated from the corresponding distributions of stochastic variables (contributing to the outcome) according to the fixed relationship relating the outcome to variables [Ratick and Schwarz 2009]. For each run of the simulation, random numbers drawn from the probability density function (PDF) of stochastic variables are substituted in the fixed relationship to calculate the outcome. If the simulation is performed repeatedly for a large number of times, a probability density function can be developed for the outcome.

Dynamics of movement is analysed frame by frame for each pair of successive vehicles as is summarized in Figure 3. In this figure, to run the sth simulation run in the nth time frame in this study, within any given deployment of vehicles travelling in a platoon on a lane, the driver on leading driver i is assumed to brake his/her vehicle in its maximum deceleration rate available (drawn randomly from the PDF of maximum available brake deceleration rates). The driver on the following vehicle i+1 is assumed to react after a response time, which is also drawn randomly from the PDF of drivers' response time.

Required deceleration rate to avoid collision between the i+1th and the ith vehicles in the sth run of simulation in the nth time frame of movement $a^n_{i+1,s}$ is calculated according to Equation 6 (adopted from Equation 1).

$$a^n_{i+1,s} = \frac{(v^n_{i+1,s})^2}{(v^n_{i,s})^2 / \tilde{a}^n_{i,s}) + 2 \times (x^n_{i+1,s} - v^n_{i+1,s} \times r^n_{i+1,s})} \tag{6}$$

In this Equation:

$v^n_{i,s}$: Speed of the i^{th} vehicle in the s^{th} run of simulation in the nth time frame,

$x^n_{i,s}$: Space gap between the i^{th} and the $i\text{-}1^{th}$ vehicles in the s^{th} run of simulation in the nth time frame,

$r^n_{i,s}$: Response time of the i^{th} vehicle in the s^{th} run of simulation in the n^{th} time frame assigned randomly from PDF, and

$\tilde{a}^n_{i,s}$: Maximum deceleration rate available in the i^{th} vehicle in the s^{th} run of simulation in the nth time frame assigned randomly from PDF.

The above deceleration rate is then compared to the maximum deceleration rate available in the i+1th vehicle. Safety of the event then can be measured in a bivariate scale, i.e. safe if the required deceleration rate is less than the available maximum deceleration rate and unsafe, otherwise. The same calculations can be repeated for the whole individual events of car following to the downstream. If the simulation is ran repeatedly, the probability that a particular vehicle (in a particular deployment in the platoon) is involved in a rear-end collision can be estimated for each time frame according to Equation 7.

$$P_i^n = \frac{\sum_{s=1}^{S} Z_{i,s}^n}{S} \quad \text{i= 1, …, K-1 , n= 1, …, N} \qquad (7)$$

In this equation,

P_i^n : Probability that vehicle i is hit by vehicle i+1in the nth time frame,

$Z_{i,s}^n$: Outcome in following vehicle i by vehicle i+1 in the nth time frame (0 if no conflict is detected between the two successive vehicles and 1 otherwise),

i: Vehicle number in platoon counter (i=1, 2, ..., K),
K: Number of vehicles in the platoon,
s: Simulation run counter (s=1, 2, …, S),
S: Number of simulation runs,
n: Time frame counter (n=1, 2, …,),
N: is the number of time frames included in the analysis

Furthermore, the probability of more than two vehicles being involved in the same rear-end collision (a chain reaction collision) can also be estimated according to Equation 8.

$$P_{i,i'}^n = \frac{\sum_{s=1}^{S} (Z_{i,s}^n \mid Z_{i,s}^n = 1, ..., Z_{i'-i,s}^n = 1)}{S} \qquad (8)$$

where:

$P_{i,i'}^n$: Probability that successive vehicles i to i' (i'>i) are involved in an i'-i+1 vehicle chain reaction collision in the nth time frame,

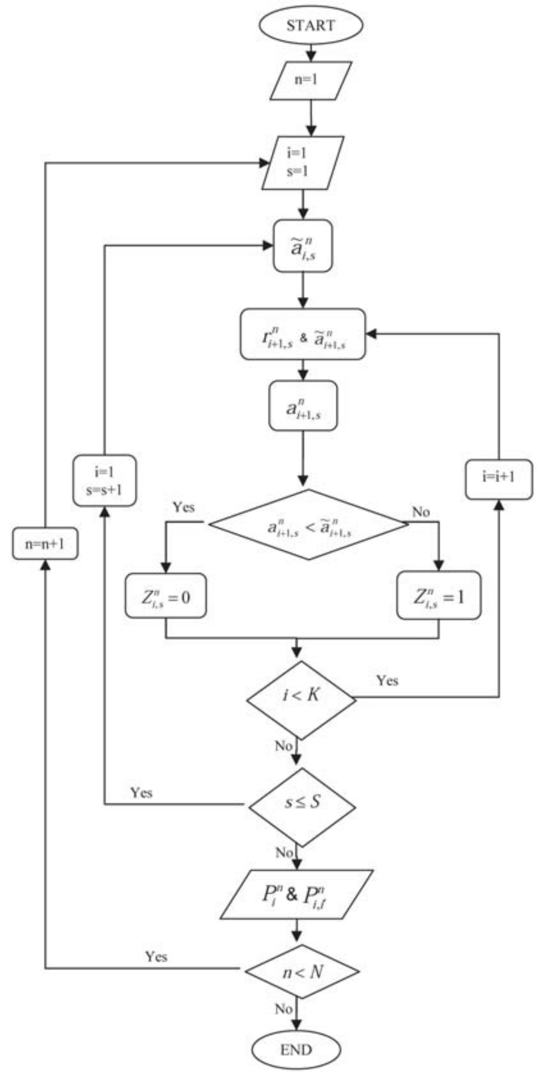

Figure 3. Proposed methodology flowchart for probability of potential single and chain reaction rear-end collision

4. Trajectory Data

Vehicular trajectory data have been analysed in this study to achieve the PDF for stochastic variable of drivers' response time. The ability of different data gathering techniques in obtaining accurate trajectory data (from using cameras attached to aerial platforms and application of Global Positioning System (GPS) to extract the attributes of a moving object to the use of mounted internet protocol (IP) based cameras at fixed and elevated locations) is vital in getting correct inferences from the analysis. For instance, research shows that contrasting results (in terms of model calibrations and comparisons against different sets of measurements) can be reported after different data sources (in terms of level of accuracies) are analysed [Brackstone and McDonald 1999]. Since the available data sources in Iran are both limited and lack the accuracy analysis studies, the authors used the database gathered under the Next Generation SIMulation (NGSIM) program in the United States, in order to develop the PDF for drivers' response time. It is argued that the NGSIM datasets are representing the most accurate field data collected to date [FHWA 2006].

The NGSIM program was initiated in 2002 by support from the Federal Highway Administration (FHWA). The program aims to develop a core of open behavioural algorithms, particularly in support of microscopic traffic simulation, with supporting documentation and validation data sets [NGSIM 2013]. In this program, several synchronized video cameras, mounted on top of high buildings adjacent to roadway, record vehicles passing through the study area and then processing of images gives exact vehicle positions and other attributes (including speed and acceleration) on the road section for every tenth of a second [Punzo, Borzacchiello, and Ciuffo 2011].

The Interstate 80 (I–80) freeway (in the San Francisco Bay area in Emeryville, CA) dataset was the first dataset collected in the NGSIM program and was used in data analysis in the present study. The I-80 freeway dataset represents traffic data in micro-level collected during the afternoon peak period (from 4:00 pm to 4:15 and from 5:00 to 5:30 pm) in 45 minutes on an approximately 500 meters length segment consisted of six freeway lanes ["NGSIM - Home" 2013]. The data were collected in 2005 by seven synchronized digital video cameras, mounted on top of a tall building adjacent to the freeway. These cameras recorded all vehicles passing through the study area during the observation period. The study area (including the lanes and the coverage area for each camera) is shown in Figure 4. The dataset consists of detailed vehicular trajectory data (including vehicular longitudinal and lateral positions, speeds, lengths and accelerations/decelerations) for vehicles moving in each lane, obtained every tenth of a second. The database is free to access source and available on NGSIM website ["NGSIM - Home", 2013].

5. Probability Density Functions

Variables of maximum available deceleration rate and response time, were shown in causative relation with the chain reaction rear-end collision. To estimate the potential rear ending collision probability, the probability density function of variables should be available to randomized generation explained in the methodology.

5.1 PDF for the Maximum Available Deceleration

Maximum available deceleration rate has been recommended by AASHTO Green Book to follow a uniform distribution (conservatively assumed to be 3.5 m/s2) in most ordinary vehicles [AASHTO, 2004]. Cunto and Saccomanno assumed that two times of maxi-

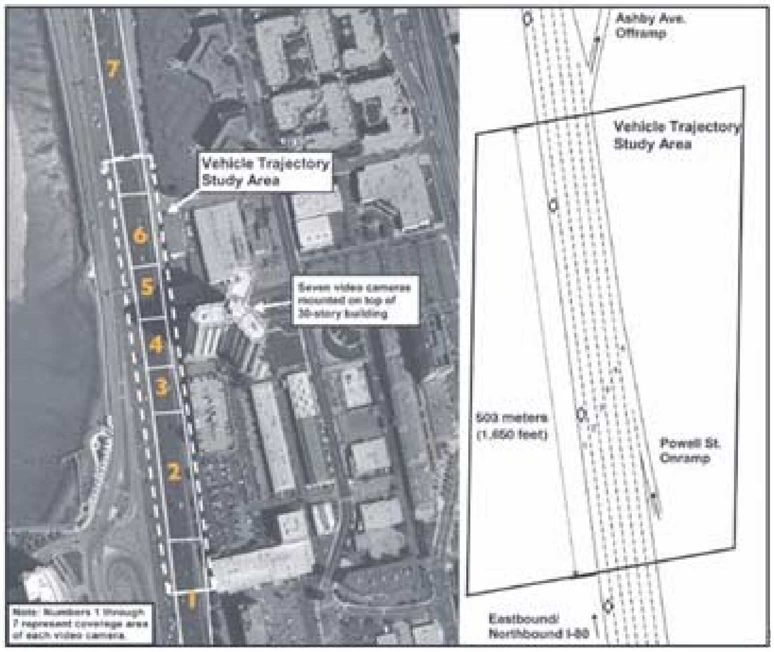

Figure 4. Aerial photograph of the I-80 freeway segments in NGSIM program including each camera coverage area (Left) and the number of lanes and position of on ramp within the I-80 study area (Right) [FHWA, 2006]

mum available deceleration rate follows a truncated normal distribution with an average of 7.42 m/s^2 and standard deviation of 0.24 m/s^2 [Cunto and Saccomanno 2007]. In another study the same authors assumed a normal truncated distribution with an average of 8.45 m/s^2, standard deviation of 1.40 m/s^2 and the upper and lower limits of 12.68 m/s2 and 4.23 m/s^2, respectively for 2 times of the maximum available deceleration rate for small vehicles on dry pavements [Cunto and Saccomanno 2008].

Searching within the literature with the aim to find the appropriate deceleration rate available in Iranian vehicles achieved no results. The industrial protocols however are not considerably different as the car industries follow relatively the same globally accepted standards in brake systems. Therefore, the PDF for maximum deceleration rate available in vehicles in this study is assumed to follow the latter distribution case adopted by Cunto and Saccomanno.

5.2 Developing PDF for Drivers' Response Time

In almost all car-following models drivers' response time is an important component [Brackstone and McDonald 1999]. In car following models the response of a driver in a following event is a function of the stimulus that driver receives from the leading vehicle. Although the first model adopted from this concept dates back to the 60 century, the general form of more recent car following equations has been remained almost the same (Equation 9).

$$a_n(t+T) = \lambda \Delta V_n(t) \tag{9}$$

In this equation $a_n(t+T)$ is the response of the following vehicle in terms of its deceleration rate delayed by a response time T. The stimulus causing this response is the relative speed of the following vehicle with respect to the leading vehicle $\Delta V_n(t)$. This equation assumes the following vehicle response to be proportionate to the stimulus (with proportionality factor of λ termed also as sensitivity factor).

The strong correlation between the stimuli (by the leading vehicle) and response (by the following vehicle) as a measure of goodness-of-fit for car following models has been argued by several researchers. Therefore, an extensive body of research has been focused on model verification and refinement of the car following models in similar forms (e.g. see [Brackstone and McDonald 1999]).

In almost all car-following models the delay term for human response time is common. The response time may not only be different for a single driver in different situations (inter-personal variation), but also it may be different from driver to driver (intra-personal variation). This is why a wide range of values for reaction time is reported in the literature (between 0.5 to 3.9 seconds). In some research the response time has been considered related to other parameters (e.g. spacing, speed and leaders acceleration [Ozaki, 1993] and traffic density [Del Castillo, Pintado and Benitez, 1994].

To adopt the PDF of the drivers' response time for Monte Carlo simulation in the present study, a graphical multistep method was applied on vehicular trajectory data described in the previous subsection. This method estimates the reaction time as the time lag that produces the maximum correlation between the stimuli (in terms of the relative speed of the following and leading vehicles) and the response by the following driver (in terms of the deceleration or acceleration rate of the following vehicle). The method has been also explained and used widely in previous work (e.g. [Ranjitkar and Nakatsuji 2010]. The full procedure to measure the instantaneous response time can be understood from Figure 5. As illustrated, using the trajectory data, stimuli and response trend over time both have been shown on the same graph. In this study

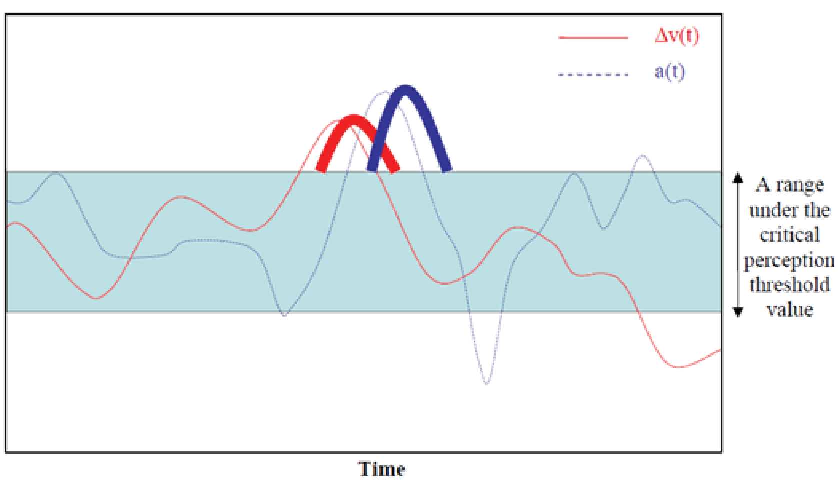

Figure 5. Schematic change in relative speed of leading and following vehicles and acceleration of the following vehicle in time to illustrate estimation of instantaneous response time in a car following [Ranjitkar and Nakatsuji, 2010]

the data is available every 0.1 second.

The instantaneous response time can be measured through the three proposed steps below:

- Detection of peak points on relative speed curve.
- Checking perceptual threshold values: Only stimuli that exceed a critical perceptual value are used to estimate the response time. That is because small amounts of stimuli are not recognized or even may not be clearly perceived by the following vehicle to proportionately response. Under the critical perception values of the stimuli is shown in Figure 5 in a dark zone, the data points in this region should be excluded from the analysis. The authors used 0.8 m/s as critical threshold value for the relative speed in this study, corresponding to what proposed in [Ranjitkar and Nakatsuji 2010].
- Search for response: Correlation between a range of data points (corresponding to 3.5 seconds in this study) on relative speed curve on ascending branch of the curve adjoining to the curve's relative maximum point and the corresponding points on acceleration values is measured, considering different amounts of time delays. The time lag value that delivers maximum correlation between the relative speed and acceleration is considered as the response time for acceleration. The same calculations can be repeated between the data points on descending branch of the relative speed curve connecting to relative minimum point and the points on deceleration curve in order to measure the instantaneous response time in deceleration.

Traffic flow was constituted of vehicles following each other. Altogether 1534 pais of vehicles following each other (following events) were recognized in the traffic flow. For each following event, a vehicle was running behind a leading vehicle on the same lane and direction, regardless of the speed and the time gap. The explained stepwise method was applied on micro level data for each following event and the corresponding maximum correlation between the stimuli and the response as well as the time lag (response time) were calculated.

From the calculated maximum correlations, some records were still very small and should be excluded from the process of developing the PDF. To select valid data, only response times corresponding to a correlation greater than a threshold were taken into the analysis. To establish the threshold, the distribution function of the response time was set as lognormal (Equation 10) as is proposed by many previous researchers [Wu et al. 2009].

$$f(x;\mu,\sigma)=\frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}} \quad , \quad x>0 \tag{10}$$

The equation returns the probability that a random variable takes the x value. μ and σ in this equation are also the mean and standard deviation of probability distribution function, respectively.

The correlation threshold is the least amount of correlations by which the chi square is still statistically significant at 95 percent confidence level in the lognormal distribution fitted to the points. The distributions corresponding to amounts of correlation threshold is depicted in Table 1. As the table shows, 0.6 is a suitable threshold for R^2. The observations and the distribution curve fitted are shown in Figure 6.

6. Application of Simulation in Real

Table 1. Estimation* results of response time fit to lognormal distribution function**

R^2 Threshold	Sample Size	Chi Square Value	Critical Chi Square
0.40	428	90.56	15.51
0.45	348	69.00	15.51
0.50	279	43.59	15.51
0.55	215	14.79	14.07
0.6	171	10.08	14.07*

* P<0.05

** the stimulus is the deceleration by the leading vehicle

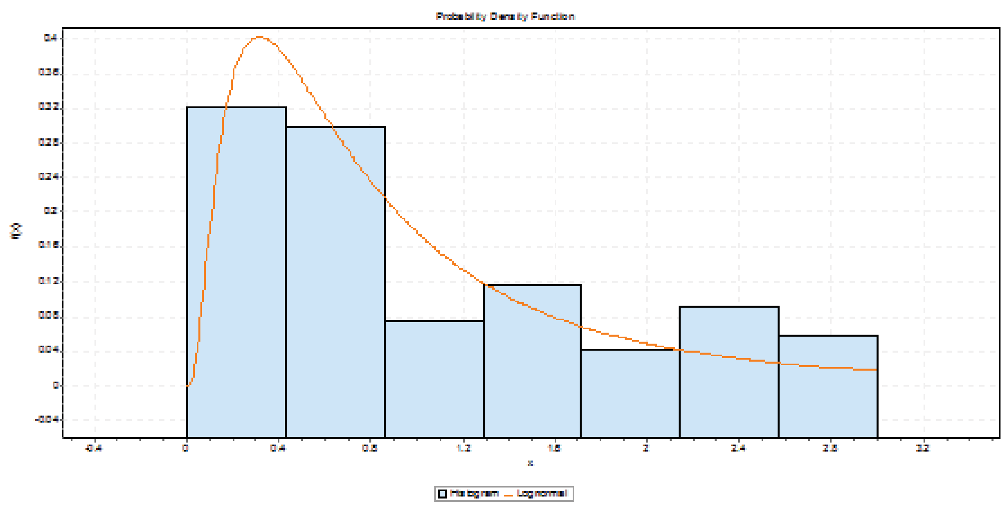

Figure 6. Distribution of following vehicles instantaneous response time, with the stimulus being deceleration of the leading vehicle

Time Estimation of Potential Rear-End Collision

To apply the Monte Carlo simulation to solve the problem of estimating rear-end potential collision risk, the trajectory data related to each frame of the video, including relative position of vehicles following each other, vehicles' lengths and vehicles' speed were substituted into the Equation 1. For each run of the simulation, drivers' response time and vehicles' maximum available deceleration rate were drawn from the corresponding PDFs. The risk of rear end collision was estimated by dividing the total number of potential collisions according to equation 1 by the total number of simulation runs as illustrated in Figure 3. In this way, the cases in which more than two vehicles are involved in rear-ending collisions are rendered by the simulation and thus the probability of N vehicles get involved in chain reaction collision can be determined. If the simulation runs frame by frame, the instantaneous risk values of N-vehicle rear-ending collision can be determined.

For example according to the data, during

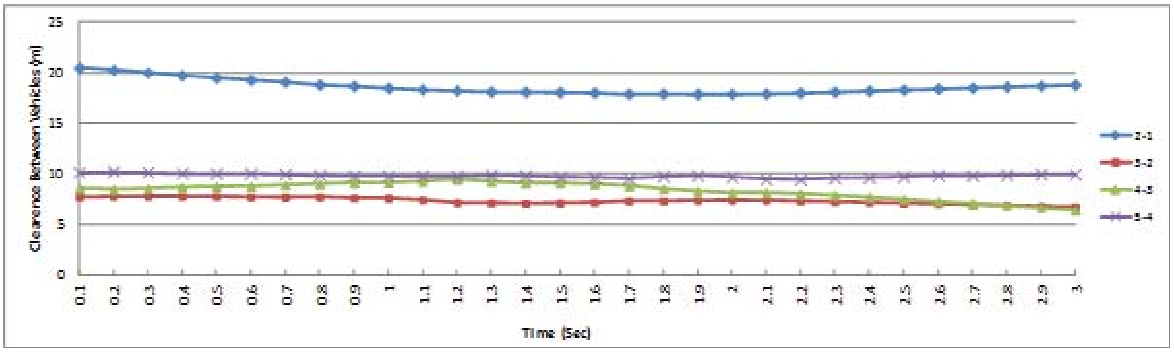

Figure 7. Clearance between successive vehicles during a three-second period movement

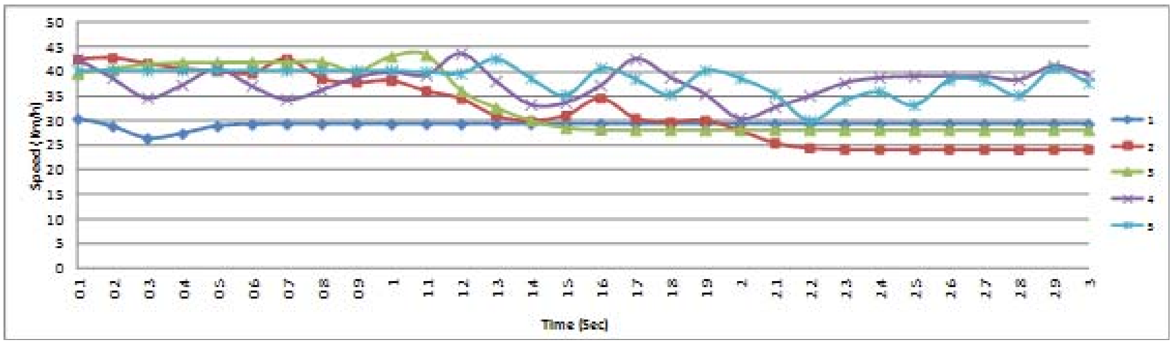

Figure 8. Speed of successive vehicles during a three-second period movement

frames 9607 to 9636 (corresponding to 3.0 seconds of movement) on the fourth lane, private vehicles 1996, 2006, 2012 and 2016 numbered from the first to last in the platoon (corresponding to vehicles 1 to 5 respectively hereinafter) have been detected in car following situation. Clearance between each pair of vehicles and speed of individual vehicles are shown in Figures 7 and 8, respectively.

Monte Carlo simulation was applied for 20,000 runs for every frame and the probability of N-vehicle collision (N is an integer varying between 2 to 5) was estimated. The number of runs was selected in such a way that for more runs, the calculated probabilities are not substantially different.

The rear-end collision risks for N-vehicle collision is depicted in Figure 9. As this figure shows, the probability of N vehicles being involved in a single collision decreases with increasing N. According to Figure 7, the clearance between vehicle #1 and its follower is substantially higher than the clearance between remaining vehicles. Therefore, vehicle 1 in the platoon has potentially a lower probability of being involved in a collision with its followers. This is shown in Figure 9 as the values in curve related to 5-vehicle collision gradually approach to zero.

On the other hand, vehicles 2022, 2016 and 2012 have almost similar speeds and very short distance in between, particularly in the last frames. Therefore, the probability curve

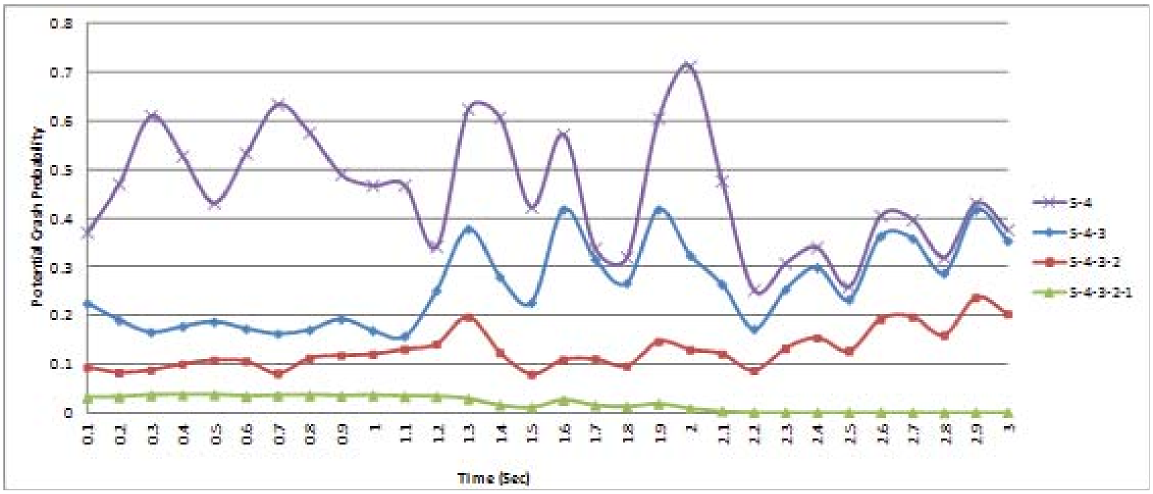

Figure 9. Real time potential collision probability for N-Vahicles (N=2 to 5) in a chain reaction rear-end collision occurs between vehicles 1 to 5 during a three second period of movement

for 2 and 3 vehicle collisions are following the same trends.

7. Summary and Conclusions

The main objective of this research is to develop a method for estimating the potential collision probability of N vehicles in successive rear-end collisions (chain reaction collision) on a freeway. A novel simple methodology was proposed based on Monte Carlo simulation, by taking the uncertainty of two variables namely the maximum available braking power in vehicles and the drivers' reaction time, into estimation. Because of the stochastic nature of the variables, they cannot be directly measured by ordinary simple instrumentation of freeway segments. Given the probability density functions (PDFs) of variables, the Monte Carlo simulation of car-following events was used to measure the probability of N vehicle rear-end collision occurrence.

To show the feasibility of developed methodology in practical applications, the proposed methodology was applied on the case of four successive following events in a five vehicle platoon, drawn from the NGSIM database. Results showed that the probability of N vehicles to collide at the same time increases with decreasing N. This is in line with intuitive expectations. The application also verified that N-vehicle rear-end collision is directly related to the clearance between the following vehicles within the platoon and the speed of individual vehicles as well as the drivers reaction time and the maximum deceleration rate available in individual vehicles. Moreover all drivers in the platoon were recognized as responsible for preventing rear-ending collisions; braking sharply or very slowly may cause collisions for other drivers in the platoon.

The current algorithms used in collision warning/avoidance systems usually isolate the following vehicle and its leader, without considering the remaining vehicles in the platoon. As shown, however, a driver on a third vehicle may be recognized as the responsible for collision without being involved in it. The proposed methodology is able to consider dynamics of all vehicles in the platoon and a more reliable potential collision risk as developed in this study, can be used to alarm the drivers about the possibility of being involved in a rear-end collision. Therefore following the recent advances in hardware, software, and communication technologies the proposed methodology may enable researchers to design and implement dynamic road traffic safety management systems.

For example networks that connect and control for moving vehicles (VANET) can benefit from the proposed methodology in detecting prevailing conditions leading to a chain reaction crash. The algorithm detects the vehicles in poor operation from a traffic safety perspective in a platoon and preset alarms and even intrusions can be timely activated upon an N vehicle crash risk is detected. Therefore, the authors believe that the proposed methodology can be effectively used in developing intelligent warning systems.

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