Rock Slope Stability Analysis Using Discrete Element Method

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Abstract

Rock slope stability depends very much on the strength features of the rock and the geometrical and strength characteristics of the discontinuities (e.g., roughness, wall strength and persistence). Since a rock mass is not a continuum, its behavior is dominated by such discontinuities as faults, joints and bedding planes. Also, Rock slope instability is a major hazard for human activities and often causes economic losses, property damage (maintenance costs), as well as injuries or fatalities. A computer program has been developed in this research study to perform the stability analysis of a rock slope using the Discrete Element Method (DEM). The rock in the present model is treated as some blocks connected together by elasto-plastic Winkler springs. This method, the formulation of which satisfies all equilibrium and compatibility conditions, considers the progressive failure and is able to find the slip surface or unstable blocks. To demonstrate the applicability and usefulness of the method, several examples have been presented for the analysis and optimization of the rock slope stabilization.

Keywords: Rock slope stability, discrete element method, limit equilibrium

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1. Introduction

Rock slope stability depends very much on the strength features of the rock and the geometrical and strength characteristics of the discontinuities (e.g., roughness, wall strength and persistence) [Lin et al. 2012]. Since a rock mass is not a continuum, its behavior is dominated by such discontinuities as faults, joints and bedding planes. In general, discontinuities (presence/absence) have profound influence on the stability of rock slopes and their behavior plays a critical part in a stability evaluation. Several authors have used the numerical discontinuum modeling method to analyze slope stability problems, [Cundall, 1987]. Easki et al. (1999) used the above method and constructed a model of a natural slope to observe the instabilities caused by excavations near the toe [Easki et al. 1999]. Zhang et al. (1997) carried out studies on the dynamic behavior of a 120 m-high rock slope of China's Three Gorges Dam ship lock using the discrete element model. They found good agreement between the numerical results and the field measurements of the residual displacements of the rock slope during the excavation unloading stage [Zhang et al. 1997]. Heuze et al. (1990) illustrated the usefulness of the discrete element approach for the analysis of rock mass mechanical behavior during wave propagation due to seismic events or rock blasting and concluded that although continuum codes are quite useful in simulating some ground shock effects, they are not adequate for representing dynamic block motion processes[Heuze et al. 1990].

It is to be emphasized that in all the above cases, use has been made of the Coulomb-slip constitutive model for joints deformations because it has been a common practice to use C and φ parameters to model the behavior of discontinuities by a linear Coulomb relation.

Discrete Element Method (DEM) can be used for the numerical analyses of different geotechnical problems too. It was presented first by C. S. Chang (1991), (1992), and (1994), as a new concept to investigate the bearing capacity of foundations and stability of slopes and retaining walls. Kim et al. (1997) used DEM and analyzed nailed earth slopes [Kim et al., 1997]. Kveldsvik et al. (2009) explained the static and dynamic loading conditions in DEM code for high rock slopes [Kveldsvik et al., 2009]. Rathod et al. (2011) used DEM for the analysis of static and dynamic response of dam abutments with a liner Coulomb slip constitutive model [Rathod, Shrivastava and Rao, 2011]. Lin et al. (2012) conducted the dynamic analysis of rock slope based on practical seismic load and performed collapse analysis of the crack development in a rock slope [Lin et al, 2012]. Kainthola et al. (2012) analyzed a 100 m high natural hill slope composed of basalt using the DEM code for dry and saturated conditions [Kainthola et al., 2012]. Stability analysis of Surabhi landslide in the Dehradun and Tehri located in India, was simulated numerically using the distinct element method by Pal et al. [Pal et al. 2012]. Shen and Abbas (2013) developed and applied the random set distinct element method in the stability analysis of a rock slope from China [Shen and Abbas, 2013].

In this method, stresses on all blocks' interfaces are compatible with the deformations and fully satisfy the stress-displacement relationship without any further assumptions. This model, a slight extension of the conventional limit equilibrium analysis, permits a solution that satisfies all equilibrium and compatibility conditions.

2. Discrete Element Method (DEM)

In this method, the rock is modelled by several solid slices connected together with Winkler springs (compression, tension and shear) (Figure 1) to establish a unique bounded system.

Normal springs behave elasto-plastically and induce rotational as well as normal stiffness; they do not yield in compression, but they do in tension cut-offs. Shear springs yield at shear capacity according to Mohr-Coulomb constitutive model (Figure 2). Blocks gravity forces are applied during the analysis.

In each calculation step, while assuming equivalent secant stiffness for the Winkler springs, the load is increased until the spring stresses exceed the allowable values, and when this happens at a certain interface, its local factor of safety is assumed as 1 and the excess stresses are redistributed among the neighboring slices through the iteration process. This continues until the stresses on all interfaces are compatible with the deformations and fully satisfy the stress-displacement relationship. Failure of rock joints shear springs depends on the joint's shear resistance properties.

The authors have applied the method to the stability analysis of rock slopes and developed a computer program with which they have worked out several examples to demonstrate the method's applicability.

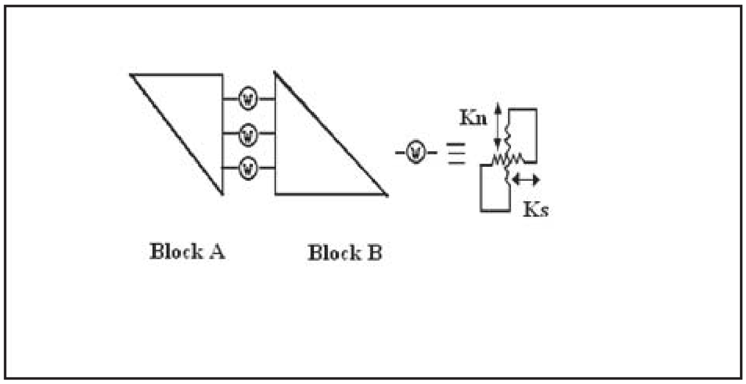

Figure 1. Winkler springs between two adjacent slices

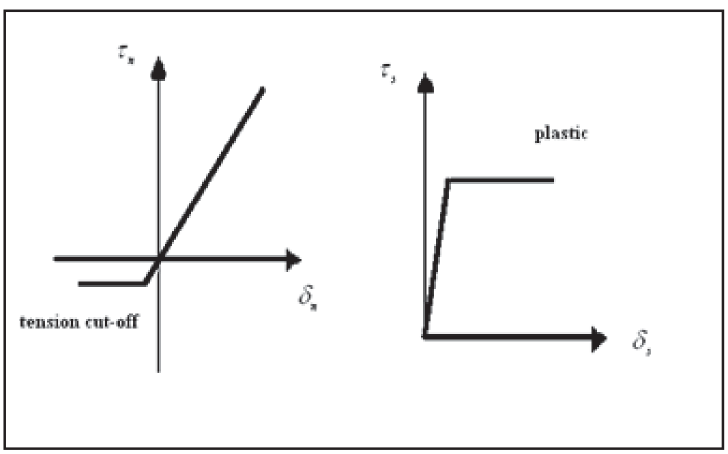

Figure 2. Behaviour of a Winkler spring

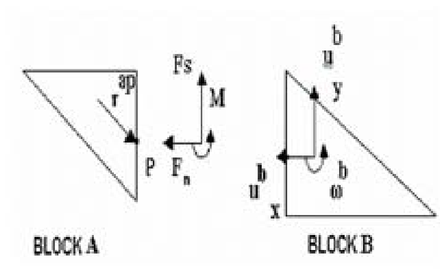

Figure 3. Two adjacent blocks

3. DEM Formulation

Consider two neighbouring blocks A and B (Figure 3). The relative displacement between the slices can be calculated using Eq. (1).

$$\begin{Bmatrix}\Delta_x^p\\ \Delta_y^p\\ \Delta_\omega^p\end{Bmatrix}=\begin{bmatrix}1 & 0 & -r_y^b\\ 0 & 1 & r_y^b\\ 0 & 1 & 1\end{bmatrix}\begin{Bmatrix}u_x^b\\ u_y^b\\ \omega^b\end{Bmatrix}-\begin{bmatrix}1 & 0 & -r_y^p\\ 0 & 1 & r_y^p\\ 0 & 1 & 1\end{bmatrix}\begin{Bmatrix}u_x^a\\ u_y^a\\ \omega^a\end{Bmatrix} \qquad (1)$$

where ua , ub and ωa, ωb are the displacements and rotations of blocks A and B, respectively, P is a point located at the middle of the interface of the two blocks, and r is a vector connecting block A's centre of gravity to point P. If one of the blocks remains immobile, its displacements are set to be zero in Eq. (1).

Vector n^p (cosα , sinα) is defined as an inward unit vector normal to the face of block A at point P wherein α is the angle it makes with the x axis; normal to it is n^p (-sinα , cosα). Now, using Eq. (2) below, the displacement vector on the left side of Eq. (1) can be transformed from the global coordinate system (x-y) to the local coordinate system (n-s).

If k_n and k_s are respectively the normal and shear constants per unit length of the Winkler spring, the interface force between the two blocks can be calculated as follows:

$$\begin{Bmatrix}\Delta_n^p\\ \Delta_s^p\\ \Delta_\omega^p\end{Bmatrix}=\begin{bmatrix}1 & 0 & -r_y^b\\ 0 & 1 & r_y^b\\ 0 & 1 & 1\end{bmatrix}\begin{Bmatrix}\Delta_x^p\\ \Delta_y^p\\ \Delta_\omega^p\end{Bmatrix}=[R]\{\Delta^p\} \qquad (2)$$

$$\begin{Bmatrix}F_n^p\\ F_s^p\\ M^p\end{Bmatrix}=\begin{bmatrix}K_n & 0 & 0\\ 0 & K_s & 0\\ 0 & 0 & K_\omega\end{bmatrix}\begin{Bmatrix}\Delta_n^p\\ \Delta_s^p\\ \Delta_\omega^p\end{Bmatrix}=[K]\{\underline{\Delta}^p\} \qquad (3)$$

where $K_\omega = k_n L^3/12$; $K_s = k_s L$; $K_n = k_n L$ and L is the interface length.

The interface forces in the global coordinates system can be determined using Eq. (4) and the forces acting on all sides of a block can be found using Eq. (5) which is derived from the equilibrium equations.

$$\begin{Bmatrix}\boldsymbol{F_x^p}\\ \boldsymbol{F_y^p}\\ \boldsymbol{M^p}\end{Bmatrix}=\begin{bmatrix}\cos\boldsymbol{\alpha} & -\sin\boldsymbol{\alpha} & 0\\ \sin\boldsymbol{\alpha} & \cos\boldsymbol{\alpha} & 0\\ 0 & 0 & 1\end{bmatrix}\begin{Bmatrix}\boldsymbol{F_n^p}\\ \boldsymbol{F_s^p}\\ \boldsymbol{F^p}\end{Bmatrix}=[\boldsymbol{T}]\{\underline{\boldsymbol{F}}\} \qquad (4)$$

$$\begin{Bmatrix}f_n^a\\ f_s^a\\ m^a\end{Bmatrix}=\begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ r_y^p & -r_y^p & -1\end{bmatrix}\begin{Bmatrix}F_x^p\\ F_y^p\\ M^p\end{Bmatrix} \qquad (5)$$

where f^a_n, f^a_s and m^a are respectively the body forces and the moment acting on a block, and can include gravity, inertia (earthquake) and loading forces.

Combining Equations (1) to (5) will result in Eq. (6) below which shows the relationship between the forces and displacements of a block.

$$\{f^a\} = -\sum [R^a]^T [T]^T [K][T] \left([R^b]\{u^b\} - [R^a]\{u^a\}\right) \quad (6)$$

It is obvious that for N blocks in the analysis, we have 3N equations and 3N unknown variables (f^a_n, f^a_s and m^a for each block).

3.1 Failure of Winkler Springs

For each block, two internal and external load vectors are introduced; the latter is established on the basis of the loads applied to the system and the former is induced in the springs as a consequence of relative displacements between slices.

Computation convergence, a necessity when incremental loading procedure is adopted in each calculation cycle, is accomplished when internal and external load vectors become equal. As mentioned earlier, when a spring fails, its stiffness is changed by the secant method; therefore, the system does not converge in the first iteration. The excess stress, appearing as the difference between the internal and external load vectors, is redistributed among the neighbouring slices. The iterative procedure continues until the stresses in the interfaces of all blocks are compatible with the deformations and fully satisfy the stress-displacement relationship.

3.2 Winkler Spring Stiffness

As shown in Eq. (3), the method requires kn and ks (Winkler spring's stiffness values); Chang [4] showed that they have insignificant effects on the computed results. The ratio (kn/ks) plays an important role (similar to that of the soil's Young and shear moduli) in the analysis and is equal to 2(1+ ν) for isotropic elastic materials (usually ranging from 2 to 3).

4. Computer Program

Based on the formulation described, the authors have developed a 2-D FORTRAN computer program that can analyse different-shape rock slopes with any sets of joints. The program has an interface that shows the slope's joints and the un-deformed shape (the failed joints are in red); so, the progressive failure and the deformed slope and its blocks can be observed in the program interface.

5. Model Verification

Model verification is done by comparing the program results with those of the analytical solution of the simple two-block example. Figure 3 shows a sliding surface that is joined with another flatter surface at the toe of the slope. The strength reserve in the toe (the passive region resting on a relatively flat sliding surface) is overcome by the excess force transmitted from the upper region (the active block that cannot remain at rest by the friction along its basal surface alone).

Analysis of the force system shown in Figure 3 yields:

$$F = \frac{W_1 \sin(\delta_1 - \varphi_1)\cos(\delta_2 - \varphi_2 - \varphi_3) + W_2 \sin(\delta_2 - \varphi_2)\cos(\delta_1 - \varphi_1 - \varphi_3)}{\cos(\delta_2 - \varphi_2)\cos(\delta_1 - \varphi_1 - \varphi_3)} \quad (7)$$

where $\varphi_1, \varphi_2, \varphi_3$ are the friction angles related to slip along the upper, lower and vertical surfaces, respectively, δ_1 and δ_2 are the inclinations of the upper and lower slip surfaces, respectively, W_1 and W_2 are respectively the weights of the active and passive blocks per unit of slip width, F is the thrust required in the passive block to reach limiting equilibrium (stable, if F > 0 and unstable if F < 0).

Assuming $W_1 = 200 \quad t$, $W_2 = 75 \quad t$, $\delta_1 = 37^\circ$, $\delta_2 = 27^\circ$, and $\varphi_3 = 35^\circ$, we modelled the example by the DEM program, compared the results with those of the analytical solutions and found good agreement between them (Figs. 1 and 2). The red colour in each joint indicates that shear failure has occurred in the joint; this gives a good insight on the condition of each joint and helps in optimizing the slope stabilization. Furthermore, the normal and shear stresses in each joint can be defined, and blocks displacements can be observed in the DEM method where stresses in all blocks interfaces are compatible with their deformations and fully satisfy the stress-displacement relationship without any further assumptions.

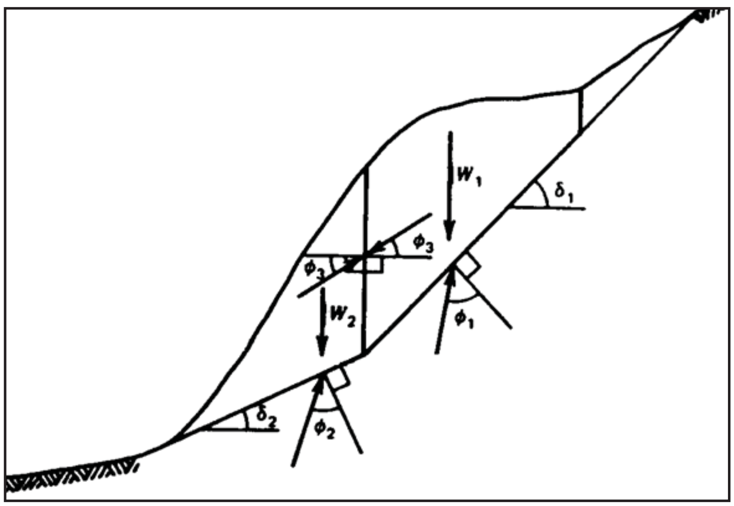

Figure 4. Model for a two-block stability analysis

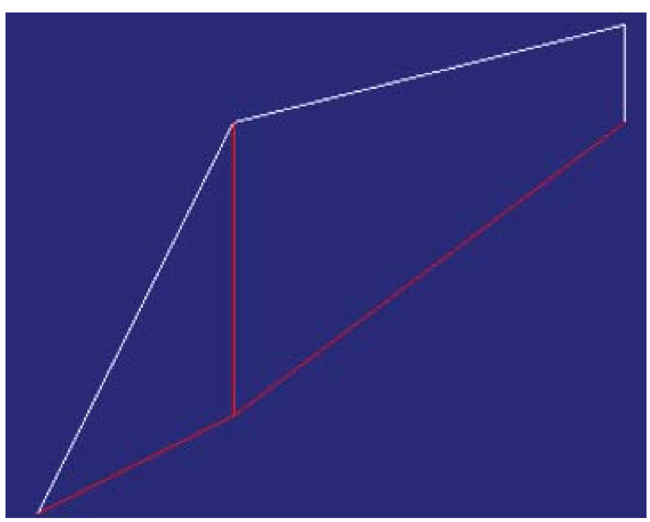

Figure 5. $\varphi_1 = 35^\circ$, $\varphi_2 = 30^\circ$, $F = 2.6 \quad t$ or $\varphi_1 = 35^\circ$, $\varphi_2 = 31^\circ$, $F = 1.2 \quad t$

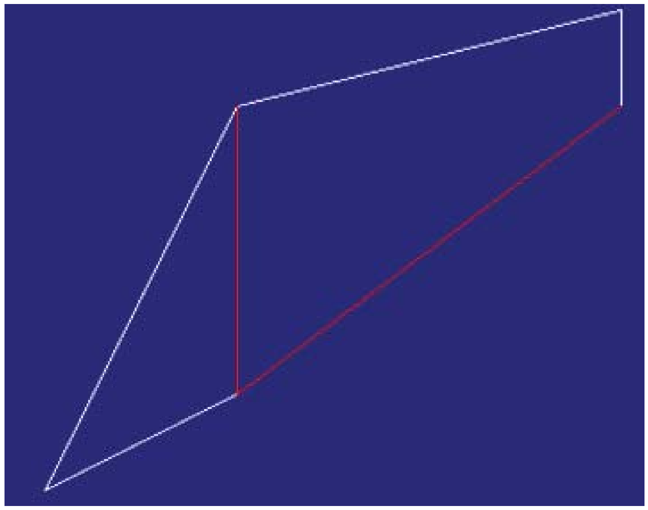

Figure 6. $\varphi_1 = 36^\circ$, $\varphi_2 = 30^\circ$, $F = -.6$ t or $\varphi_1 = 35^\circ$, $\varphi_2 = 32^\circ$, $F = -.2$ t

6. Rock Slope Analysis using the DEM

Results of several examples are presented to show the DEM's applicability for the stability analysis of rock slopes and its usefulness in optimizing the slope stabilization. Examples include the stability analyses of a slope with two sets of perpendicular joints, a slope with inclined layers, and a toppling mode.

6.1 Stability Analysis of a Slope with Two Sets of Perpendicular Joints

This example considers a slope with two sets of perpendicular joints with the following data:

H=5 m c=0 t/m^2 φ=30° γ=2.6 t/m^3

Figure 7 shows the rock slope model and the joints directions. The red colour in each joint indicates that shear failure has occurred at the joint.

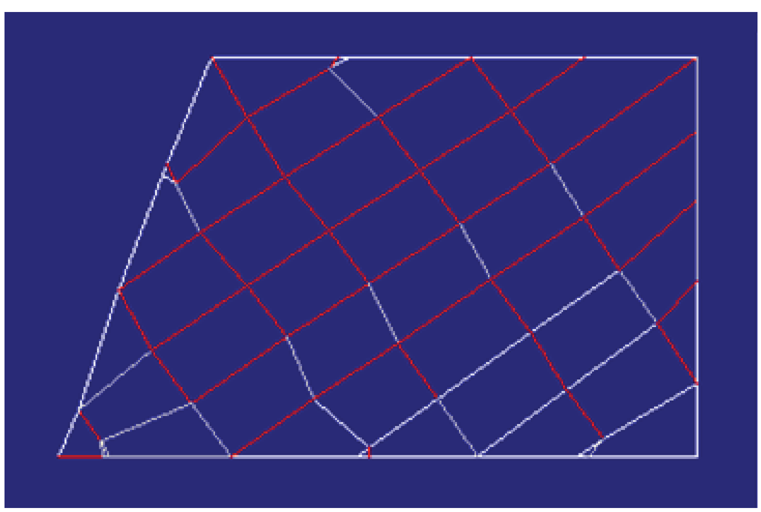

Figure 7. Stability analysis of a slope with two sets of perpendicular joints

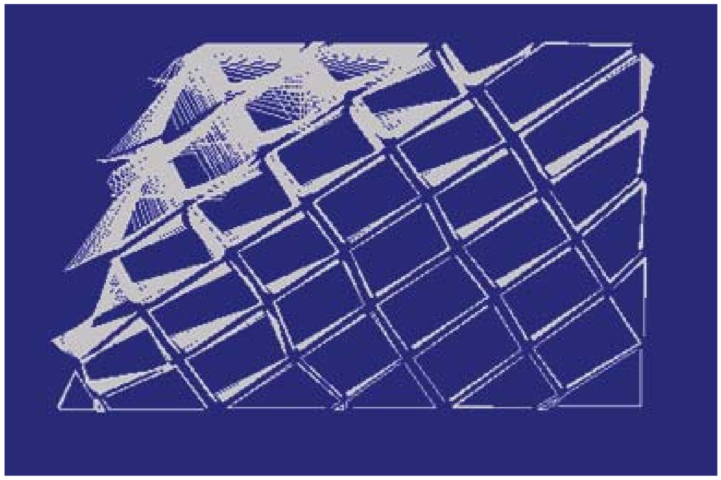

Figure 8. Rock blocks displacements in a slope with two sets of perpendicular joints

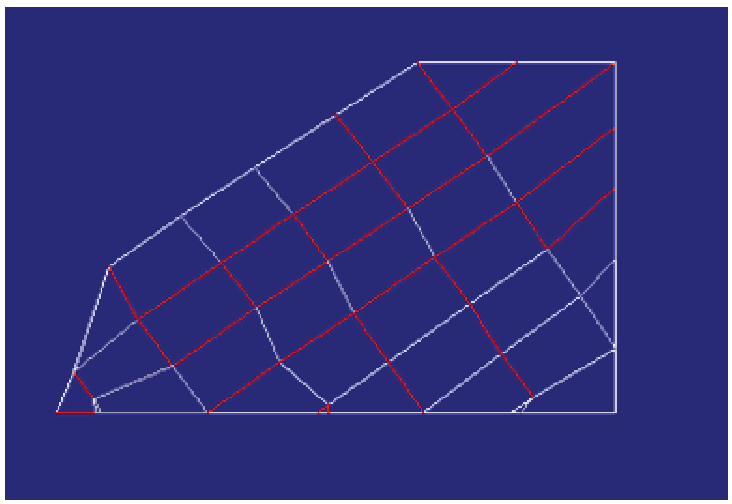

Figure 9. Stability analysis of the slope with the removal of unstable blocks (slope is stable)

Figure 8 shows blocks displacements and indicates that failure has occurred in this example. Figs. 7 and 8 help us decide how we can stabilize the slope by the removal of unstable blocks. The six upper blocks have produced a failure surface and have large displacements, so we can easily decide to remove them and re-examine the slope stability; Figure 9 shows that blocks removal has caused the slope to become stable.

6.2 Stability Analysis of a Slope with Inclined Layers

The rock slope in this analysis is the same as that in the previous section, but with inclined layers. During the analyses, blocks weights were applied in steps. The rock slope model and joints directions are shown in Figure 10 and blocks displacements (in the steps) in Figure 11.

Block-removal stabilization was examined with three examples; the first one (Figure 12) shows that the slope is still unstable, the second one (Figure 13) implies that block removal has caused the slope to become stable, and the third one (Figure 14) shows the effects of a berm on the slope and reveals how applying a berm can increase the stability of the slope. Blocks displacements during several steps in the analyses are shown in Figure 15. Comparison reveals that the slope in Figure 14 needs more block removal than the one in Figure 13 to become stable.

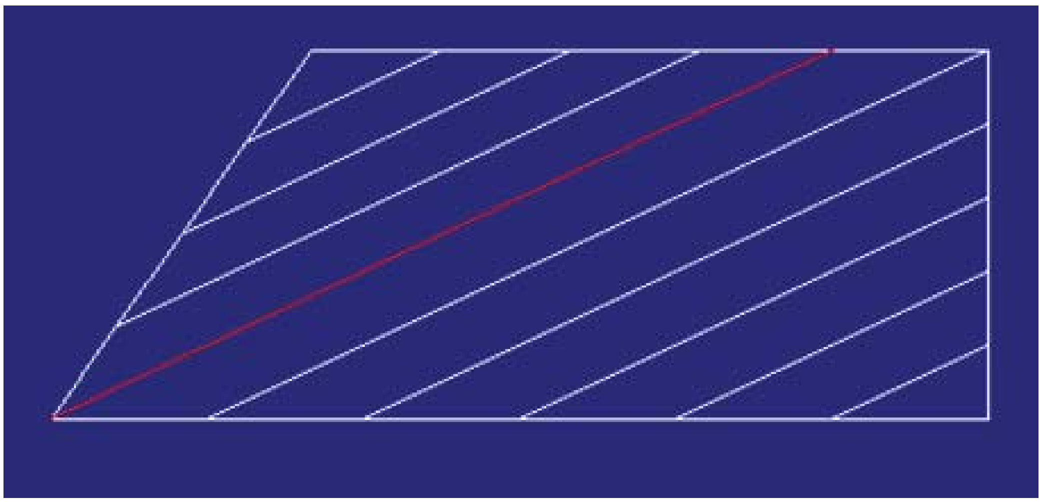

Figure 10. Rock slope model and joints directions (red colour indicates that shear failure has occurred at the joint)

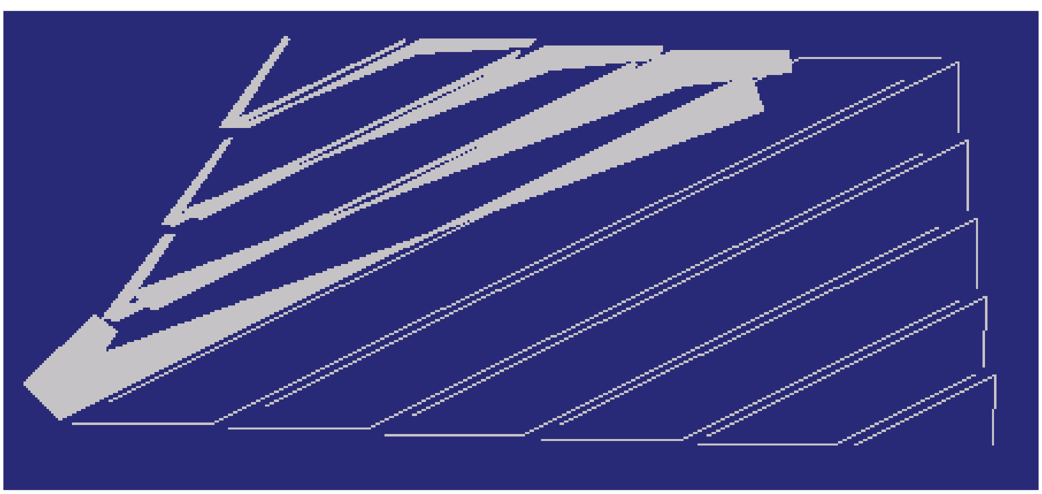

Figure 11. Rock blocks displacements

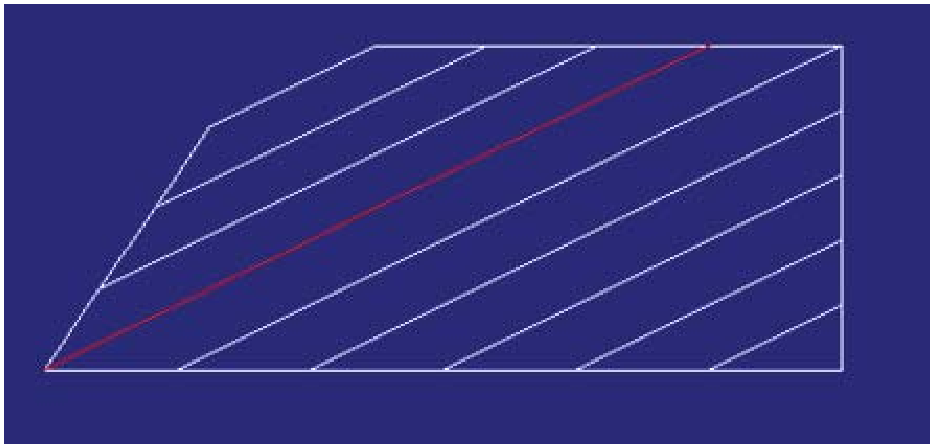

Figure 12. Stability analysis of slope with block removal (the slope is still unstable)

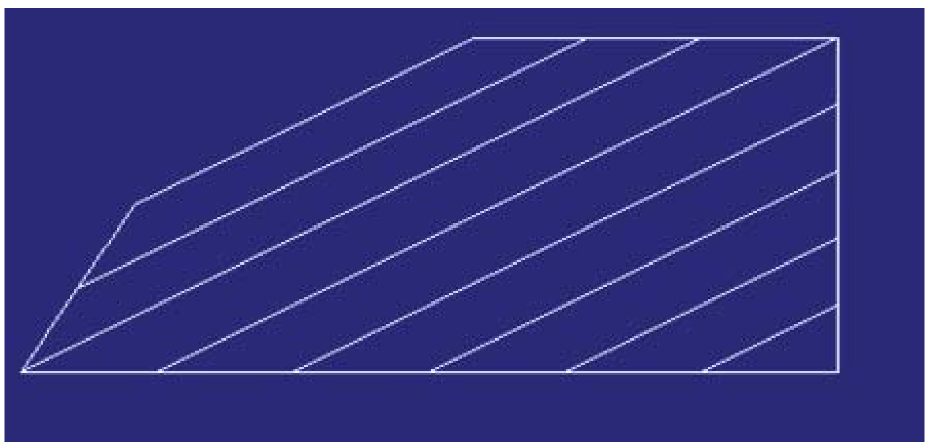

Figure 13. Stability analysis of slope with block removal (the slope is stable)

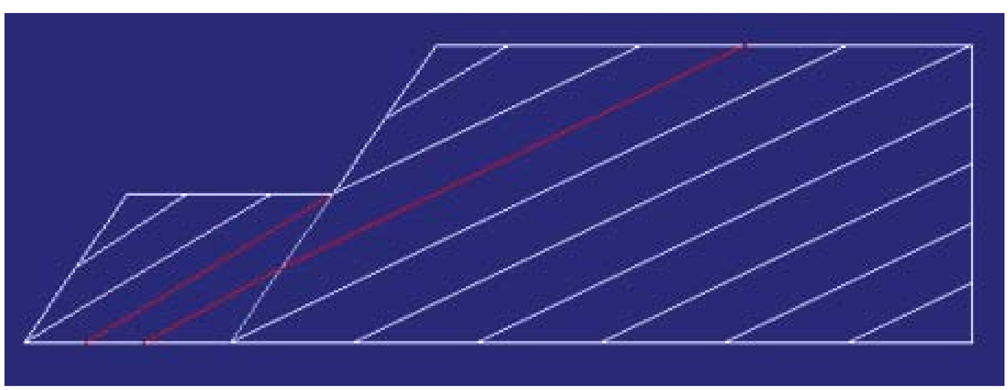

Figure 14. Stability analysis of a bermed slope with inclined layers

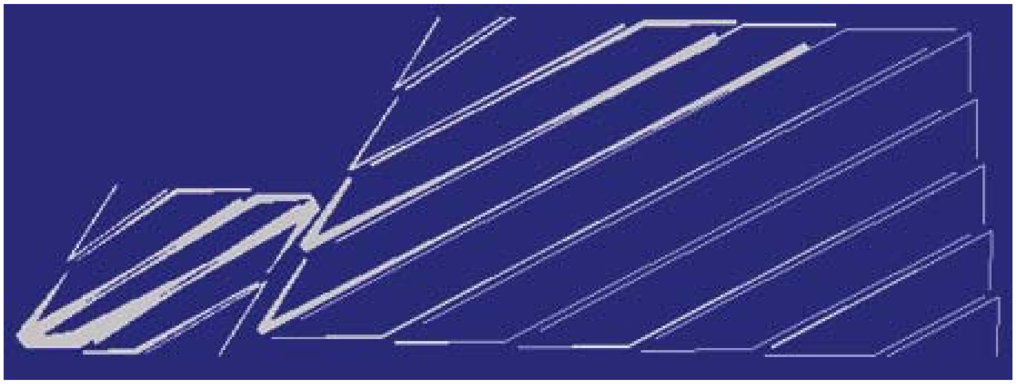

Figure 15. Displacements of rock blocks in a bermed slope with inclined layers

6.3 Toppling Mode Stability Analysis

The rock slope for the modelling of the toppling mode is similar to that in Section 6.1, but with near vertical joints. Figure 16 shows the model and joints directions and Figure 17 shows the blocks displacements and indicates that failure has occurred in this example. It is evident from Figure 16 that block removal cannot stabilize the slope because shear failure has occurred in all the joints.

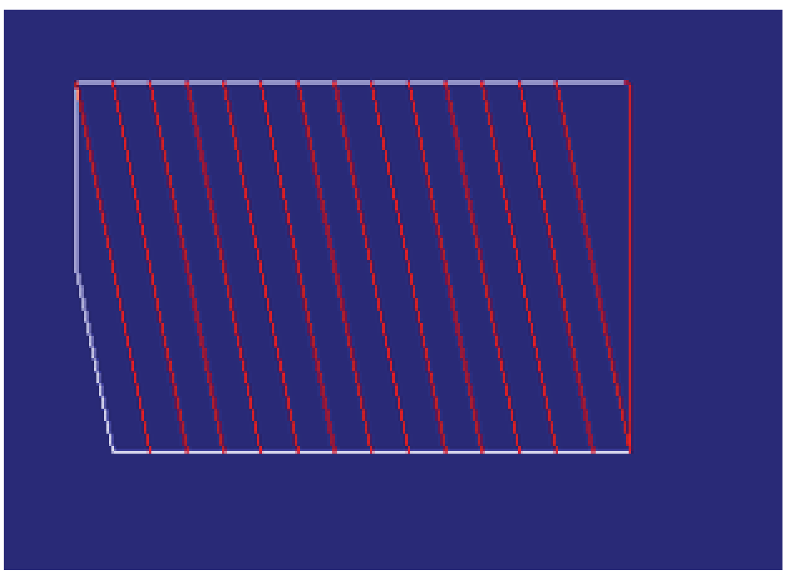

Figure 16. Stability analysis of a toppling mode

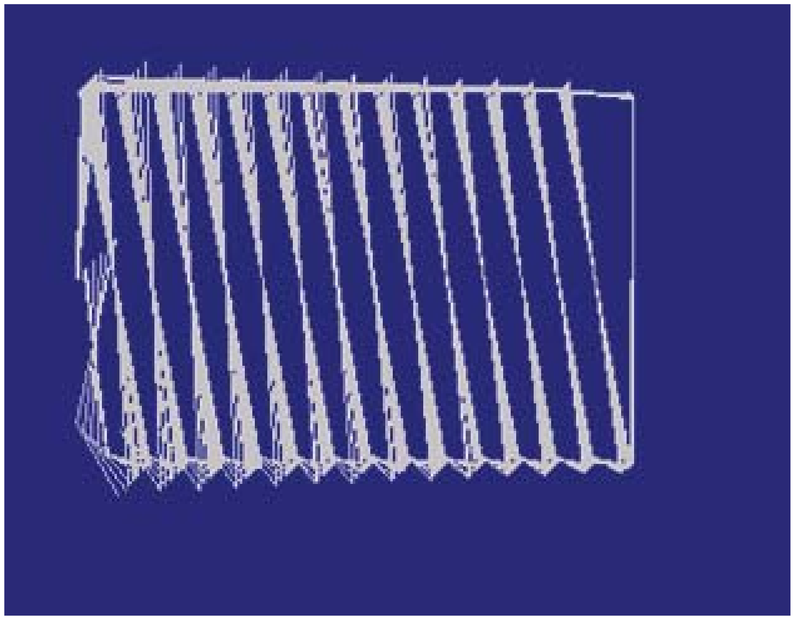

Figure 17. Displacements of rock blocks in the stability analysis of a toppling mode

7. Conclusions

In this study, the Discrete Element Method (DEM) was adopted for the stability analyses of rock slopes. To demonstrate its applicability and usefulness in analyzing and optimizing the slope stabilization, several examples including the stability analyses of a slope with inclined layers, a slope with two sets of perpendicular joints, and a toppling mode have been presented and its advantages over the conventional limit equilibrium method has also been discussed. Progressive failure with which the slip surface or unstable blocks can be defined, is a subject considered in this method; therefore, the method helps in optimizing the rock slope stabilization. The proposed method is theoretically rigorous and simple; it can easily treat such more complicated problems as a slope with inhomogeneous properties and forces acting or foundations resting on a slope.

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