Evolutionary Approach for Energy Minimizing Vehicle Routing Problem with Time Windows and Customers’ Priority

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Abstract

A new model and solution for the energy minimizing vehicle routing problem with time windows (EVRPTW) and customers’ priority is presented in this paper. In this paper unlike prior attempts to minimize cost by minimizing overall traveling distance, the model also incorporates energy minimizing which meets the latest requirements of green logistics. This paper includes the vehicles load as an additional indicator of the cost in addition to the distance traveled cost. Moreover, this paper tries to maximize the customers' satisfaction using their preference and considers the customers' priority for servicing. Every customer is assigned to a group (e.g., very important, important, casual and unimportant) and the customers’ preference is represented as a convex fuzzy number with respect to the satisfaction for service time. The detailed mathematical formulation of proposed model is provided and it is interpreted as multi-objective optimization where, the energy consumed and the total number of vehicles are minimized and the total satisfaction rate of customers is maximized. In general, the relationship between these defined objectives is unknown until the problem is solved in a proper multi-objective manner. Thus, a multi-objective evolutionary algorithm is proposed and its performance on several completely random instances is compared with Non-dominated Sorting Genetic Algorithm II (NSGA II) and CPLEX Solver. The hypervolume indicator is used to evaluate the two Pareto set approximations found by NSGA-II and the proposed approach. The performance proposed evolutionary is further demonstrated through several computational experiments and the results indicate the good quality of the method.

Keywords: Vehicle routing problem, Energy consumption, Customers' priority, Multi-objective, evolutionary algorithm

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1. Introduction

One of the most important tasks in supply chain management (SCM) is transportation of goods or the delivery of services from a supply point to various customers and it often represents the most important costs for most firms. To reduce transportation costs and also to improve customers’ services finding the best routes that a vehicle should follow through a network is frequently an important decision problem. One of the most important and widely studied combinatorial optimization problems in this area is the vehicle routing problem with time windows (VRPTW) as popular NP-hard models [Dondo and Cerda, 2007]. Although there are different forms of VRPTWs, most of them minimize cost by minimizing the overall traveling distance or the traveling time. In fact, it is the amount of fuel or energy consumed, is the greater concern to transportation companies in comparison with the traveled distance and meets the latest requirements of green logistics. Statistics show that energy cost is a significant part of total transportation cost [Xiao et al. 2012]. Therefore, it is important to decrease energy consumption by improving operational efficiency. A reduction in energy consumption also benefits the whole of society since the emission of carbon dioxide can be correspondingly reduced. Thus, this paper studies the proposed model in terms of energy consumption in order to be further representative of real-life situations. Moreover, this paper considers the customers’ priority for service by maximizing the customers’ satisfaction (especially those with higher priorities) and routing according to their preference. In many realistic applications, the concept of classical time windows does not model the preference of customers very well. Even though customers provide a fixed time window for service, they really hope to be served at a desired time if possible. This preference information of customers can be represented as a convex fuzzy number with respect to satisfaction for service time.

Besides, the proposed model in this paper is interpreted as multi-objective optimization problem. In real-life, for instance, there may be several costs associated with a single tour. However, when multiple objectives are identified, these differing objectives are frequently in conflict. For this reason, adopting a multi-objective point of view can be advantageous and the optimization process needs to determine the trade-offs between the objectives, rather than a single solution.

2. Literature Review

2.1 Vehicle Routing Problem with Time Windows (VRPTW)

Due to the NP-hardness of the VRPTW and its wide applicability in real-life situations, optimization techniques (i.e., meta-heuristics), which are capable of producing high quality solutions within a limited amount of time, are of major importance. It is shown that heuristics based on decomposition techniques (e.g., column generation and Lagrangian relaxation) may provide very good quality solutions when sufficient computational time is available [Pepin et al. 2009]. Thus, various heuristic approaches have been developed, ranging from local search methods to methods based on mathematical programming decomposition techniques and meta-heuristics. Applying different meta-heuristics, mainly evolutionary approaches, to
solve the VRPTW can be extensively found in the literature [Chiang and Hsu, 2014; Ghoseiri and Ghannadpour, 2010; Ombuki et al, 2006 and Tan et al. 2006]. In addition, the basic features of each method and experimental results for the benchmark test problems have been presented and analyzed. Other very good techniques and applications of the VRP and its developments can be found in [Alinaghian and Naderipour, 2016; Hosseini-Nasab and Lotfalian, 2017; Ghannadpour et al. 2014; Euchi et al. 2015; Yu et al. 2017; Nikkhah Qamsari et al. 2017; Yu et al. 2016 and Fernández et al. 2018].

2.2 Energy Minimizing VRPTW

In addition to studies on VRPTW and distance traveled, there are several studies on other factors that also impact energy consumption [Lin et al. 2014; Montoya et al. 2016 and Qian and Eglese, 2016]. Tavares et al. [Tavares et al. 2008] took into account the effect of both road inclination and vehicle load on energy consumption in waste collection. In this regard, Bektaş and Laporte [Bektaş and Laporte, 2011] studied the pollution-routing problem (PRP) and considered the tradeoffs between various parameters such as vehicle load, speed and total cost, and offered insight on the economics of "environmental-friendly" vehicle routing. Kara et al. [Kara et al. 2007] proposed a new cost function in terms of energy consumption for CVRP according to the distance and load of the vehicle. The effectiveness of proposed model was analyzed by classical Capacitated VRP instances from the literature using CPLEX 8.0. Minimizing the fuel consumption in VRPs is also considered in [Gaur and Mudgal, 2013] in a similar way and on a well-known heuristic of partitioning the traveling salesman tours and the use of the averaging argument. Moreover, the fuel consumption optimization model for the capacitated VRP was developed by [Xiao et al. 2012]. Based on the results, the fuel consumption could be reduced by 5% on average compared to the classical VRP model. In this regard, [Zhang et al. 2014] introduced a similar model called environmental vehicle routing problem (EVRP) with the aim of reducing the adverse effect on the environment caused by the routing of vehicles and by using a hybrid artificial bee colony algorithm.

2.3 Fuzzy VRPTW

Using the fuzzy approach for VRPTW, [Li et al. 2005] solved the VRP with fuzzy demands using a hybrid differential evolution algorithm. In this study, a fuzzy chance-constrained program model was designed and a hybrid intelligent algorithm was proposed to solve the model. In this regard, [Erbao and Mingyong, 2010] used a similar approach and designed fuzzy chance-constrained program model based on fuzzy credibility theory and applied it for the open VRP. Moreover, [Tanga et al. 2009] proposed and solved a VRP with fuzzy time windows (VRPFTW). Service level issues associated with violation of time windows in a vehicle routing problem were described using fuzzy membership functions, and the concept of fuzzy time windows was proposed. In our paper the proposed fuzzy time windows can reflect the customers’ satisfaction for service and for different kind of customers. Another efficient use of the fuzzy approach in routing networks is related to [Feng and Liao, 2014] who developed a hybrid evolutionary fuzzy learning algorithm that automatically determines the near optimal traveling path in large-scale traveling salesman problems.

2.4 Multi-objective VRPTW

In the multi-objective area, [Tan et al. 2006 and Ombuki et al. 2006] proposed a hybrid multi-objective evolutionary algorithm (MOEA) and the concept of Pareto’s optimality for solving the multi-objective VRPTW and [Lqbal et al. 2015...
and Zhou & Wang, 2015] proposed a multi-objective approach for VRP with soft time windows and with the help of bees. Similarly, [Tan et al. 2007] proposed an approach for the vehicle routing problem with stochastic demand with minimum travel distance, driver remuneration, and number of vehicles. [Ghoseiri and Ghannadpour, 2010] used a direct interpretation of the VRPTW as a multi-objective problem where both the total required fleet size and total traveling distance are minimized. [Sivaram Kumar et al. 2014] solved the multi-objective VRPTW by an approach named fitness aggregated genetic algorithm (FAGA). This method is a genetic algorithm with fitness aggregation approach and specialized operators. Moreover, [Garcia-Najera et al. 2015] have proposed an improved multi-objective evolutionary algorithm (MOEA) for solving the VRP with backhauls that simultaneously minimizes any number of objectives.

Finally, this paper makes the following major research contributions:

- We develop a new objective function for energy minimizing VRPTW by including the load of the vehicles as an additional indicator of the cost in addition to the distance traveled cost.
- We involve the routing model (energy minimizing VRPTW) to the preferences of customers and attempt to maximize the satisfaction level of customers (specially the high-priority customers).
- We formulate the proposed model as a multi-objective problem.
- We develop a multi-objective problem based on an evolutionary algorithm where, the energy consumed by the vehicles and the total number of vehicles used to serve the customers are minimized and the total satisfaction rate of customers is maximized.

The remainder of this paper is organized as follows. Section 3 defines the model description. The structure of the solution technique is discussed in Section 4. Section 5 describes the computational experiments carried out to investigate the performance of the proposed method, and finally Section 6 provides concluding remarks.

3. Model Description

The problem considered here is an energy minimizing vehicle routing and scheduling problem with customers’ priority as a multi-objective optimization. Before explaining the proposed model, it is necessary to describe the classical concept of vehicle routing problem with time windows (VRPTW). VRPTW is given by a special node called depot, a set of customer $C = \{0, 1, 2, \ldots, N\}$ to be visited and a directed network connecting the depot and the customers. Also a set of fleet $V = \{1, 2, \ldots, K\}$ located at the depot is available. They must leave from and return to the central depot. Each vehicle has a limited capacity and each customer has a predefined demand. A distance $d_{ij}$ and travel time $t_{ij}$ are associated with each arc of the network. On the other hand, any customer $i$ must be serviced within a pre-defined time interval$[e_i, l_i]$. Vehicles arriving earlier than the earliest arrival time incur waiting ($w_i$ is the waiting time at node $i$). Each vehicle $k$ is also supposed to complete its individual route within the total route time ($r_k$) which is essentially the time window of the depot. The model has two types of decision variables. The objective of the classical VRPTW is to serve the customers such
that the total distance traveled by the vehicles is minimized. The typical output of this problem is illustrated in Figure 1.

![Figure 1. Typical output of VRPTW](image)

All the notations and decision variables used in this paper are defined as follows:

- $C$: Set of customers as $\{0, 1, \ldots, N\}$; depot is denoted as customer 0
- $C_C$: Set of casual customers
- $C_I$: Set of important customers
- $N + 1$: Number of customers
- $i$: Index of each customer ($i \in C$)
- $j$: Index of each customer ($j \in C$)
- $V$: Set of vehicles as $\{1, \ldots, K\}$
- $k$: Index of vehicle ($k \in V$)
- $q_k$: Capacity of vehicle $k$
- $m_i$: Demand of customer $i$
- $f_i$: Service time at customer $i$
- $d_{ij}$: Distance travel between customers $i$ and $j$
- $t_{ij}$: Travel time between customers $i$ and $j$
- $\theta_{ij}$: Slope of the route between customers $i$ and $j$
- $e_i$: Earliest arrival time at customer $i$
- $l_i$: Latest arrival time at customer $i$
- $u_i$: Desired time of customer $i$ to service
- $\mu_i$: Membership function of customer $i$
- $w_i$: Waiting time at customer $i$
- $PR_i$: Importance degree of customer $i$
- $r_k$: Total route time of vehicle $k$
- $y_i$: Control variable for each customer $i$
- $\mu_{ij}$: Coefficient of friction on link $(i, j)$
- $u_i$: Load of vehicle upon leaving customer $i$
- $u_{ij}^k$: Load of vehicle $k$ when moves from customer $i$ to customer $j$
- $tare_k$: Weight of the empty vehicle $k$
- $S$: A feasible solution
- $M$: Very large number
- $\delta$: Constant violation value from hard time windows
- $x_{ijk}$: Is equal to 1 if vehicle $k$ drives from vertex $i$ to vertex $j$ and 0 otherwise ($i \neq j, i, j \neq 0$)
- $at_i$: Arrival time at customer $i$
- $t_i$: Start time of service for each customer $i$

### 3.1 Energy Minimizing VRPTW

It has been recognized that the real cost of a vehicle traveling along a route depends on many factors. These include the load of vehicles, fuel consumption per mile (kilometer), fuel price, time spent or distance traveled up to a given node, time spent visiting all customers, total distance traveled, depreciation of tiers and the vehicles, maintenance, driver wages, etc. Most of the mentioned factors are actually distance or time base and can be approximated by the distance. However, some variables cannot be considered as the distance between nodes or involve travel costs that may not be taken as constant. These types of variables may be represented as a function of the flow, especially, as a function of the load of vehicles on the corresponding arc. Although energy consumption is largely determined by distance, other factors such as load also have a considerable impact on fuel costs. In other words, if the other factors are kept constant, the energy consumption then mainly depends on distance and load. For example, the energy consumption of an empty vehicle is always less than the cost of a fully loaded vehicle when traveling along a specific route at the same speed.
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The classical cost function of VRPTW as $\text{Min } \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} d_{ij} x_{ijk}$ should be modified to a new cost function for the above consideration as $\text{Min } f_1^{(1)}(\text{Load, distance traveled})$ where Load is the weight of the vehicle over each link $(i,j)$ and distance traveled is the distance of link $(i,j)$. Evidently, the weight of the vehicle equals the weight of the empty vehicle (tare) plus the load of the vehicle. It should be noted that this cost function is mainly focused on energy consumption and can be calculated based on the work done by a vehicle over a route (arc) of network. In physics, mechanical work is a quantity that can be described as the product of a force and the distance through which it acts in the direction of the force. It is assumed that the movement of vehicles is considered as an impending motion where the force causing the movement is equal to the friction force ($\text{work} = \text{force} \times \text{distance} =$ (coefficient of friction $\times$ weight $\times$ $\cos(\theta)) + \text{weight} \times \sin(\theta)$ where $\theta$ is the slope of route). Thus, a new objective function to minimize the work done by vehicles or the energy used (equivalent to fuel consumed by vehicles) is obtained and should be considered instead of the classical cost function as follows:

$$\text{Min } \sum_{i=0}^{N} \sum_{j=0, j \neq i}^{N} \sum_{k=1}^{K} \left[ (\mu_{ij} \times (u_i + \text{tare}_k)) \times g \times \cos(\theta_{ij}) \right] \times d_{ij} \times x_{ijk} \rightarrow \text{if } \theta_{ij} = 0 \rightarrow \text{new function}$$

Moreover, $u_i$ is the continuous variable and shows the load of the vehicle upon leaving customer $i$. In addition to this objective function, the constraint $(2)$ & $(3)$ should also be added to the model as follows:

$$u_0 = 0$$

$$\sum_{i=0, i \neq j}^{N} \sum_{k=1}^{K} \left( u_i + m_j \right) \times x_{ijk} = u_j \forall j \in C \setminus \{0\}$$

These new constraints and objective functions are non-linear and should be approximated to a linear equation. For this purpose a new variable $u_{ij}^k$ is defined instead of $u_i$ which means the load of vehicle $k$ when moving from customer $i$ to customer $j$. So the equation $(1)$ and $(3)$ is changed to equation $(4)$ and $(18)$ that is described later.

3.2 VRPTW with Customers’ Priority

The proposed routing model considers the customers’ priority according to customer-specific time windows, which is highly relevant to the customers’ satisfaction level. This concept was proposed in our recent paper [Ghannadpour et al. 2014] and it is developed here through mathematical formulation. This model attempts to maximize the satisfaction level of customers (especially high-priority customers) and involves routing vehicles according to the preferences of customers. In these many realistic applications, the concept of classical time windows does not model the preference of customers very well. Even though customers provide a fixed time window for service, they really hope to be served at a desired time if possible. This preference information of customers can be represented as fuzzy time windows with respect to the satisfaction for service time. Figure 2 shows the satisfaction rate of each customer for service in the conventional approach. This figure also depicts the typical fuzzy time window that can
reflect the customers’ satisfaction for service for different kinds of customers. Every customer can be assigned to a group (e.g., very important, important, casual and unimportant) which are predefined by the expert. The more important the customer, the tighter the time window is. In an extreme case, the fuzzy time window is tighter than the classical counterpart. Thus, this figure shows an example of fuzzy time windows for casual ($C_C$) and important customers ($C_I$) where $C_C \cup C_I = C \setminus \{0\}$. According to this figure, the proposed model is able to cope with the evaluation of deliveries for casual customers that violate hard time windows.

For example, if a customer is served at its desired time, the grade of its satisfaction is 1; otherwise, the grade of satisfaction gradually decreases along with the increase of difference between the arrival time of vehicle and desired times. The grade of satisfaction will be 0 (i.e., no satisfaction) if the arrival time falls in outside of the time interval. For simplicity, the membership function of an important customer $i$ or $\mu_i(t_i)$, which represents the grade of satisfaction when the start of service time is $t_i$ defined as triangular fuzzy membership function. The start time of service for each customer $i$ is calculated by $t_i = at_i + w_i$. Therefore, a new objective function should be considered and the model tries to maximize the customers' satisfaction along with the classical cost function. When all customers are important, the new objective function is $\text{Max } \sum_{i \in C \setminus \{0\}} \mu_i(t_i)$. However, when there are the customers with different importance classes, the above objective can be written as $\text{Max } \sum_{i \in C \setminus \{0\}} PR_i \times \mu_i(t_i)$ where, $PR_i$ is the importance degree of customer $i$.

According to Figure 2, the classical time window is changed to the triple $[e_i, u_i, l_i]$ and $[\hat{e}_i, \hat{u}_i, \hat{l}_i]$ for important and casual customers, respectively where, $[\hat{e}_i, \hat{u}_i, \hat{l}_i] = [e_i - \delta, u_i, l_i + \delta]$ and $\delta$ is the constant violation value from hard time windows.

### 3.3 Mathematical Formulation

The mathematical formulation of the proposed model is as follows:
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Classical Time window

Time window for an important customer

Time window for a casual customer

Figure 2. Conventional and fuzzy time window for each customer

Minimize \( f_1 = \sum_{j=0}^{N} \sum_{j=0, j \neq i}^{N} \sum_{k=1}^{K} (\text{tare}_k \times x_{ijk} + u_{ij}) \times d_{ij} \) \hspace{1cm} (4)

Minimize \( f_2 = \sum_{k=1}^{K} \sum_{j=1}^{N} x_{0jk} \) \hspace{1cm} (5)

Maximize \( f_3 = \sum_{i=1}^{N} PR_i \times \mu_i(t_i) \) \hspace{1cm} (6)

S.t:

\[ \sum_{k=1}^{K} \sum_{j=1}^{N} x_{ijk} \leq K \hspace{1cm} \forall \ i = 0 \] \hspace{1cm} (7)

\[ \sum_{j=0, j \neq i}^{N} x_{ijk} = \sum_{j=0, j \neq i}^{N} x_{jik} \leq 1 \hspace{1cm} \forall \ i \in C, \ \forall \ k \in K \] \hspace{1cm} (8)

\[ \sum_{k=1}^{K} \sum_{i=0, i \neq j}^{N} x_{ijk} = 1 \hspace{1cm} \forall \ j \in C \setminus \{0\} \] \hspace{1cm} (9)

\[ at_0 = w_0 = f_0 = \mu_0(t_0) = t_0 = 0 \] \hspace{1cm} (10)

\[ at_i + w_i + f_i + t_{ij} - (1 - x_{ijk})M \leq r_k \hspace{1cm} \forall \ i \in C \setminus \{0\}, \ \forall \ k \in K, \ j = 0 \] \hspace{1cm} (11)
\[ at_i + w_i + f_i + t_{ij} - (1 - x_{ijk})M \leq at_j \quad \forall j \in C\{0\}, \forall i \neq j \in C, \forall k \in K \]  

\[ \mu_i(t_i) = \left( \frac{(at_i + w_i) - e_i}{\hat{u}_i - e_i} \right) * (1 - y_i) + \left( \frac{\hat{u}_i - (at_i + w_i)}{\hat{u}_i - e_i} \right) * y_i \quad \forall i \in C_i \]  

\[ \mu_i(t_i) = \left( \frac{(at_i + w_i) - (e_i - \delta)}{\hat{u}_i - (e_i - \delta)} \right) * (1 - y_i) 
+ \left( \frac{\hat{u}_i - (at_i + w_i)}{(\hat{u}_i + \delta) - \hat{u}_i} \right) * y_i \quad \forall i \in C_C \]  

\[ (\hat{u}_i - (at_i + w_i)) * y_i + ((at_i + w_i) - \hat{u}_i) * (1 - y_i) \leq 0 \quad \forall i \in C\{0\} \]  

\[ e_i \leq (at_i + w_i) \leq l_i \quad \forall i \in C_i \]  

\[ e_i - \delta \leq (at_i + w_i) \leq l_i + \delta \quad \forall i \in C_C \]  

\[ \sum_{j=0,j \neq i}^{N} \sum_{k=1}^{K} u^k_{ij} - \sum_{j=0,j \neq i}^{N} \sum_{k=1}^{K} u^k_{ji} = m_i \quad \forall i \in C\{0\} \]  

\[ u^k_{ij} \leq q_k \times x_{ijk} \quad \forall i \neq j \in C \quad \forall k \in K \]  

\[ x_{ijk} \in \{0,1\}, u^k_{ij} \geq 0 \quad \forall i, j \in C, \forall k \in K \]  

Formulas (4-6) are the objective functions of the proposed model in scenario (I). Formula (4) minimizes total energy consumed by the vehicles, formula (5) minimizes the total number of vehicles used to serve the customers and formula (6) maximizes the total satisfaction rate of customers. Constraint (8) secures that every route starts and ends at the central depot. Constraints (8) and (9) define that every customer node is visited only once by one vehicle. Constraint (11) is the maximum travel time constraint. Constraints (12), (16) and (17) define the arrival time and the time windows for different kinds of customers. Constraints (13-15) compute the satisfaction level of each customer based on a control variable \( y_i \). This variable is used to determine whether the start of service time is before or after the desired time. Constraints (13-15) are non-linear and should be transformed to linear constraints. For instance the constraint (13) and (15) could be rewritten as follows: the first part of constraint (13) is \( \mu_i^{(1)} = [(at_i + w_i) - e_i]/(\hat{u}_i - e_i) \) and the second part is \( \mu_i^{(2)} = [l_i - (at_i + w_i)]/(l_i - \hat{u}_i) \). So the satisfaction level of this customer is \( \mu_i \leq 1 \) where \( \mu_i \leq \mu_i^{(1)} \) and \( \mu_i \leq \mu_i^{(2)} \). Constraint (18) indicates the load of vehicle after it visits a customer. Constraint (19) limits the maximal load carried by the vehicle and force \( u^k_{ij} \) to zero when \( x_{ijk} = 0 \).
4. Evolutionary Method

This section designs an efficient method for tackling the proposed model in the previous section such that the objectives are met and the constraints are satisfied. In this paper, among the number of the approaches explored in the literature, evolutionary algorithms are examined in greater depth and employed to tackle the proposed model. Evolutionary algorithms like GA have many advantages in finding an easy way of the solution’s representation, implementation for multi-objective models and ability of incorporation with the different operators that improve the solutions [Zhang et al, 2015; Shim et al, 2015 and Li et al. 2015]. In addition, the appeal of these methods can be explained by their simplicity and elegance as robust search algorithms as well as from their power to discover good solutions rapidly for difficult high dimensional problems. It is also easy to adapt the GA scheduler to the particular requirements of a very wide range of possible overall objectives. The hybrid genetic algorithm used in this paper is as follows.

4.1 Chromosome Representation

In this method each chromosome which is a solution to the problem, is represented by an integer string of length $N$. This string of customer identifiers represents the sequence of deliveries that must be covered by vehicles during their routes. All routes are encoded together, with no special route termination characters in between, and they are decoded back into routes based on the feasibility conditions, namely maximum allowable operating time for vehicles, servicing without delay time and vehicles capacity constraint.

4.2 Pareto ranking procedure

In the GA for the evaluation of each chromosome, a special fitness function is defined. However, in multi-objective applications of the GA, the Pareto ranking scheme has been often used [Ghoseiri and Ghannadpour, 2010; Ombuki et al. 2006; Tan et al. 2006; Ghannadpour et al. 2014 and Zhang et al. 2015]. The Pareto ranking process tries to rank the solutions to find the non-dominated solutions. It should be noted that the lower ranks are preferable and the chromosomes within rank 1 are the best in the current population. So according to this process, each solution gives a rank with respect to different objective values that shows its quality in comparison with other solutions. It is easily incorporated into the fitness evaluation process within a GA, by replacing the raw fitness scores with Pareto ranks (for more detailed theoretical descriptions, see [Ghoseiri and Ghannadpour, 2010]). Therefore, chromosomes assigned rank 1 are non-dominated, and inductively, those of rank $i+1$ are dominated by all chromosomes of ranks 1 through $i$. It should be noted that the non-dominated solutions in each population are identified based on the concept of dominance as follows:

**Definition:** A solution $y = (y_1, y_2, \ldots, y_n)$ dominates (denoted $<$) a solution $z = (z_1, z_2, \ldots, z_n)$ if and only if $\forall i \in \{1 \ldots n\}, y_i \leq z_i$ and $\exists i \in \{1 \ldots n\}, y_i < z_i$.

For instance, the three objectives of proposed model define three independent dimensions in a multi-objective fitness space. Thus, each chromosome in the population is associated with a vector $\vec{v} = (c^-, v^-, s^+)$ where $c^-$ and $v^-$ are the objective values for energy consumed by vehicles and number of vehicles, respectively. Also, $s^+$ is the objective value for the total satisfaction rate of customers. These three
dimensions are retained as independent values, to be eventually used by the Pareto ranking procedure to create for the population a set of integral ranks \( \geq 1 \). These ranks are then used by the proposed GA as fitness values for subsequent actions.

4.3 Population and initialization

In this paper the modified method of PFIH (originally proposed by [Solomon, 1987]) is used to create the first chromosome. PFIH method defines the relation of \( c_i = \alpha d_{oi} + \beta l_i + \gamma ((p_i/360)d_{oi}) \) to find the first customer in each new route where; \( d_{oi} \) is the distance from customer \( i \) to the central depot; \( l_i \) is the latest time and \( p_i \) is the polar coordinate angle of the customer \( i \). Once the first customer is selected for the current route, the method selects from the set of unrouted customers the one customer which minimizes the total insertion cost between every edge in the current route without violating the time and capacity constraints. Besides the first chromosome determined by the simple heuristic, each other chromosome is initialized as a random permutation of customers and using \( \lambda \)-interchange mechanism which discussed later. When all the initial chromosomes are generated, the corresponding solution costs \( \vec{v} = (c^-, v^-, s^+) \) and the Pareto rank (fitness) of each chromosome is also identified using the Pareto ranking procedure.

4.4 Selection

This paper uses a standard \( k \)-tournament selection where a tournament set of size \( k \) is randomly drawn from the population and the chromosome with a lower rank is selected and will then be recombined via the recombination operators to create a potential new population.

4.5 Recombination

One of the unique and important aspects of the GA is the important role of the crossover operator. The classical crossovers (e.g., one-point crossover and \( n \)-point crossover) are not appropriate for this sequencing model because of duplication and omission of vertices [Zhu et al. 2016]. This paper uses the modified best cost-best route crossover (BCBRC), which is similar to the BCBRC and selects a best route from each parent and then for a given parent, the customers in the chosen route from the opposite parent are removed. The final step is to locate the best possible locations for the removed customers in the corresponding children. This procedure is illustrated in Figure 3. According to this figure, the third route of parent \#1 is selected and the customers on this route are removed from the routes of parent \#2. This process is done similarly for another parent. Hereinafter for each parent the best locations for removed customers are determined by the insertion procedure one at a time. This procedure is continued until two feasible offspring are produced.

4.6 Local search

The local search (LS) is employed to the child chromosome with a probability \( p_l \). This paper uses a \( \lambda \)-interchange mechanism as local search method that moves customers between routes to generate neighborhood solution. Given a feasible solution for the model represented by \( S = \{R_p^1, ..., R_p^r, ..., R_q^r\} \) where \( R_p^r \) and \( R_q^r \) represent the set of customers served by routes \( p \) and \( q \) respectively. A \( \lambda \)-interchange between a pair of routes \( R_p^r \) and \( R_q^r \) is a replacement of subset \( S_1 \subseteq R_p^r \) of size \( |S_1| \leq \lambda \) by another subset \( S_2 \subseteq R_q^r \) of size \( |S_2| \leq \lambda \), to get the new route sets \( \hat{R}_p^r, \hat{R}_q^r \) and a new neighboring solution \( \hat{S} = \{R_p^1, ..., \hat{R}_p^r, ..., \hat{R}_q^r, ..., R_q^r\} \) where \( \hat{R}_p^r = (R_p^r - S_1) \cup S_2 \) and \( \hat{R}_q^r = (R_q^r - S_2) \cup S_1 \).
Evolutionary Approach for Energy Minimizing Vehicle Routing Problem with Time Windows and Customers’ Priority

neighboring $N_\lambda(S)$ of a given solution $S$ is the set of all neighbors $\{\tilde{S}\}$ generated by the $\lambda$-interchange method for a given $\lambda$. In one version of the algorithm called GB (global best), the whole neighborhood is explored and the best move with lower rank is selected. In another version, FB (first best), the first improving move is selected if it exists; otherwise the best move is implemented. In this paper 1-interchange (FB) or 2-interchange (GB) is employed to the child chromosome with the special probability.

![Parent #1](image1.png) ![Parent #2](image2.png)

Figure 3. Modified best cost-best route crossover

5. Computational Experiments

In this section, since there is no prior work on the proposed model, a set of complete randomly generated instances with different sizes is considered as numerical examples. Moreover, the effectiveness of proposed method is also evaluated on the well-known Solomon’s benchmark problem instances [Solomon, 1987], which have been extensively used for benchmarking different heuristics in the literature over the years. First, the usefulness of the new mathematical formulations is examined by CPLEX Solver (with a time limit of 2 hours). It is found that CPLEX Solver cannot find an optimal solution even for small-case instances for each model in a reasonable amount of computational time. Then, the performance of the proposed method is evaluated on the Solomon’s problems consist of 56 data sets and the gap is studied and reported. Finally, the proposed model in section 2 and the quality of proposed evolutionary method is evaluated. For more appropriate comparison, the performance of proposed evolutionary method is also compared with other existing multi-objective optimization algorithm frameworks like NSGA-II which is the popular non-domination based genetic algorithm for multi-objective optimization.

5.1 Analysis of the usefulness of the new mathematical formulations

In this section, the new mathematical formulation of VRPTW with customers’ priority and energy minimizing VRPTW for small and medium instances are implemented by CPLEX Solver separately and the results are analyzed.

5.1.1 Energy Minimizing VRPTW
Table 1 presents a summary of results obtained by CPLEX Solver when the VRPTW with minimizing the total energy consumption is considered. The column labeled “with classical cost function” gives the findings of model when it tries to minimize the total distance travelled by vehicles (distance oriented); column “with new cost function” gives the findings of model when it tries to minimize the total energy (fuel) consumption (fuel oriented). For each instance, the vehicles’ total traveling distance (indicated by Dis.) and the related energy or fuel consumption (indicated by Related FC) are calculated when the distance-oriented model is implemented. Moreover, the fuel consumption (FC) and the related traveling distance (Related Dis.) are also obtained by the fuel-oriented model. The times marked with an asterisk show the time limit of 2 hours for the CPLEX Solver and the solver is interrupted after this time. For some instances there is no integer solution up to this time limit. Deviation between the results obtained by distance & fuel oriented models is listed in the last two columns. Below, the first instance in Table 1 is taken as an example to show how the deviations are calculated:

$$\text{FC dev.} (%) = \frac{2438.131 - 2847.909}{2438.131} \times 100 = -16.81$$

It can be observed from Table 1 that for the small/medium – scale instances, the FC obtained by fuel oriented model is on average 5.6% lower than the FC obtained by the distance oriented model, but with a 10.6% increase in distance traveled. In other words, by increasing the distance traveled by 10.6%, the fuel cost which is a significant part of the total transportation cost can be reduced by 5.6%. We take the first instance as an example in order to check with more details. The solution obtained by the distance-oriented model has a total distance of 115.3761 traveled by two vehicles. Moreover, the related fuel consumption of this solution is 2847.909.

### Table 1. Energy minimizing VRPTW by CPLEX Solver

<table>
<thead>
<tr>
<th>Instance</th>
<th>N</th>
<th>With classical cost function</th>
<th>With new cost function</th>
<th>Deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Dis.</td>
<td>CPU t. (Sec.)</td>
<td>Related FC.</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>115.3760</td>
<td>0.2030</td>
<td>2847.909</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>140.1070</td>
<td>0.2180</td>
<td>2427.428</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>226.6523</td>
<td>2.8750</td>
<td>4392.852</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>321.6250</td>
<td>37.765</td>
<td>9817.879</td>
</tr>
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<td>6</td>
<td>20</td>
<td>497.100</td>
<td>7200*</td>
<td>14827.87</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>------</td>
<td>7200*</td>
<td>------</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>------</td>
<td>7200*</td>
<td>------</td>
</tr>
<tr>
<td>Ave.</td>
<td></td>
<td>221.4018</td>
<td>5118.586</td>
<td>4873.631</td>
</tr>
</tbody>
</table>

When the fuel-oriented model is implemented the visiting routes by three vehicles and with the smallest fuel cost are obtained. Eventually, although the fuel-oriented model generally provides a schedule with a longer distance, it reduces the energy consumed or fuel cost significantly by 16.81%. It should be noted that the choice of either solution (fuel & distance oriented) depends on the DM’s preference and it
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is based on the cost of fuel consumption and the distance traveled.

5.1.2 VRPTW with Customers’ Priority

Table 2 presents a summary of results obtained by CPLEX Solver when the VRPTW with customers’ priority is considered. In this model the total distance traveled by the fleets is minimized and the total satisfaction rate of customers is maximized as follows:

\[
\min w^1 \times \left( \frac{f_1}{f_1^{\text{max}}} \right) - w^2 \times \left( \frac{f_2}{f_2^{\text{max}}} \right)
\]  \( (21) \)

where, \( w^i \) is the weight of objective function \( f_i \) estimated by DM and \( \sum_i w^i = 1 \). The various solutions (non-dominated solutions) can be created by changes in weights of objective functions. For each obtained solution, the value of distance cost (indicated by Dis.) and the total satisfaction rate of customers (indicated by Sat.) are shown separately in this Table. These values are equivalent to objective functions \( f_1 \) and \( f_2 \) respectively. Moreover, the bolded numbers indicate the solutions of the single objective of the model. The times marked with an asterisk show when the CPLEX solver is interrupted after the 2-hour time limit. For some instances the best integer solution found up to this time is reported here. For more analysis, we take the second instance as an example in order to show how solutions change when the weights of objective functions are changed. The results of second instance in Table 2 for different weights of \((w^1, w^2)\) are illustrated in figure 4. This behavior shows a Pareto frontier of two objectives (Min–Max) in this figure.

![Pareto Frontier](image)

Figure 4. Results of second instance in Table 2 for different weights

Table 2. VRPTW with customers’ priority by CPLEX Solver

<table>
<thead>
<tr>
<th>Instance</th>
<th>N</th>
<th>( w^1 )</th>
<th>( w^2 )</th>
<th>Results</th>
<th>CPU t. (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Dis.</td>
<td>Sat.</td>
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<td>1</td>
<td>5</td>
<td>0.00</td>
<td>1.00</td>
<td>172.301</td>
<td><strong>15.000</strong></td>
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<tr>
<td></td>
<td></td>
<td>1.00</td>
<td>0.00</td>
<td><strong>140.107</strong></td>
<td>2.8200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.50</td>
<td>0.50</td>
<td>143.943</td>
<td>15.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.75</td>
<td>0.25</td>
<td>140.107</td>
<td>14.438</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.25</td>
<td>0.75</td>
<td>143.943</td>
<td>15.000</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>0.00</td>
<td>1.00</td>
<td>237.870</td>
<td><strong>26.000</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.00</td>
<td>0.00</td>
<td><strong>226.652</strong></td>
<td>2.7900</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.95</td>
<td>0.05</td>
<td>226.652</td>
<td>22.419</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.75</td>
<td>0.25</td>
<td>230.002</td>
<td>26.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.25</td>
<td>0.75</td>
<td>230.002</td>
<td>26.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.90</td>
<td>0.10</td>
<td>228.013</td>
<td>25.093</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>0.00</td>
<td>1.00</td>
<td>347.392</td>
<td><strong>30.000</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.00</td>
<td>0.00</td>
<td><strong>303.246</strong></td>
<td>2.7900</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.95</td>
<td>0.05</td>
<td>304.606</td>
<td>29.093</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.75</td>
<td>0.25</td>
<td>306.595</td>
<td>30.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.25</td>
<td>0.75</td>
<td>306.766</td>
<td>30.000</td>
</tr>
</tbody>
</table>
According to this table, when the vector of weights \((w_1, w_2)\) is changed from \((1,0)\) to \((0.95,0.05)\), the total customers’ satisfaction is increased from 2.8 to 22.4 while the distance cost remains the same. This means that the waiting time imposed on vehicles is increased to get a better satisfaction rate for customers. Due to the high weight of the first objective function, the distance cost is not changed and it is the same as the previous cost. In the same way, when the importance weight of the second objective becomes more important and the vector of weights are changed to \((0.90,0.10)\); the customers' satisfaction rate is improved and increased to 25.1 as the total travelling distance cost is slightly deteriorated. Eventually, while increasing the importance weight of second objective, the total travelling distance cost further deteriorates as the customers' satisfaction rate is improved.

5.1.3 Medium-sized Case Study

This section presents a randomly medium sized case of proposed model to improve the usefulness of the new formulations and usability of the proposed model. This case includes the following assumptions and parameters:

- 70 geographically dispersed customers located around a depot with different size of demands and priority.
- The customer priority is randomly assigned between numbers 1 through 10. The larger number represents the higher priority for the service.
- Fuzzy time windows of each customer are randomly created and triangular fuzzy membership function which represents the grade of customers' satisfaction on service time is calculated.
- There is a set of vehicle located at the central depot with limited capacity of 200.
- Service time on each customers is assumed to be 90.
- The weight of the empty vehicle (tare) is assumed to be 50.
- The maximum allowable operating time of vehicles to end the route at the central depot is set to 1236.

Figure (5-A) illustrates the output of the above mentioned problem when the classical cost function (distance oriented) of VRPTW is used. According to this figure, the solution obtained by the distance-oriented model has a total distance of 1091.6 traveled by 17 vehicles. Moreover, the related fuel consumption and customers' satisfaction of this solution are 105320 and 38.498 respectively.
Evolutionary Approach for Energy Minimizing Vehicle Routing Problem with Time Windows and Customers’ Priority

Figure (5-B) shows the result of the model when the energy minimizing VRPTW (with the proposed new cost function of equation (4)) is implemented. Energy consumed by this solution is 94691 and the total distance cost is 1201.8 with 18 vehicles. It means that the oriented model generally provides a schedule with a longer distance (increase by 9.2%) but reduces the energy consumed significantly by 11.2%.

In addition to improving energy consumption, it is necessary to consider another other issues to improve the customers' satisfaction and fleet size and make a trade-off between objectives. Thus, the Figure 6 shows one output (one Pareto solution) of the above mentioned problem when the proposed multi-objective model is used. As mentioned before, the proposed model tries to minimize the energy consumed and the total number of vehicles and maximize the total satisfaction rate of customers.
5.2 Analysis the proposed method on the well-known benchmarks

In this section, the effectiveness of proposed method is evaluated on the Solomon’s benchmark problem instances [Solomon, 1987]. The Solomon’s problems consist of 56 data sets, which have been extensively used for benchmarking different heuristics in the literature over the years. The problems vary in a fleet size, vehicle capacity, traveling time of vehicles, spatial and temporal distribution of customers. Table 3 presents a summary of results where the columns labeled “Best known” gives the best known published solutions ([Chiang et al., 2014; Ombuki et al., 2006 and Ghannadpour et al., 2014]). Bolded numbers in this table indicate that the solutions are obtained by the proposed method are almost same as the best currently known results from the literature. The relative percentage Gap is also presented in this table for each instance and it is analyzed later.

In this table, the best solutions found by proposed method are reported over 10 runs. In order to know the performance of the proposed method, the findings are compared with the best known solutions for each category of Solomon’s problems. Table 4 summaries the results of Table 3 for each instance category. The average travel costs of the best known results and those found by the proposed method are presented in this Table. Additionally, the last column presents the total cost distance over whole 56 instances. The last row indicates the percentage difference between the results.

Table 3. Results of the proposed method and Solomon's instances for certain travel times

<table>
<thead>
<tr>
<th>Pro.</th>
<th>Best Known Travel Cost</th>
<th>Proposed Method Travel Cost</th>
<th>% diff.</th>
<th>Best Known Travel Cost</th>
<th>Proposed Method Travel Cost</th>
<th>% diff</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6. Proposed Model: Energy Minimizing VRPTW with Customers’ Priority
Table 4. Average results of proposed method and the best known solutions

<table>
<thead>
<tr>
<th>Results</th>
<th>C1</th>
<th>C2</th>
<th>R1</th>
<th>R2</th>
<th>RC1</th>
<th>RC2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best known</td>
<td>828.38</td>
<td>589.77</td>
<td>1195.15</td>
<td>904.19</td>
<td>1360.47</td>
<td>1052.03</td>
<td>55761.47</td>
</tr>
<tr>
<td>Proposed method</td>
<td>828.38</td>
<td>591.49</td>
<td>1220.23</td>
<td>944.20</td>
<td>1381.99</td>
<td>1104.85</td>
<td>57111.04</td>
</tr>
<tr>
<td>% diff. D</td>
<td>0.00</td>
<td>0.29</td>
<td>2.05</td>
<td>4.23</td>
<td>1.55</td>
<td>4.78</td>
<td>2.36</td>
</tr>
</tbody>
</table>

According to this table, the proposed method obtained superior results for the class of C1 & C2. On the other hand, for the remaining categories, solutions from the proposed method are between 2% and 4.7% larger in travel cost than the best known results. Moreover, the difference between the results of the proposed method and best known solutions for all 56 instances is only 2.36%. Therefore, the good quality results obtained by the model in general compare favorably, with respect to time and quality, to the best published results.

One of the most important points to implement the proposed method is relevant to parameters tuning and the set values are presented in the next section. In this paper the changes of each
parameter in the pre-specified variation range are considered when the other parameters are fixed. For instance, when all parameters of proposed method are fixed on a specific value, the rate of crossover operator is tuned according to Fig.7. According to this figure the variation of crossover rate between 0.4, 0.6 and 0.8 is checked over all instance problems in Class R1 (repeated runs is not shown in this figure).

![Tuning of crossover rate when other parameters are fixed](image)

**Figure 7. Tuning of crossover rate when other parameters are fixed**

Each slice of this figure shows how the gap between the results of proposed method and best known (reported in Table 4) varies over the range of problem instances. The performance is also calculated based on the gap between the results of proposed method and Best Known reported in Table 3. According to this figure the average gap over all instances is 30.05% for crossover rate 0.4; 7.56% for crossover rate 0.6 and 2.07% for crossover rate 0.8. This procedure is done for all other variation of parameters and the best value for applying crossover rate is chosen based on the average performance over all instances. The set value of all parameters are reported in the next section.

### 5.3 Analysis of proposed model and solution

In this section, the quality of the proposed evolutionary method is evaluated. First, the instances with larger size are considered and the results found by the proposed method and CPLEX Solver are analyzed. Then, the performance is compared with other existing multi-objective optimization algorithm frameworks like NSGA-II. It should be noted that the proposed method is coded and run on a PC with Core 2 Duo CPU (3.00 GHz) and 2.9 GB of RAM. Moreover, the model is implemented under parameters of Population size = 30 - 100, Generation number = 500-1000, Crossover rate = 0.80, Mutation rate = 0.40, Selection rate of improvement operators = 0.5 and Repetition for experiments = 10. The population size and the generation number is adopted with the problem size. Table 5 presents a summary of results when the proposed mathematical formulation by CPLEX solver is compared to proposed evolutionary approach. The times marked with an asterisk show the time limit of 2 hours after which the solver was interrupted. For some instances there is no integer solution up to this time limit. The required computation time (indicated by CPU t.) in seconds is also reported for each instance when the proposed evolutionary method is implemented. This table gives the findings of proposed model using the weighting approach of defined objectives as follows:

\[
\min w^1 \times \left( \frac{f_1}{f_1^{\text{max}}} \right) + w^2 \times \left( \frac{f_2}{f_2^{\text{max}}} \right) - w^3 \quad (22)
\]

\[
\times \left( \frac{f_3}{f_3^{\text{max}}} \right)
\]

For each obtained solution, the value of fuel cost (indicated by FC.), the size of fleet (indicated by \( K \)) and the total satisfaction rate of customers (indicated by Sat.) are shown separately in this Table. It is noteworthy that the above mentioned weighting approach is only used for implementation of proposed mathematical formulation by CPLEX Solver.
and finding the Pareto solutions by changing the weights. However, the proposed evolutionary uses Pareto ranking procedure to find the non-dominated solutions.

**Table 5. Results of proposed model and solution**

<table>
<thead>
<tr>
<th>#</th>
<th>N</th>
<th>Mathematical formulation by CPLEX Solver</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>w₁</td>
<td>w₂</td>
</tr>
<tr>
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</tr>
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</tr>
<tr>
<td></td>
<td></td>
<td>85024.2</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>95744.4</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>95873.7</td>
<td>19</td>
</tr>
</tbody>
</table>
defined objectives is unknown until the problem is solved in a proper multi-objective manner. These objectives may be positively correlated with each other or they may be conflicting with each other. According to Table 5, the customers' satisfaction rate is improved as the total fuel consumed is deteriorated. Moreover, the waiting time imposed on vehicles is increased in these instances due to the better satisfaction rate of customers. These behaviors for the 7th instance of Table 5 are illustrated in Figure 8.

Figure 8. Population distribution of the 7th instance

A varying behavior is observed for the relationship between total fuel consumption and the required fleets. They are positively correlated with each other in some instances like instance #3 and they are conflicting to each other in others (like instance #2). By adding a vehicle to the schedule, the load of vehicles could be decreased along a route but the total distance traveled by vehicles may be increased or decreased and it is related to the geographical location and time windows of customers [Ghoseiri and Ghannadpour, 2010]. Thus, by increasing the number of vehicles the load of vehicles for each route is decreased and when the distance cost of the solution is changed in the opposite direction, the total fuel consumed by vehicles is decreased. This behavior is deduced from instance 2, where the fuel consumption of fleets is reduced, but at the expense of adding extra vehicles (from 6 to 7). On the other hand, in instance 3, the total fuel consumed by vehicles is increased as the number of vehicles is increased from 8 to 9. In this instance, although adding a vehicle provides a schedule with a lower load of vehicles for each route, the distance cost is much higher than that of the basic model. Therefore, the total fuel consumed by all fleets is increased.

It should be noted that the Repetition of experiments is 10 runs and the non-dominated solutions of each instance reported in Tables 5 are the solutions obtained in the first experiment. So, Table 6 presents the average and best values over 10 runs and compares the results to the finding of CPLEX Solver. The column labeled "h - ave" gives the total average findings of proposed method over 10 runs and it is divided into three columns where the each of them represents the average of each objective function (indicated by $\overline{FC}$, $\overline{K}$ and $\overline{Sat}$); column "h - best" gives the best results of each objective function obtained by the proposed method over 10 experiments (indicated by $FC^b$, $K^b$ and $Sat^b$). Difference between the average and best results of proposed evolutionary method are listed in the columns labeled $D^{\overline{FC}}$, $D^{\overline{K}}$ and $D^{\overline{Sat}}$. Moreover, $D^i$ ($i = 1,2,3$) represents the deviation between the best value of objective function $f_i$ obtained by
proposed method over 10 runs and the best value found by the CPLEX Solver (reported in Table 5). It should be noted that the listed values of deviations represent the amount of difference between the best and average results of proposed method over 10 experiments and could illustrate the consistency and reliability of results. Moreover the deviations between the best results of proposed method and CPLEX Solver represents the quality of the obtained results, where a negative value represents the amount of improvement obtained by the proposed approach. According to this table we can see the results obtained from proposed method are rather consistent and the average deviations over 10 experiments are lower than 8%. Moreover, the average difference between the best values of proposed method and CPLEX Solver illustrates the improvement of 0.2% in the first objective for the first three instances and for the others the CPLEX Solver cannot find any solution in a reasonable amount of computational time. Therefore, according to Table 5 & 6 it is evident that the proposed evolutionary method outperforms the CPLEX solver in terms of the quality of solutions and computing time. The superiority of proposed method is more apparent in large-scale test instances.

For more appropriate comparison, the performance of proposed evolutionary method is also compared with NSGA-II which is the popular non-domination based genetic algorithm for multi-objective optimization. The principals and the concept of this method could be found in [Deb,2002]. The concept of this method is briefly based on population initialization, non-dominated sort, crowding distance, selection and genetic operators and recombination. For more details please refer to [Deb, 2002]. So, the performance of the proposed evolutionary method is compared with the above mentioned method (NSGA-II) and a summary of results and the list of non-dominated solutions of two approaches is presented in Table 7. The required computation time (indicated by CPU t.) in second is also reported for each instance.

**Table 6. Average and best results over 10 experiments**

<table>
<thead>
<tr>
<th>#</th>
<th>h – ave</th>
<th>h – best</th>
<th>Deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FC, K, Sat</td>
<td>FC, K, Sat</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4424.69</td>
<td>4382.37</td>
<td>0.960</td>
</tr>
<tr>
<td>2</td>
<td>4382.37</td>
<td>4382.37</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>4382.37</td>
<td>4382.37</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>4382.37</td>
<td>4382.37</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>4382.37</td>
<td>4382.37</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>4382.37</td>
<td>4382.37</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>4382.37</td>
<td>4382.37</td>
<td>6</td>
</tr>
<tr>
<td>Ave.</td>
<td>7.340</td>
<td>7.340</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

**Table 7. Comparison between the proposed evolutionary method and NSGA-II**

<table>
<thead>
<tr>
<th>#</th>
<th>N</th>
<th>Proposed Evolutionary</th>
<th>NSGA-II</th>
</tr>
</thead>
</table>

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To assess the performance of two these multi-objective methods various aspects could be involved such as the quality of the outcome, the computation time required, the parameter settings, etc. According to the Table 7, the computational efforts of proposed method are near to the fast multi-objective genetic algorithm (NSGA-II). Moreover, similar to investigation of Table 6, the average and best values of proposed model over 10 runs are compared with the finding of NSGA-II and a summary of results is presented in Table 8. In this Table, the column labeled "NSGA – ave" gives the total average findings of NSGA-II over 10 runs and it is divided into three columns where the each of them represents the average of each objective function; column " NSGA – best " gives the best results of each objective function obtained by NSGA-II over 10 experiments (indicated by FC, K and Sat). In addition, the deviation between the average and best results of proposed metaheuristic and NSGA-II are listed in the last columns.

The deviations are calculated based on the finding of proposed method and the negative values represent the improvement achieved by the proposed method in comparison with NSGA-II. According to this table the average difference between the best values of proposed method and NSGA-II illustrates the improvement of 4.8% in the first objective and 1.1% in the second. Moreover, the competitive improvement by proposed evolutionary is also reported in this table over 10 experiments.

Table 8. Average and best results of NSGA-II over 10 experiments
Evolutionary Approach for Energy Minimizing Vehicle Routing Problem with Time Windows and Customers’ Priority

<table>
<thead>
<tr>
<th>#</th>
<th>NSGA – ave</th>
<th>NSGA – best</th>
<th>Deviation (%) of proposed method from NSGA-II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FC</td>
<td>K</td>
<td>Sat</td>
</tr>
<tr>
<td>1</td>
<td>14760.6</td>
<td>8.7</td>
<td>49.1</td>
</tr>
<tr>
<td>2</td>
<td>19208.2</td>
<td>12.5</td>
<td>77.2</td>
</tr>
<tr>
<td>3</td>
<td>32201.0</td>
<td>16.7</td>
<td>102.5</td>
</tr>
<tr>
<td>4</td>
<td>70050.1</td>
<td>15.2</td>
<td>118.2</td>
</tr>
<tr>
<td>5</td>
<td>95146.6</td>
<td>19.4</td>
<td>262.7</td>
</tr>
<tr>
<td>Ave.</td>
<td>-8.7</td>
<td>-3.3</td>
<td>-0.3</td>
</tr>
</tbody>
</table>

For precise analysis on the performance of proposed method, the Wilcoxon signed ranks test method is used as the simplest and widely used kind of nonparametric statistical tests [Derrac et al., 2011]. This method is used to perform individual comparisons between the Pareto results (22 solutions of Table 7) found by the proposed method and NSGA-II. In this method the difference between the Pareto results of two algorithms on \(i\)th out of \(i\) instances \((d_i)\) is computed and then the following sum of ranks for the problems in which the one method outperformed the other is calculated:

\[
R^+ = \sum_{d_i > 0} rank(d_i) + \frac{1}{2} \sum_{d_i = 0} rank(d_i)
\]

(23)

\[
R^- = \sum_{d_i < 0} rank(d_i) + \frac{1}{2} \sum_{d_i = 0} rank(d_i)
\]

(24)

Eventually, the smaller of the sums, \(T = \min(R^+, R^-)\) is computed and when \(T\) is less than or equal to the value of the distribution of Wilcoxon for \(n\) degree of freedom, the null hypothesis of equality of means is rejected. This will mean that a given algorithm out performs the other one, with the p-value associated. By using the Wilcoxon’s test in our experimental study, the proposed method shows a significant improvement over the results of NSGA II with a level of significant \(\alpha = 0.01\) \((R^+ = 43, R^- = 210 and p – value = 0.0067)\).

For better comparison, there is also an assessment of Pareto set approximations of two multi-objective methods by quality assessment methods of Pareto Set Approximations [Zitzler et al. 2008]. The quality indicator methods map each Pareto set approximation to a number, and perform statistics on the resulting distribution(s) of numbers. That means we can compare the outcomes of two multi-objective methods, i.e., two Pareto set approximations, by comparing the corresponding indicator values. One of the most important indicators is the hypervolume indicator (IH) that gives the hypervolume of each subspace. According to relationship between the defined objectives functions mentioned before, there is a meaningful relationship between the two objectives of fuel consumption and total customers' satisfaction. As discussed before, the customers' satisfaction rate is improved as the total fuel consumed is deteriorated. This means that the distance traveled by vehicles and consequently the load of vehicles is deteriorated to meet the customers' satisfaction level. So, the hypervolume indicator is used here to evaluate the two Pareto set approximations found by NSGA-II and the proposed evolutionary based on these two objective functions. Figure 9 illustrates the hypervolume indicator on the Pareto set approximations of the proposed evolutionary method on the last instance of Table 7 when two mentioned objectives are considered.

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It can be seen that the hypervolume indicator reveals differences in quality that cannot be detected by the dominance relation. Based on the results $I_H(\text{proposed method}) = 10.04 \times 10^6$ and $I_H(\text{NSGA-II}) = 7.09 \times 10^6$. Moreover, the average hypervolume indicator over 10 runs for proposed evolutionary method is also more than the value of NSGA-II. Every quality indicator represents certain assumptions about the decision maker’s preferences. Due to $I_H(\text{proposed method}) > I_H(\text{NSGA-II})$, we can state that "proposed evolutionary" is better than "NSGA-II" with respect to the hypervolume indicator; however, the situation could be different for another quality indicator. So, when we use the hypervolume indicator in our comparative study, the proposed evolutionary outperforms NSGA-II in terms of quality of the generated Pareto set approximation under the assumption of $I_H$ that reflects the decision maker’s preferences.

6. Conclusion
This paper presents a new model and solution for the multi-objective energy minimizing vehicle routing and scheduling problem with customers’ priority. A reduction in fuel consumption is also considered in modeling the proposed routing plan. In fact, it is the amount of fuel consumed, not the traveled distance that is the greater concern to transportation companies that pursue fuel cost savings. Statistics show that fuel cost is a significant part of total transportation cost. Moreover, this paper considers the customers’ priority according to customer-specific time windows, which are highly relevant to the customers’ satisfaction level.

Furthermore, the proposed model is interpreted as a multi-objective optimization problem where, the fuel consumed by the vehicles and the total number of vehicles used to serve the customers are minimized and the total satisfaction rate of customers is maximized. A new solution based on
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the evolutionary algorithm was proposed and its performance on several completely random generated instance problems was compared with the CPLEX Solver and NSGA II as a popular non-domination based genetic algorithm for multi-objective optimization. According to the findings and the hypervolume indicator in our comparative study, the proposed evolutionary outperforms NSGA-II. Based on the results, the waiting time imposed on vehicle has increased in some instances and it maybe costs in some transportation systems. Thus, the future research on the proposed model of this paper can be directed by adding another new objective function for minimization of total waiting times. The interested practitioners can also develop the model with concept of vehicles heterogeneity. Because the main issues of this paper like fleet size and energy consumption could be varied when the heterogeneous fleet is employed. The concept of heterogeneity maybe concerned with the ownership of vehicles and the available private fleet. The inclusion of Multi depot VRPTW and VRP with fuzzy travel times in addition to fuzzy time windows can also be another issues appealing to the researchers.

7. References


Seyed Farid Ghannadpour


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Operational Research, Vol. 177, No. 2, pp.813-139.


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