The Special Application of Vehicle Routing Problem with Uncertainty Travel Times: Locomotive Routing Problem

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Abstract

This paper aims to study the locomotive routing problem (LRP) which is one of the most important problems in railroad scheduling in view of involving expensive assets and high cost of operating locomotives. This problem is assigning a fleet of locomotives to a network of trains to provide sufficient power to pull them from their origins to destinations by satisfying a rich set of operational constraints and minimizing the total operational cost. This problem is the special application of vehicle scheduling and it is modeled by using the vehicle routing problem with time windows (VRPTW) to optimal assignment of locomotives to assembled trains. Almost all of the prior models were deterministic and an important issue, widely ignored in prior research in locomotive optimization, is the presence of significant sources of uncertainty in transit times, travel times and changes to the train schedule. Therefore, in this paper unlike most of the work where all the times are deterministic, uncertainty in travel time is considered. Because travel times in reality fluctuate due to a variety of factors and its understanding and management in transportation networks is very important. The concepts of fuzzy sets and fuzzy control systems are considered to model the uncertainty in travel times. Besides, a genetic algorithm (GA) with various heuristics is proposed to tackle the proposed model and its performance is evaluated in different steps on various test problems generalized from a set of instances in the literature. The computational experiments on data sets illustrate the efficiency and effectiveness of the proposed approach.

Keywords*:* Locomotive Routing Problem, Vehicle Routing and Scheduling, Fuzzy Travel Time, Genetic Algorithm.

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1. Introduction

Locomotive routing problem (LRP) is one of the most important problems in railroad planning in view of involving expensive assets and high cost of operating locomotives. A large railroad may have billions invested in their fleet of locomotives. Too many locomotives mean that hundreds of millions are invested in equipment that is not yielding a return. Given the size of the investment, railroads have tried for decades to tap the power of optimization tools to manage their fleets more effectively [Powell et al. 2012]. Therefore Railroads face the challenge of determining the right fleet size and mix, as well as operating the fleet in an efficient way [Bouzaiene-Ayari et al. 2014]. In the literature review this problem has been reviewed under following topics as mostly similar concepts:

- Locomotive Assignment Problem (LAP)
- Locomotive Routing Problem (LRP)
- Locomotive scheduling problem
- locomotive Planning Problem (LPP)

The position of this problem in the scheduled railroads is illustrated in Figure 1. [Yaghini and Lessan, 2010] where, the planning stage is divided into strategic, tactical and operational horizon. These stages are in accordance with the length of the respective planning horizon and the temporal impact and relevance of the decision. In strategic planning, a variety of strategic planning studies for freight & passenger transportation and regarding to rolling stocks the objective followed is usually to minimize the required fleet size. But the locomotive routing problem (LRP) mainly discussed at the tactical and operational levels where, the available rolling stock is given and one usually wants to minimize the costs incurred by light running. In the operational planning, the activities are planned in the short term and it is much more sensitive to errors in either the train plan or the locomotive snapshot. The train schedule should be given as an input of locomotive assignment problem and then a locomotive routing problem should be solved. If the train schedule is affected by delays and disruptions events, the dynamic LAP should be considered and the model should be re-optimized based on the new situations. In this paper it is tried to consider delays or disruptions events in another point of view and by using the uncertainty travel times. It could be achieved by the records and historical data of a railroad companies and the concept fuzzy sets and fuzzy control systems. Every day many railroad companies assign thousands of locomotives to thousands of trains and every year they invest considerable cost to manage and operate the locomotives according to a properly assignment plan. Due to the size of real-life problems, even a small percentage improvement toward a better efficiency in the use of locomotives, can lead to significant economic savings. It is also important for countries where their national railroad proceeds toward a privatization and they pay more and more attention on operating cost, punctuality and performance and customers' satisfaction. So find a way to design a proper locomotives routing policy can be an important decision in railroad companies and it has attracted considerable attention from academics, consultants, and operations research groups within the railroads [Bouzaiene-Ayari et al. 2014]. Considering the Reducing of deadhead movement time of locomotives, waiting time to receive a locomotive and their associated costs is one of the important issues in planning, routing and assignment of locomotives. This issue determines the importance of LRP especially in uncertain conditions when there are limitations in fleet size (locomotives), costs of purchasing and adding new locomotives to the network, limited efficiency in utilization and etc. [Yaghini and Lessan, 2010]. Therefore, a growing interest for using optimization techniques in railroad problems has appeared in the operation research literature (see e.g. [Yaghini, Sharifian and Akhavan, 2012; Piu and Speranza, 2014; Pashchenko et al. 2015 and Piu et al. 2015]). Unfortunately, the Locomotive Routing Problem (LRP) is a very complex from a mathematical modeling and (even more) computational point of view, because of its detail richness and its size in real-life applications. The most of the studies formulated the LPP as a mixed-integer programming problem that is quite complex and accordingly, their solution technique is time consuming [Vaidyanathan et al. 2008 and Bouzaiene-Ayari et al. 2014].

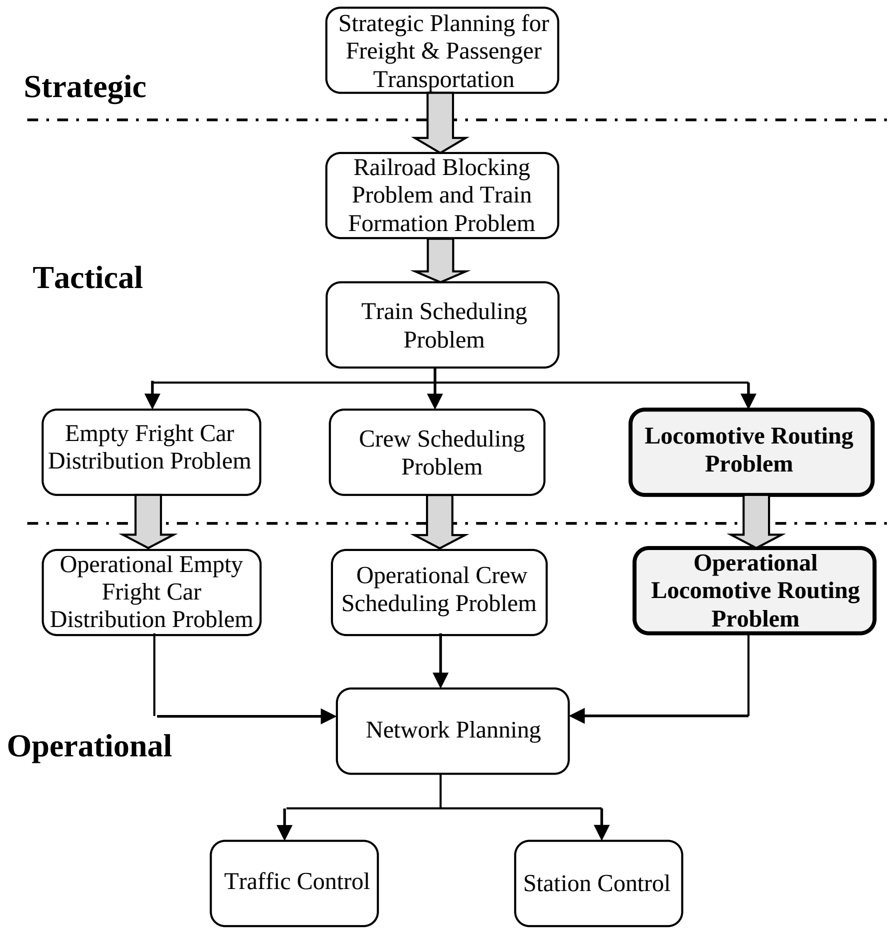

Figure 1. Planning stages in Railroad scheduling

The locomotive problem is not just a large integer program, it is a hard integer program that involves difficult features [Bouzaiene-Ayari et al. 2014].But this problem could be generally modeled as the vehicle scheduling problem to find the locomotive routing & assigning plan and improve the computational effort [Ghoseiri and Ghannadpour, 2009; Ghoseiri and Ghannadpour, 2010; Teichmann et al. 2015; Torres, 2003]. So this paper in continuation of previous researches tries to consider the locomotive routing problem as special application of vehicle scheduling and model it by using the vehicle routing problem with time windows (VRPTW) to find the optimal assignment of locomotives to assembled trains.

Almost all of the prior models were deterministic and an important issue, widely ignored in prior research in locomotive optimization, is the presence of significant sources of uncertainty in transit times, travel times and changes to the train schedule [Bouzaiene-Ayari et al. 2014]. So the plans being used do not satisfy real conditions and may has a negative impact on operations [Vaidyanathan et al. 2008].

Powell et al. [Powell et al. 2012] studied different issues and types of uncertainty that are widely discussed but rarely solved as follows:

- Dynamic schedule changes including uncertainty in transit times, travel times and changes to the train schedule: according to Figure 1, the train schedule is as an input of locomotive routing problem and any delays and disruptions events in train scheduling (or adding/dropping trains to/from the schedule) have major effects on planning and locomotive routing problem.
- Transit time delays: these can be as long as six to 12 hours for the shorter movements of an Eastern railroad, to

more than a day for the long movements of the western railroads. This issue mainly includes deadhead locomotive movements which should properly managed in order to have available locomotive in the right time for attaching to the train; Therefore, since the train traffic schedule is an uncertain concept, this is uncertain too.

- Shop delays: maintenance managers will provide estimates of when a locomotive will be ready to leave a shop, but these are just estimates
- Equipment failures: locomotives may fail unexpectedly, and this represents an additional source of uncertainty.

Therefore, in this paper unlike most of the work where all the times are deterministic, uncertainty in travel times is considered. Because travel times in reality fluctuate due to a variety of factors and its understanding and management in transportation networks is very important. The concepts of fuzzy sets and fuzzy control systems are considered to model the uncertainty in travel times.

Various locomotive scheduling models have been appeared in the literature. The papers by Cordeau et al. [Cordeau, Toth and Vigo, 1998] and Piu & Speranza [Piu and Speranza, 2014] present an excellent survey of the existing locomotive planning models and algorithms for the locomotive planning problem. There are two kinds of locomotive planning models: Single and Multiple ones. Single locomotive planning models assume that there is only one type of locomotive available for the assignment. These models can be formulated as minimum cost flow problems with side constraints [Kasalica, Mandić and Vukadinović, 2013]. Some papers on single locomotive planning models are due to Forbes et al. [Forbes, Holt and Walts, 1991], Booler [Booler, 1980], and Fischetti and Toth [Fischetti, Toth, 1997]. Forbes et al. [Forbes, Holt and Walts, 1991] addressed a version of the problem where a single locomotive must be assigned to each train, no deadhead is allowed, and no maintenance requirements are taken into account. Single locomotive planning models are better suited for some European railroads rather than North American railroads since most North American railroads assign multiple locomotive types to trains.

Multiple locomotive planning models have been studied in [Florian et al. 1976; Cordeau et al. 2001; Rouillon, Desaulniers, and Soumis, 2006; Ziarati, Chizari and Mohammadi Nezhads, 2005]. Florian et al. [Florian et al. 1976] introduced an integer programming model based on a multi-commodity network for the case where several locomotives can be assigned to each train. Ziarati et al. [Ziarati, Chizari and Mohammadi Nezhads, 2005], solved the locomotive assignment model using train delays and proposed an evolutionary approach to solve the cyclic locomotive assignment planning problem. Regarding to multiple locomotive planning models, Ahuja et al. [Ahuja et al. 2005] presented the real-life locomotive scheduling faced by CSX transportation, a major US railroad company. They formulated the locomotive planning problem as a mixed-integer programming problem and solved it using techniques from Very Large Scale Neighborhood Search (VLSN), linear programming-based relaxation heuristics, and integer programming. Vaidyanathan et al. [Vaidyanathan et al. 2008] extended this approach to several dimensions by adding new constraints to the planning problem required by railroads and by developing additional formulations namely *consist formulation* and *hybrid formulation* necessary to transfer solutions of the models to practice. In this area, the robust optimization methods to solve the locomotive planning problem (LPP) have been developed in [Vaidyanathan, Ahuja, and Orlin, 2008]. In this paper two major sets of constraints were considered as *locomotive fueling and locomotive servicing constraints*. first constraints group requires visiting a fueling station at least once for every F miles of travel, and the second group requires visiting a service station at least once for every S miles of travel. This problem was formulated as an integer programming problem on a suitably constructed space-time network and it was shown that this problem is NP-Complete. Recently an extended locomotive assignment problem has been modeled by Teichmann et al. [Teichmann et al. 2015] where, a transport operator can use different classes of the locomotives to serve individual connections, some connections must be served by a predefined locomotive class, and the locomotives can be allocated to several depots at the beginning.

As mentioned earlier, the considered LAP in this paper is modeled by the vehicle routing problem with time windows (VRPTW) as one of the most important and widely studied combinatorial optimization problems. This problem seeks to determine the optimal number of routes and the optimal sequence of customers (from a set of geographically dispersed locations that pose a daily demand for deliveries) visited by each vehicle, taking into account constraints imposed by the vehicle capacity, service times and time windows, and defined by the earliest and latest feasible delivery time. The literature of the VRPTW, due to its inherent complexities and usefulness in real life is rich in different solution approaches. Different types of heuristic methodologies, which seek approximate solutions in polynomial time instead of exact solutions with an intolerably high cost, are available in the literature of the VRPTW. In addition, it is shown that heuristics based on decomposition techniques (e.g., column generation and Lagrangian relaxation) may provide very good quality solutions when sufficient computational time is available [Pepin et al. 2009; Qmasari, Hosseini Motlagh and Jokar, 2017; Samani and Hosseini-Motlagh, 2017]. Thus, various heuristic approaches have been developed, ranging from local search methods to methods based on mathematical programming decomposition techniques and meta-heuristics. Applying different meta-heuristics to solve the VRPTW can be extensively found in the literature and there are many papers used evolutionary algorithms [Chiang and Hsu, 2014; Dhahri, Zidi and Ghedira, 2014; Ghannadpour, Noori and Tavakkoli-Moghaddam, 2014]. In addition the basic features of each method and experimental results for the benchmark test problems have been presented and analyzed. Other very good techniques and applications of the VRPTW and its developments can be found in [Majidi, Hosseini Motlagh, Ignatius, 2017; Hosseini Motlagh et al. 2017; Hiermann et al. 2016; Miranda and Conceição, 2016].

The remaining parts of paper are organized as follows. Section 2 defines the locomotive assignment problem with uncertainty travel times. Section 3 introduces the hybrid genetic search algorithm to solve the problem. Section 4 discusses the model validation and results and Section 5 provides the concluding remarks.

2. Locomotive Routing Problem with Uncertainty Travel Times

As mentioned earlier, the proposed locomotive routing problem (LRP) is considered as a special application of vehicle scheduling and modeled using the vehicle routing problem with time windows (VRPTW) to optimal assignment of locomotives to assembled trains. In this model the trains act as customers of a VRPTW that should be serviced in their time windows and the locomotives are equivalent to vehicles. Moreover, this problem generally includes a set of homogeneous locomotives, a set of depots where locomotives are initially located, and a set of pre-scheduled trains (C_i) which their origins and destinations (O_i, D_i) are known during the T-days planning horizon. The distance (or travel time) between origin and destination of each train is corresponds the service time (f_i) of each customer in VRPTW and the distance (or travel time) between two trains C_i and C_j is the distance (travel time) between the destination node of train i (D_i) and the origin node of train j (O_i). Figure 2 illustrates the locomotive routing problem as VRPTW. In this figure $d_{O_iD_i}$is the service time of train (i) and $t_{D_iO_j}$is the travel time required to dispatch a locomotive (without train) from destination node of train (i) to origin node of train (j) and it is named deadhead (inactive) movement. In other word, this locomotive has been assigned to train (i) to haul it to its destination D_i and after that it is dispatched to origin node of train (j) to provide sufficient power for pulling. In this case the locomotives are not attached to a train and it just need to be moved from on station to another. Deadhead movements are the input of model and they should be calculated for each pair of trains before. For calculation, the standard scheduling model [Higgins and Kozan, 1997] which consider the limitations of blocks, positions and destinations of the stations is used. The right hand side of this figure shows the trains schedule graph resulted from train scheduling problem and according to Figure 1 should be given as another input of locomotive routing problem to identify the origins, destinations and time windows of trains $[e_i, l_i]$. So, all the operational limits as the positions of the stations, stationary stops, prayer times stops, single and double tracks, and etc. have been carefully considered. The time windows assigned to each train is one the most important constraints where the trains must be serviced within their time windows and the locomotives should be assigned to pull them on time. The time windows width could be varied for trains and it is according to punctuality and operating class of railroads. If each locomotive k arrives to pull each train (i) before the earliest time of its service initiation (e_i), it is incurred the waiting time and it is assumed the delay in service is not allowed. Eventually, Figure 3 is a typical output of the defined problem and for more details about the classical problem see [Ghoseiri and Ghannadpour, 2010]. According to this figure each locomotive starts its journey from a depot and reaches to the origin of a train and hauls the train to its destination. Afterward, it is decided for the locomotive whether should be returned to its home depot or be dispatched to the origin of another train. The factor which forces the locomotives to return to its home depot is the maximum allowable operating time. In this regards the total travelled time of each locomotive should be less than the pre-defined maximum operation time. Finally, it is tried to design the proper routing and assignment policy that in which the total travelled time of locomotives is minimized and following constraints are satisfied:

- Hard Time windows of trains should be observed.
- Each train is served exactly once by one locomotive
- Each locomotive is starting from depot and ending at the same depot.
- Maximum allowable travel times and operation time should be observed for each locomotive

Moreover, the most important assumptions of the proposed model are as follows:

- Total travel time is considered as the goal while the main purpose of this type of issue is cost reduction and it could be considered a part of this objective function.
- The homogeneous VRPTW is assumed to be considered and so the locomotives of the same type are used for planning
- The model tries to find the routes with shortest travel time.

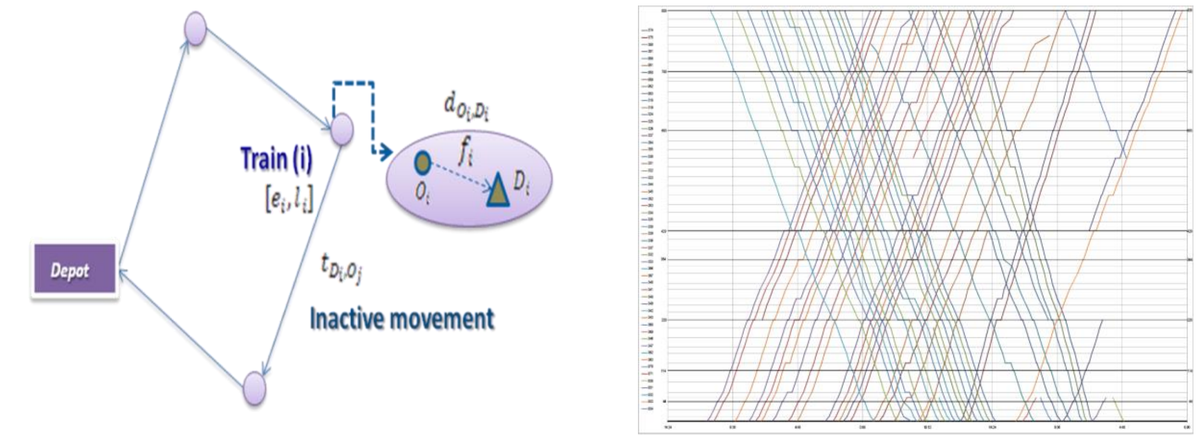

Figure 2. VRPTW & locomotive Routing

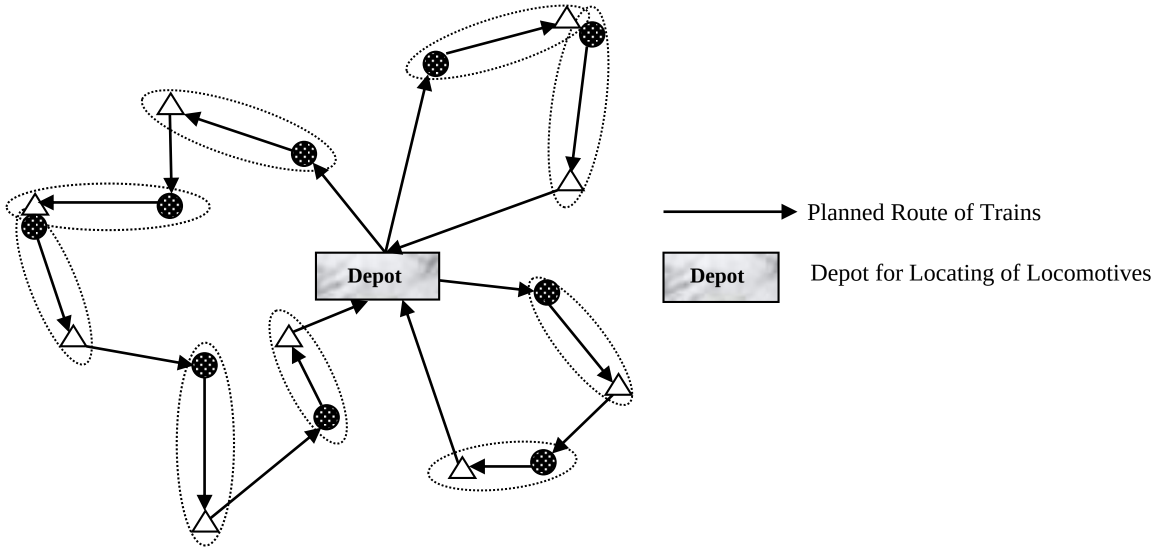

Figure 3. Typical Output for the Locomotive Routing Problem

As mentioned earlier, in this paper unlike most of the work where all the times are deterministic, uncertainty in travel time is considered. Because travel times in reality fluctuate due to a variety of factors and its understanding and management in transportation networks is very important. This parameter can be modeled by the probability theory and based on the historical data as well as real times on railway networks. The probability theory can be used when the representative historical data are available. In contrast, fuzzy sets do not require such assumptions and can be used with little knowledge about the historical data. They express intuitive knowledge rather than exact uncertainty distribution. Moreover, the use of fuzzy sets is simple and naturally an extended version of arithmetic on real numbers. For these reasons, the fuzzy numbers are used as the representation of these uncertain values. So this value is modeled as triangular fuzzy numbers $(\underline{x}, x^*, \overline{x})$ where value x^* (one-element core) represents knowledge of type "travel time is around x^*" and support $[\underline{x}, \overline{x}]$ models statement "travel time lays between $\underline{x}$ and $\overline{x}$". The uncertainty travel times naturally is propagated to arrival times of locomotives to each train (at_i) and start of service times (t_i) or start time of pulling trains to their destinations. When the fuzzy travel time is considered, these parameters need to be modified to $\tilde{t}_i$ and $\widetilde{at}_i$. Therefore the possible arrival time of each locomotive to train i is as follows:

$$\widetilde{at}_i = \tilde{t}_{i-1} \oplus \tilde{t}_{O_{i-1}D_{i-1}} \oplus \tilde{t}_{D_{i-1}O_i} \qquad (1)$$

Where $\tilde{t}_{O_{i-1}D_{i-1}}$ is the time required to pull train i to its destination and $\tilde{t}_{D_{i-1}O_i}$ is the fuzzy travel time between trains $i-1$ and i. Moreover, the waiting time imposed on each locomotive when it arrives to each train (i) is $\tilde{w}_i = \tilde{t}_i - \widetilde{at}_i$. Therefore, the most important point to consider these parameters, is relevant to calculate the service start time $(\tilde{t}_i)$ based on the uncertainty travel times and locomotives' arrival times $(\widetilde{at}_i)$ which is discussed in details at the next section.

3. Solution Procedure

The proposed model is solved by an evolutionary method based on genetic algorithm. Genetic algorithm is a class of adaptive heuristics based on the drawing concept of evaluation – "survival of the fitness", and it has been developed by J.Holland [Holland, 1975] at the University of Michigan in 1975. The using Hybrid-GA is designed as follow.

3.1 Chromosome Representation

A solution to the problem is represented by an integer string of length N, where N is the number of customers which need to be served. All routes are encoded together, with no special route termination characters in between;

chromosomes are decoded back into routes based on the feasibility conditions namely maximum allowable operating time and servicing without delay time.

3.2 Fuzzy Logic for Uncertainty Travel Time

As mentioned before, the service start time at each origin point of train *i* (start time for pulling train *i* to its destination) cannot start before the earliest start time (e_i) and it is also uncertain because it is directly related to uncertainty travel times. After beginning of e_i, service can be started and there is no possibility to start service before e_i even if the locomotive is already there. Moreover, this paper uses the hard time windows and in this case the delay in service is not allowed and each locomotive must be assigned to train (i) before latest service time (l_i). So the service start time on train i ($\tilde{t_i}$) is more depended to earliest service time (e_i), latest service time (l_i) and the possible arrival time ($\widetilde{at_i}$). In this regard, the value of $\tilde{t_i}$ can be calculated based on four different relation between e_i, l_i and $\widetilde{at_i}$ as $max\{\widetilde{at_i}, e_i\}$ and $min\{\widetilde{at_i}, l_i\}$. Figure 4 illustrates some possible relationships as an example. In this figure, the dashed lines are the approximation made to adjust the fuzzy number to triangular fuzzy representation..

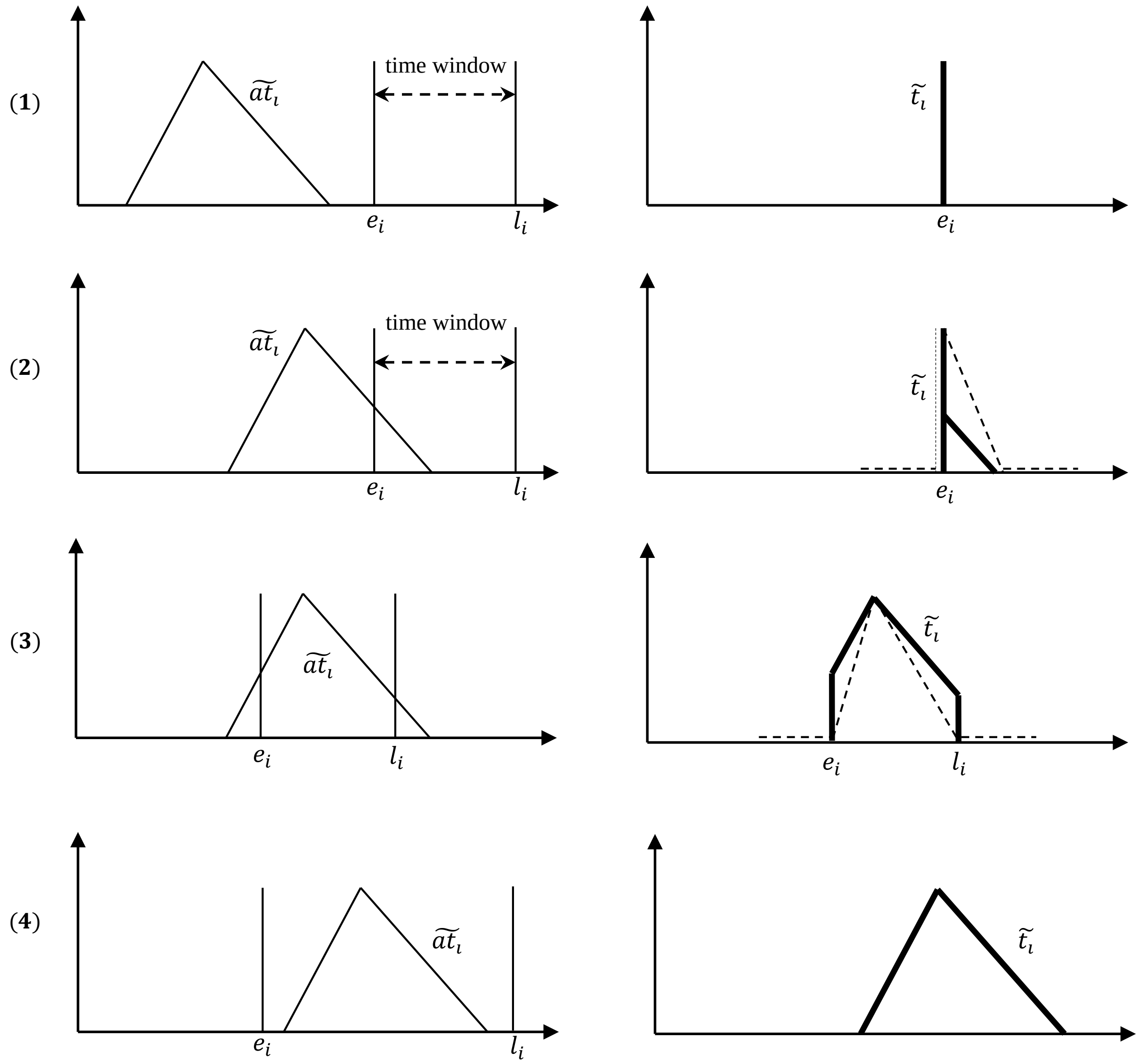

Figure 4. Service start time based on the arrival time, earliest and latest start time

The first case of this figure identifies the service start time when all possible arrival time is before the beginning of time window. In the second case, the possible arrival is before and after the earliest start time; however, the modal value is before it. The third case is similar the previous case; however, the modal value is within the time window. In the last case, all possible arrival time occur within the time window and it is considered as the service start time.

Therefore, according to estimated service start time for each train and the fuzzy travel time to next one the possible arrival time of other trains is approximated and it could be continued up to end of a route. One important factor to force the locomotives to return to its home depot is the maximum allowable operating time along each route. In this regards the total travelled time of each locomotive which is equivalent to the length of its travelled route, should be less than the pre-defined maximum operation time. This concept is tackled by the concept of fuzzy control systems and discussed in the next section

3.3 Fuzzy control systems for length of tours

As mentioned earlier, another important point considered here is the maximum allowable travel time of locomotives that forces them to return to depot. This feasibility condition is easily controlled for crisp travel times and detailed investigation is needed when the fuzzy travel time is used. According to previous sections the uncertainty travel times have been modeled by fuzzy travel times as triangular fuzzy number and clearly, the total travelled time of locomotives is a triangular fuzzy number as well. So, when the maximum allowable travel time of vehicles is denoted by *R*, the available time of each locomotive *k* after giving services to *n* trains ($\widetilde{AT}_n^k$) is as follows where $\widetilde{TO}_n^k$ is the total travelled time by locomotive *k* after serving the n^{th} train.

$$\widetilde{AT}_n^k = R \ominus \widetilde{TO}_n^k \tag{2}$$

$$\widetilde{TO}_n^k = \tilde{t}_{n-1} \oplus \tilde{t}_{O_{n-1}D_{n-1}} \oplus \tilde{t}_{D_{n-1}O_n} \tag{3}$$

It is clear that the "strength" of preference for this locomotive to serve the next train after serving *n* trains depends on available time $\widetilde{AT}_n^k$. This preference can be "LOW", "MEDIUM" or "HIGH" and the preference index is denoted by $p_n \in [0,1]$, which describes the strength of this preference to send the locomotive to the next train. When $p_n = 1$, the locomotive is absolutely certain to serve the next train and when $p_n = 0$, the locomotive must return to the depot. Available time ($\widetilde{AT}_n^k$) can also be subjectively estimated as "SMALL", "MEDIUM" and "LARGE". It is assumed that the strength of preference depends on available time, and hereby three main rules can be considered:

- Rule 1: if $\widetilde{AT}_n^k = SMALL$ then $p_n = LOW$,
- Rule 2: if $\widetilde{AT}_n^k = MEDIUM$ then $p_n = MEDIUM$,
- Rule 3: if $\widetilde{AT}_n^k = LARGE$ then $p_n = HIGH$,

Every rule represents a fuzzy relation between the available time and preference strength. So for known available time $\widetilde{AT}_n^k$ that remains after serving *n* trains, the strength of preference (p_n^*) to send the locomotive to the next train is easily calculated. This approximate reasoning procedure is graphically shown in Figure 5. This figure presents the membership function of the index preference obtained by applying the approximate rules and its center of gravity. Finally, based on the value of chosen preference index (p_k^*), a decision should be made whether to send a locomotive to the next train or return it to the depot. This decision is made as follows: the locomotive should be sent to the next train when $p_k^* \geq p^{**}$ where p^{**} is given from interval [0,1]. Otherwise, it should be return to the depot. It should be noted that the lower values of p^{**} represent the endeavor to use the travelled time of locomotive as much as possible.

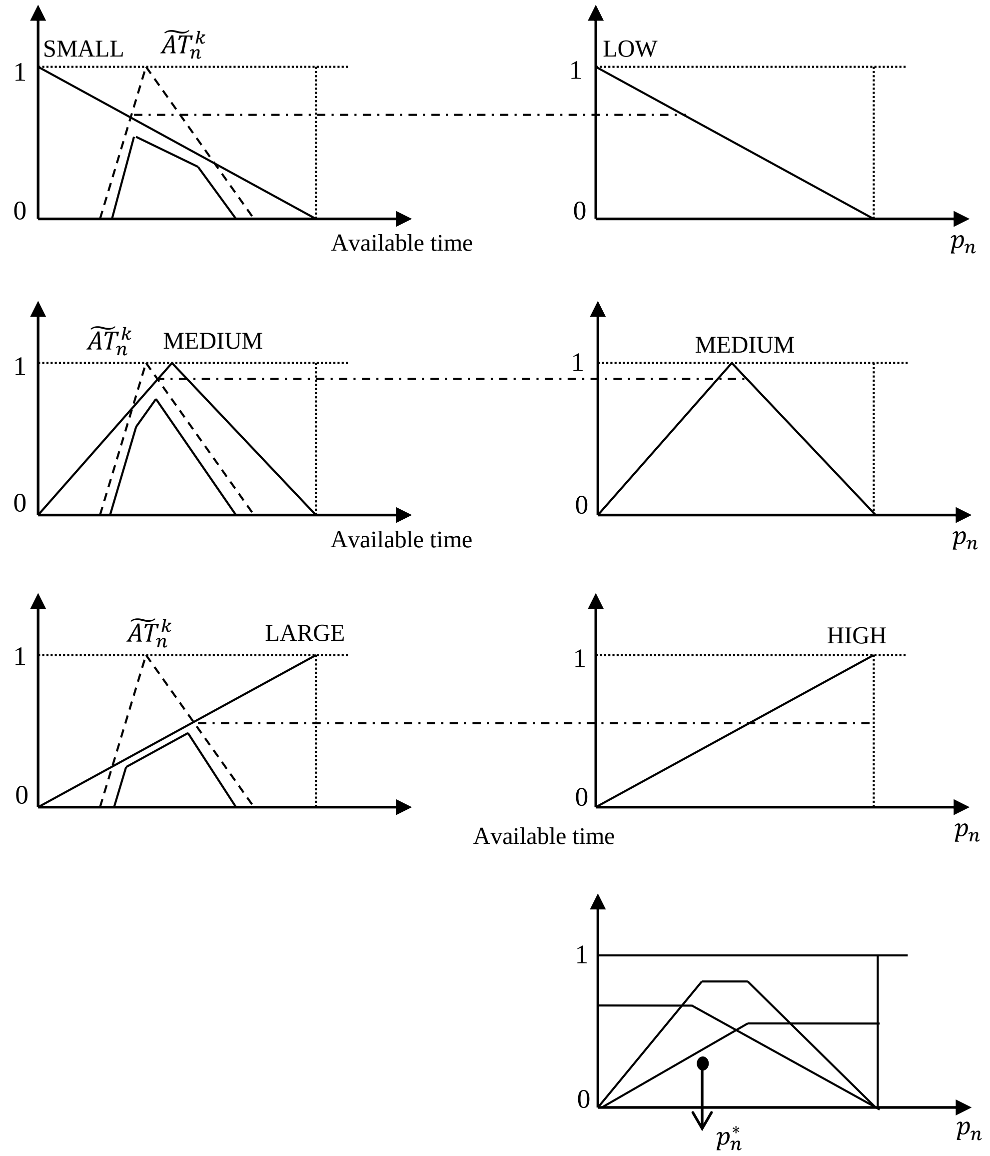

Figure 5. Determining preference strength (p_n^*) for known available time ($\widetilde{AT}_n^k$)

3.4 Initial Population

Part of the population is initialized using modified Push-Forward Insertion Heuristic (PFIH) method and λ-interchange mechanism, and part is initialized randomly. The PFIH method, first introduced by Solomon [Solomon, 1987] to create an initial route configuration. This paper uses the modified PFIH method according to defined problem that cost function for inserting a customer into a new route is as follows:

$$Cost\ (C_i) = -\alpha\tilde{t}_{(0)O_i} + \beta l_i + \gamma\left(\frac{\left|\theta_{O_i} - \theta_{D_j}\right|}{360}\right) \times \tilde{t}_{(0)O_i} \quad (4)$$

Where θ_{O_i} and θ_{D_j} are the polar angle of the train in question and the last visited train in the last formed route, $\tilde{t}_{(0)O_i}$ is the fuzzy travel time between the home depot and train i and l_i is latest arrival time at train i. Therefore the unrouted train with the lowest cost is selected as the first train to be visited. Once the first train is selected for the current route, the heuristic selects from the set of unrouted trains the train j^* which minimizes the total insertion cost between every edge $\{k, l\}$ in the current route without violating the time and maximum route time constraints. The cost function of the proposed model is assumed to be minimization

of total travel time consumed by locomotives. It should be noted that the above mentioned cost value is a fuzzy value and the ranking concept of fuzzy numbers is used to determine the lowest cost [Parandin and Fariborzi Araghi, 2008].
This paper uses a λ-*interchange mechanism* to move trains between routes to generate neighborhood solution for the problem (for more details see [Ghoseiri and Ghannadpour, 2010]). In one version of the algorithm called *GB* (global best), the whole neighborhood is explored and the best move is selected. In the other version, *FB* (first best), the first admissible improving move is selected if one exists; otherwise the best admissible move is implemented.

3.5 Selection

This paper uses a standard *k*-tournament selection where a tournament set of size *k* is randomly drawn from the population and the chromosome with a lower cost (according to ranking concept of fuzzy numbers) is selected and will then be recombined via the recombination operators to create potential new population.

3.6 Crossover & Mutation

One of the unique and important aspects of the GA is the important role of the crossover operator. The classical crossovers (e.g., one-point crossover and n-point crossover) are not appropriate for this sequencing model because of duplication and omission of vertices. This paper uses the best cost-best rout crossover (BCBRC), which selects a best route from each parent and then for a given parent, the customers (trains) in the chosen route from the opposite parent are removed. The final step is to locate the best possible locations for the removed trains in the corresponding children. This procedure is illustrated in Figure 6. According to this figure, route 4 from parent #1 is selected and the customers on this route are removed from the routes of parent #2. This process is done similarly for another parent. Hereinafter for each parent the best location of removed customers are determined by the insertion procedure one at a time. This procedure is continued until two feasible offspring are produced. According to this figure, route (3) from parent #1 is selected randomly and the trains on this route are removed from the routes of parent #2. This process is done similarly for another parent. Hereinafter for each parent the best location of removed trains are determined by the insertion procedure one at a time. Moreover, the mutation schemes that used here are swap node and swap sequence.

3.7 Hill-Climbing & Recovery

Also the hill-climbing is used in order to improve the chromosomes obtained through crossover and mutation. Hill-climbing is a scheme for randomly selecting a portion of the population and improving them by a few iterations of removal and reinsertion. At the end, to additionally improve the quality of the population, the worst portion of the population will be replaced with the best of the parent population.

4. Numerical Example and Results Analysis

This section describes computational experiments carried out to investigate the performance of the proposed GA. Due to lack of the prior work on the proposed model, we have to analyze the validity and effectiveness of proposed method in three different sections. Moreover, the first idea of using these models for LRP has been considered in our recent paper [Ghoseiri and Ghannadpour, 2009 and Ghoseiri and Ghannadpour, 2010] where, the effectiveness of the perspective was tested meticulously by an exact solver. The first section analyzes the model on the pure VRPTW when the travel times are certain and in the second one the changes of using proposed uncertainty travel times would be analyzes. Finally we applied the model on a complete randomly generated instance for locomotive assignment problem as a case study. So, at the first section it is not expected that the proposed method yields new best known results for this general existing benchmarks, but the gap should be studied and reported to be able to evaluate the effectiveness of the proposed method.

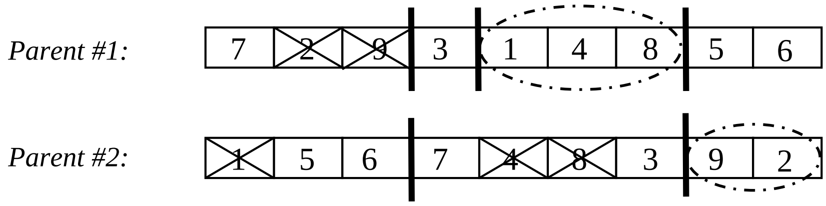

Figure 6. Best cost route crossover

Table 1. Results of the Solomon's instances for certain travel times

Pro.	Best Known	Proposed Method		Pro.	Best Known	Proposed Method	
	Travel Cost	Travel Cost	% diff.		Travel Cost	Travel Cost	% diff
C101	828.94	828.94	0.00	C201	591.56	591.56	0.00
C102	828.94	828.94	0.00	C202	591.56	591.56	0.00
C103	828.06	828.06	0.00	C203	591.17	591.17	0.00
C104	824.78	824.78	0.00	C204	590.60	599.96	1.56
C105	828.94	828.94	0.00	C205	588.16	588.88	0.12
C106	828.94	828.94	0.00	C206	588.49	588.88	0.07
C107	828.94	828.94	0.00	C207	588.29	591.56	0.55
C108	828.94	828.94	0.00	C208	588.32	588.32	0.00
C109	828.94	828.94	0.00				
R101	1650.80	1668.0	1.03	R201	1206.42	1210.8	0.36
R102	1434	1494.7	4.06	R202	1091.21	1091.22	0.00
R103	1237.05	1252.1	1.20	R203	935.04	995.8	6.10
R104	974.24	974.24	0.00	R204	789.72	832.2	5.10
R105	1377.11	1382.5	0.39	R205	994.42	1087.8	8.58
R106	1252.03	1270.3	1.44	R206	833	902.11	7.66
R107	1100.52	1108.8	0.75	R207	814.78	904.90	9.96
R108	960.26	971.91	1.20	R208	726.823	774.18	6.12
R109	1169.85	1206.7	3.05	R209	855	875.6	2.35
R110	1112.21	1156.5	3.83	R210	938.58	938.58	-0.00
R111	1096.72	1120.1	2.09	R211	761.10	773.03	1.54
R112	976.99	1036.9	5.78				
RC101	1636.92	1668.7	1.90	RC201	1134.91	1205.1	5.82
RC102	1470.26	1493.2	1.54	RC202	1181.99	1196.1	1.18
RC103	1261.67	1331.8	5.27	RC203	1020.1	1060.0	3.76
RC104	1135.48	1177.2	3.54	RC204	798.46	901.46	11.43
RC105	1589.4	1589.4	0.00	RC205	1300.25	1342.3	3.13
RC106	1425.3	1425.3	0.00	RC206	1153.93	1194.8	3.42
RC107	1222.1	1222.1	0.00	RC207	1040.67	1040.6	-0.01
RC108	1142.66	1148.2	0.48	RC208	785.93	898.50	12.53

4.1 Analysis the proposed method on the well-known benchmarks

In this section, the effectiveness of the proposed method is evaluated on the Solomon's VRPTW benchmark problem instances [Solomon, 1987]. These samples which have various kinds of assumptions have been considered consequently in literature review in order to check the effectiveness of the suggested algorithm. It should be noted that the proposed evolutionary method is coded and run on a PC with Core 2 Duo CPU (3.00 GHz) and 2.9 GB of RAM. Moreover, the model is implemented under parameters of Population size = 100, Generation number = 1000, Crossover rate = 0.80, Mutation rate = 0.40, Repetition for experiments = 10. These parameters have been tuned and deeply analyzed in our recent studies [Ghannadpour, Noori and Tavakkoli-Moghaddam, 2014].

Table 1 presents a summary of results when the proposed method is applied on the well-known VRPTW instances where the total travel cost by the vehicles are minimized and when the certain travel times are used. The columns labeled "Best Known" give the best known published solutions in [Ghannadpour, Noori and Tavakkoli-Moghaddam, 2014]. The relative percentage Gap is also presented in this table for each instance and it is analyzed later.

In this table, the best solutions found by proposed method are reported over 10 runs. Moreover, the average computational time for classes C1, R1 and RC1 varies between 1.5 and 2.5 hours within 1000 generations and is between 3 and 5 hours for classes C2, R2 and RC2. The second classes require a larger CPU time due to the longer time windows, to allow a more flexible arrangement in the routing construction process.

In order to know the performance of the proposed method, the findings are compared with the best known solutions for each category of Solomon's problems. Table 2 summaries the results of Table 1 for each instance category. The average travel costs of the best known results and those found by the proposed method are presented in this Table. Additionally, the last column presents the total cost distance over whole 56 instances. The last row indicates the percentage difference between the results.

Table 2. Average results of proposed method and the best known solutions

Results	C1	C2	R1	R2	RC1	RC2	Total
Best known	828.38	589.77	1195.15	904.19	1360.47	1052.03	55761.47
Proposed method	828.38	591.49	1220.23	944.20	1381.99	1104.85	57111.04
% diff. D	0.00	0.29	2.05	4.23	1.55	4.78	2.36

According to table (2), the proposed method obtained superior results for the class of C1 & C2. On the other hand, for the remaining categories, solutions from the proposed method are between 2% and 4.7% larger in travel cost than the best known results Moreover, the difference between the results of the proposed method and best known solutions for all 56 instances is only 2.36% Therefore, the good quality results obtained by the model in general compare favorably, with respect to time and quality, to the best published results.

4.2 Analysis the proposed method with uncertainty travel times

Now, the results should be reconsidered, when the uncertainty travel times (fuzzy travel times) are used. To use this concept on the Solomon's instances, the predefined travel time between each customers i and j is changed to the triple $[t_{ij}^{(1)}, t_{ij}^{(2)}, t_{ij}^{(3)}]$ where $t_{ij}^{(2)}$ is equivalent to predetermined time in data set of Solomon's instances and $t_{ij}^{(1)}$ and $t_{ij}^{(3)}$ are selected randomly. As mentioned before, one important parameter that should be determined in this stage is p^{**} where is given from [0,1]. For instance, the distance costs of instance RC101 for different values of p^{**} are illustrated in Figure 7. According to this figure , when $p^{**} = 0$, one big route is permitted to be planned and just the capacity constraint is considered. In this regard, when such vehicle has used all its capacity, it should be returned to the depot. When $p^{**} = 1$, the number of routes is equal to the number of customers and each route consists of only one customer. Based on this figure, the least total expected distance to be covered by the vehicles is realized when $0.35 < p^{**} < 0.6$. So it is assumed that the preference value for the instance RC101 is 0.45 and then the related cost is calculated as Table 3 that presents the results of applying proposed uncertainty model on some randomly selected Solomon's instance problems from Table 1 and in classes of R & RC.

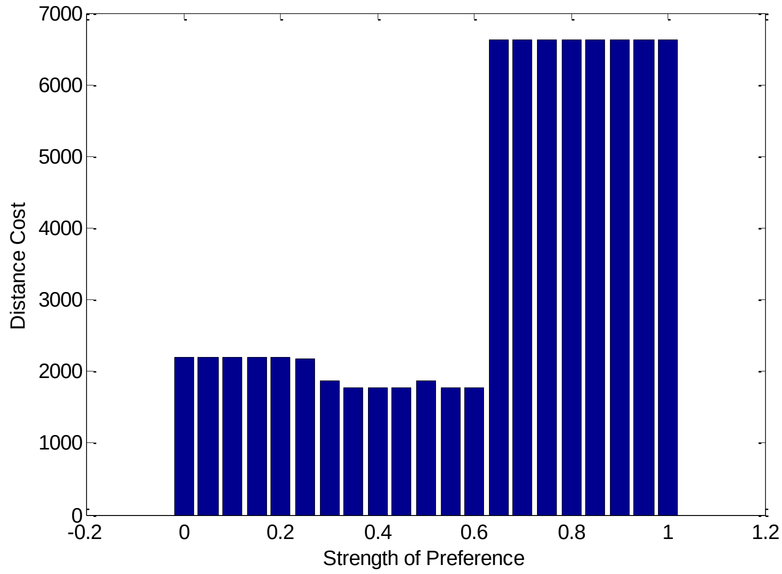

Figure 7. Different distance costs based on different strength of problem RC101

Table 3. Results of proposed method for VRPTW with uncertainty travel time

Pro.	VRPTW with Certain Travel Time (from Table 1)		VRPTW with fuzzy Travel Time	
			Travel cost	
	Distance cost (best known)	Distance cost (proposed method)	Total distance cost	Total travelled time
R104	974.24	974.24	1120.9	(1188,1191,1201)
R108	960.26	971.91	1250.3	(1290,1310,1346)
R206	833	902.11	1056.4	(1168,1181,1195)
R208	726.823	774.18	988.7	(1069,1154,1159)
RC101	1636.92	1668.7	1778.9	(1801,1882,1885)
RC106	1425.3	1425.3	1795.1	(2089,2187,2191)
RC203	1020.1	1060.0	1506.5	(1840,1862,1905)
RC208	785.93	898.50	990.1	(1060,1079,1144)

In this Table, the column labeled "Travel cost" is divided into two columns that show the total travelled distance and the total travelled time that they are equivalent in classical point of view. It should be noted that the total travelled time is considered as the objective function and the total distance cost is only reported for the obtained solution. According to this table, the minimum total distance travelled by vehicles is deteriorated when the fuzzy travel times are considered.

4.3 Case study

In this section, a complete randomly generated medium size problem is considered as a case study. It includes 80 nodes and 40 trains per day in a weekly planning horizon and they have to be serviced without any delay. A maximum permissible operating time of eighteen hours is defined for all the locomotives running in the planning horizon and maintenance time for each locomotive is assumed to be 6 hours. The decision maker is faced with a transportation network with uncertainty travel times and the travel times depend on the locomotives' speed. The locomotives' speed is variable at different points of the routes and could be defined by the historical data and in accordance with a normal probability distribution (for example it is assumed for one route from 45 to 65 km per hour). So, the uncertainty (fuzzy) travel times could be easily calculated by the distances and the probability Distribution of speed and by using the *min*, *max* and *mean* values of the related probability distribution.

One important required input is the trains schedule graph resulted from train scheduling problem that is prior to locomotive routing & assignment models. The trains schedule graph is necessary to identify the origins, destinations and time windows of trains$[e_i, l_i]$. The trains must be serviced within their assigned time windows and the related locomotives would available on these predefined time windows to pull trains on time.

Figure 8 summarizes the results of the above mentioned model and the related order of servicing. Based on the results 10 locomotives are needed to service all trains within their time windows to minimize the total travel time of locomotives and satisfy the predefined trains schedule graph. The total traveling time of this solution is (88.04, 95.11, 98.03) and the related traveling distance is 3016.8.

Locomotive 1	[	0	15	22	30	0	-	-	]
Locomotive 2	[	0	23	27	13	16	35	0	]
Locomotive 3	[	0	40	3	33	24	7	0	]
Locomotive 4	[	0	6	1	0	-	-	-	]
Locomotive 5	[	0	18	5	9	0	-	-	]
Locomotive 6	[	0	2	38	4	10	12	0	]
Locomotive 7	[	0	26	19	29	36	0	-	]
Locomotive 8	[	0	20	14	11	31	0	-	]
Locomotive 9	[	0	32	21	28	39	0	-	]
Locomotive 10	[	0	8	17	25	34	37	0	]

Figure 8. Result of locomotive assignment model

According to Figure 10, each locomotive serves the trains en route. For instance, the first locomotive starts its journey from depot and pulls trains 15, 22 and 30 to their destinations and then returns to the central depot for daily maintenance and inspection. In fact, this locomotive goes to the origin node of the 15th train zone and hauls the train to its destination and then goes to the origin node of the 22th train zone and keeps on going. i.e., the detailed order of servicing for this route is as $\{Depot, O_{15}, D_{15}, O_{22}, D_{22}, O_{30}, D_{30}, Depot\}$.

According to this plan the efficiency indices could also be calculated and analyzed. One important index related to locomotives within their availability is performance index (PI) [Yaghini and Lessan, 2010; Ghoseiri and Ghannadpour, 2009 and Ghoseiri and Ghannadpour, 2010] and it is calculated as follows:

$$PI = \frac{total\ travel\ time - total\ waiting\ time}{total\ travel\ time} \times 100 \qquad (5)$$

As mentioned earlier, the total travel time of locomotive k who serves to n trains during its planned route is as equation (6) and the total waiting time imposed on this locomotive during the mentioned route is as equation (7).

$$\widetilde{TO}^k = \tilde{t}_{n-1} + \tilde{t}_{O_{n-1}D_{n-1}} + \tilde{t}_{D_{n-1}O_n} \qquad (6)$$

$$\tilde{w}^k = \sum_{i=1}^{n} \tilde{t}_i - \widetilde{at}_i \qquad (7)$$

To improve the performance indices of locomotives it is better to consider the minimization of total waiting time as another objective function and develop the proposed model to the multi-objective concept.

5. Conclusion

This paper presented the locomotive routing problem (LRP) which is very important for railway companies, in view of high cost of operating locomotives. This problem was to determine the minimum cost assignment of homogeneous locomotives to a set of pre-schedules trains in order to provide sufficient power to pull the trains from their origins to their destinations. This problem was modeled by using of vehicle routing problem with time windows (VRPTW) where trains performed as customers and they should have serviced in pre-specified hard time windows. In this paper unlike most of the work where all the times are deterministic, uncertainty in travel time has been considered. The concepts of fuzzy sets and fuzzy control systems have been considered to model the uncertainty in travel times. A genetic algorithm (GA) with various heuristics has been proposed to tackle the model. The concepts of fuzzy sets and fuzzy control systems have also been considered to model for uncertainty travel times and checking the maximum allowable travel time of locomotives. The performance of the algorithm has been evaluated in different steps and on various test problems generalized from a set of instances in the literature. First, the model has been analyzed on the pure VRPTW and then a complete randomly generated instance for locomotive assignment problem was considered as a case study. The computational experiments on data sets illustrate the efficiency and effectiveness of the proposed approach. Moreover, to improve the performance of locomotives it is better to consider the minimization of total waiting time as another objective function and develop the proposed model to the multi-objective concept. So the applicability of this idea would be considered in our future study and it would be tried to analyze the multi-objective models and solutions in this direction.

6. References

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