Application of Support Vector Machine Regression for Predicting Critical Responses of Flexible Pavements

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Received: 24.08.2016 Accepted: 20.02.2017

Abstract

This paper aims to assess the application of Support Vector Machine (SVM) regression in order to analysis flexible pavements. To this end, 10000 Four-layer flexible pavement sections consisted of asphalt concrete layer, granular base layer, granular subbase layer, and subgrade soil were analyzed under the effect of standard axle loading using multi-layered elastic theory and pavement critical responses including maximum tensile strain at the bottom of asphalt layer and maximum compressive strain at the top of subgrade soil were calculated. Then the support vector machine regression was used to predict these two critical responses. Results of this study show that the SVM can be used as a reliable tool to predict critical responses of flexible pavements. Analysis of flexible pavements using SVM needs less computing time and the SVM can be used as a quick tool for predicting fatigue and rutting lives of different pavement sections in comparison with multi-layer elastic theory and finite element method.

Keywords: pavement analysis, Support Vector Machine, critical responses, standard axle load.

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1. Introduction

The first step in pavement design by means of mechanistic – empirical method is analysis of pavement structure and computation of pavement critical responses subjected to different loadings. Several methods have been proposed to analyze flexible pavements. Boussinesq (1885) was the first person, who obtained responses of a semi-infinite system affected by point loading [Boussinesq, 1885]. Equations provided by Boussinesq were developed in next years by other researchers for uniform distributed loads [Newmark, 1947; Sanborn and Yoder, 1967]. Equivalent thickness method was presented by Odemark (1949). He assumed that the deflection of a multi-layer pavement is equal to the deflection of an equivalent semi-infinite system, such that thickness and modulus of this equivalent system is equal to H and E, respectively. After conversion of multi-layer system into a semi-infinite system, stresses, strains, and deflections can be computed using Boussinesq equations [Odemark, 1949]. For the first time, Burmeister proposed stress and deformation equations for two and three-layer systems affected by circular loading [Burmister, 1945]. Schiffman proposed general solution to analyze stresses and strains in a multi-layer elastic system and this method is known as multi-layered elastic theory [Schiffman, 1962]. Now, most of the flexible pavements analysis programs use multi-layered elastic theory method to analyze the pavement structure. For example, we can refer to the programs of BISAR [Jong and Peutz, 1979], JULEA [Uzan, 1994], LEAF [Hayhoe, 2002], KENLAYER [Huang, 2004], Mnlayer [Khazanovich and Wang, 2007], and NonPAS [Ghanizadeh and Ziaie, 2015]. Pavement modeling using multi-layered elastic theory is simpler and solving system using computer requires less time compared to finite elements method. In addition, for non-professional users working with applications based on multi-layer elastic theory is easier than finite elements methods [Huang, 2004]. For the first time, Duncan et al. (1968) used finite elements method (FEM) to analyze pavement structure [Duncan, 1968]. The most common programs, which use finite element method to analyze flexible pavements are MICHPAV and ILLIPAV [Harichandran et al., 1990; Raad and Figueroa, 1980]. General finite element programs such as ANSYS and ABAQUS have also been used successfully for the analysis of pavement structure [Kim et al., 2009; Ahmed et al.,2015; Maitra, Reddy and Ramachandra, 2010; Zheng et al. 2013].

In finite element method, choosing correct form of elements has a major influence on desired accuracy. Finite element method is more capable in modeling of systems with specified dimensions, because the layered method has been proposed with the assumption of the infinity of layers in the radial direction. In addition, the finite element method is advantageous to the programs based on multilayered system theory for the nonlinear analysis of pavement [Huang, 2004]. However, in practical applications, it might not be possible to use finite element method due to the increase of analysis time; therefore, multi-layer elastic theory is preferred in comparison with finite element analysis method.

In order to design the pavement under the influence of standard axle loading using a mechanistic – empirical method, we need to analyze the pavement structure under the influence of this loading and to determine maximum horizontal principal tensile strain at the bottom of asphalt layer and also maximum vertical compressive strain on the top of subgrade soil. For this purpose, it is necessary to determine pavement responses at 10 different points. Then the fatigue and rutting lives can be estimated with respect to critical strain values.

Artificial intelligence (AI) techniques, such as, Artificial Neural Networks (ANN), Fuzzy Logic (FL), Genetic Algorithm (GA), Support Vector Machines (SVM) or hybrid methods of these techniques are successfully used to solve complex problems associated with Pavement engineering [Goktepe, Agar and Lav, 2006; Maalouf et al. 2008; Gopalakrishnan and Kim 2010; Lin and Liu, 2010; Patil, Mandal and Hegde, 2012; Terzi 2013; Gopalakrishnan et al. 2013; Fakhri and Ghanizadeh 2014; Soltani et al 2015].

If we can determine the critical responses of pavement using AI techniques, it is possible to increase the speed of pavement analysis several times faster than that of analysis using software based on multi-layered elastic theory or software based on FEM.

Support Vector Machine (SVM) is a machine learning technique that has gained enormous popularity in the field of classification, pattern recognition and regression.

SVM works on structural risk minimization principle that has greater generalization ability and is superior to the empirical risk minimization principle as adopted in conventional neural network models [Patil, Mandal and Hegde, 2012].

In the present study, SVM method has been proposed to predict critical responses of flexible pavements and the results obtained from this model have been compared with those of JULEA program.

2. Support Vector Machine

The main idea of support vector machines is to map the original data x into a feature space of higher order through a non-linear mapping function and construct an optimal hyper-plane in new space.

Assuming a set of data $S = \{(x_i, d_i)\}_{i=1}^{N}$, where x_i is the input data set, d_i is the desired result, and N corresponds to the size of the data set; the SVM regression function is expressed as follows [Smola and Scholkopf, 2004]:

$$y = f(x) = w_i \phi_i(x) + b \tag{1}$$

where ϕ_i(x) is the non-linear function of input *x*, and both w_i and *b* are constant coefficients. The constant coefficients (w_i and *b)* are determined by minimizing the regularized risk function as follows:

$$Minimize: R(c) = \frac{1}{2} \| w \|^2 + C \frac{1}{N} \sum_{i=1}^{N} L_\varepsilon(d_i, y_i) \tag{2}$$

Where

$$L_\varepsilon(d_i, y_i) = \begin{Bmatrix} |d_i - y_i| - \varepsilon, & |d_i - y_i| \geq \varepsilon \\ 0, & others \end{Bmatrix} \tag{3}$$

and C and ε are used defined parameters and y_i is the predicted value at period i. In Eq. (2), the first term is called regularized term and the L_ε(d, y) is called the ε-insensitive loss function. Loss function will be zero if the predicted value is within the ε – tube (Eq. (3) and Fig. 1).

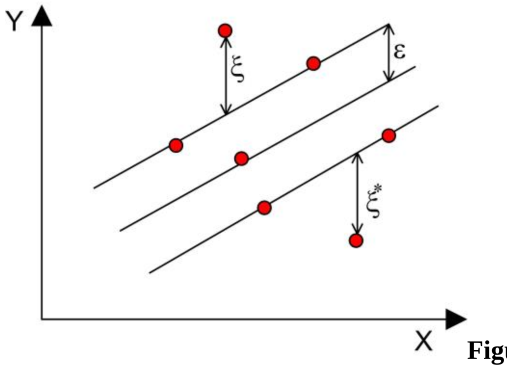


Figure 1. The concept of ε.

Hence, C specifies the trade-off between the empirical risk and the model flatness. By assuming two positive slack variables ξ and ξ*, which represent the distance from actual values to the corresponding boundary values of ε-tube (Fig. 1), the Eq. (2) is transformed into the following constrained form:

$$Minimize: R(w, \zeta_i, \zeta_i^*) = \frac{1}{2} \|w\|^2 + C \sum_{i=1}^{N} (\zeta_i + \zeta_i^*) \tag{4}$$

Subject to:

$$d_i - w_i \phi(x_i) - b \leq \varepsilon + \zeta_i$$

$$w_i \phi(x_i) + b - d_i \leq \varepsilon + \zeta_i^*$$

$$where \ \ \zeta_i^*, \zeta_i^* \geq 0, \quad i = 1,2,..., N$$

This constrained optimization problem can be solved using the primal Lagrangian form as the follows:

$$\begin{aligned} L(wi, \zeta_i, \zeta_i^*, \alpha_i, \alpha_i^*, \beta_i, \beta_i^*) = \frac{1}{2} \|w\|^2 + C \sum_{i=1}^{N} (\zeta_i + \zeta_i^*) \\ - \sum_{i=1}^{N} \alpha_i \left[w_i \phi(x_i) + b - d_i + \varepsilon + \zeta_i \right] \\ - \sum_{i=1}^{N} \alpha_i^* \left[d_i - w_i \phi(x_i) - b + \varepsilon + \zeta_i^* \right] \\ - \sum_{i=1}^{N} (\beta_i \zeta_i + \beta_i^* \zeta_i^*) \end{aligned} \tag{5}$$

Eq. (6) is minimized with respect to primal variables w_i, b, ζ and ζ*, and maximized with respect to non-negative Lagrangian multipliers

α_i; α_i^*; β_i and β_i^*. Applying Karush–Kuhn–Tucker conditions to the regression, and Eq. (6), yields the dual Lagrangian form as follows:

$$J(\alpha_i,\alpha_i^*)=\sum_{i=1}^{N}d_i(\alpha_i-\alpha_i^*)-\varepsilon\sum_{i=1}^{N}(\alpha_i+\alpha_i^*)-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(\alpha_i-\alpha_i^*)(\alpha_j-\alpha_j^*)K(x_i,x_j) \quad (2)$$

Subject to:

$$\sum_{i=1}^{N}(\alpha_i-\alpha_i^*)=0,\qquad 0\le\alpha_i\alpha_i^*\le C,\qquad i=1,2,...,N$$

In Eq. (7), α_i and α_i^* are called Lagrangian multipliers which satisfy equalities, $\alpha_i\times\alpha_i^*=0$. optimal desired weights vector of the regression hyper-plane with respect to α_i and α_i^* is written as,

$$w^*=\sum_{i=1}^{N}(\alpha_i-\alpha_i^*)K(x_i,x_j) \quad (3)$$

Therefore, the regression function is represented as,

$$f(x,\alpha,\alpha^*)=\sum_{i=1}^{N}(\alpha_i-\alpha_i^*)K(x_i,x_j)+b \quad (4)$$

where, $K(x_i,x_j)$ denotes the kernel function. The most famous kernels are linear kernel, polynomial kernel, radial basis function (RBF), or Gaussian kernel and sigmoid kernel. Linear kernel, polynomial kernel, RBF kernel, and sigmoid kernel are as follows:

Linear kernel

$$K(x_i,x_j)=x_i^T x_j \quad (5)$$

Polynomial kernel

$$K(x_i,x_j)=(1+x_i.x_j)^d \quad (6)$$

RBF kernel,

$$K(x_i,x_j)=\exp(-\gamma\|x_i-x_j\|^2) \quad (7)$$

Sigmoid kernel,

$$K(x_i,x_j)=\tanh[\nu(x_i,x_j)+\alpha] \quad (8)$$

Where d, γ, ν and α are kernel parameters. The kernel parameters should be set properly because it affects the regression accuracy. This study uses the RBF kernel function because it is the best choice in case of most predicting problems [Wu and Chen, 2010]. The RBF kernel is also effective and has fast training process [Xin et al, 2012]. For the RBF kernel function, there are three important parameters including Regularization parameter (C), Kernel parameter (γ) and tube size of ε-insensitive loss function (ε).

In this study, the optimum value of these parameters was determined using try and error procedure.

3. Establishment of Synthetic Database

In this study, two critical responses of pavement including maximum tensile strain at the bottom of asphalt layer and maximum vertical compressive strain on the top of subgrade soil are taken into consideration. These two critical responses control the bottom-up fatigue cracking and rutting depth of pavement [Huang 2004, NCHRP 2004, Austroad 2010, IRC 2012].

In order to develope a comprehensive dataset for training and testing of SVM, 10000 different pavement sections subjected to standard axle loading, were analyzed and maximum tensile strain at the bottom of asphalt layer and maximum compressive strain on the top of subgrade soil were calculated. The maximum value of each response was determined according to analysis results for five different points at the bottom of asphalt layer and five different points on the top of subgrade. Standard axle load (single axle with dual wheel with the weight of 8.2 tons), pavement section, and the position of the response points are shown in Figure (1). Statistical specifications of inputs and outputs parameters for synthetic database are given in Table (1). Also the interface between two succeeding layers was assumed as fully bonded. In all the analyses, the Poisson's ratio of asphalt concrete, granular base, and granular subbase was assumed as 0.35 and the Poisson's ratio of subgrade was assumed as 0.4. These values are typical values of Poisson's ratio for Hot Mix Asphalt, untreated granular materials and fine-grained soils [Maher and Bennert 2008]. Previous findings have also shown that the selection of Poisson's ratio has a small effect on pavement responses [Huang 2004]. The minimum thickness of granular base and granular subbase was selected as zero which means that the database covers the pavement structures without granular base or granular subbase in addition to conventional flexible pavements with base and subbase layers. Minimum resilient modulus of granular base and granular subbase was selected based on the minimum allowable value of CBR for these two layers (30% for granular subbase and 80% for granular base)[IMPO, 2010]. Range of resilient modulus of subgade soil was selected between 30 to 200 MPa which is equivalent to CBR of 3% to 60% for subgrade soil [IMPO, 2010].

In order to analyze different pavement sections, NonPAS software was employed, which has the capability of linear and nonlinear analysis of pavement using multi-layered elastic theory. Detailed verification of NonPAS code using Kenlayer program proved that the the NonPAS can accurately predict the pavement responses subjected to single and multiple loading [Ghanizadeh and Ziaie 2015].

4. The Determination of the Appropriate Values for Kernel Parameters

In this study, STATISTICA 12.0 was used for training and testing SVM. 80 percent of records (8000 records) were considered as training set and 20 percent of records (2000 records) were considered as testing set. Input or independent variables in SVM model were considered as thickness and modulus of different layers of pavement and output or dependent variable was assumed as critical response of pavement (maximum horizontal principal tensile strain at the bottom of asphalt layer or maximum vertical compressive strain on the top of subgrade). In this research, two SVM model were trained and tested for predicting two critical responses. The optimum value of parameters in case of each SVM model including regularization parameter (C), kernel parameter (γ), and the tube size of ε-insensitive loss function (ε) was determined based on the try and error method as 21, 2 and 0.0001, respectively

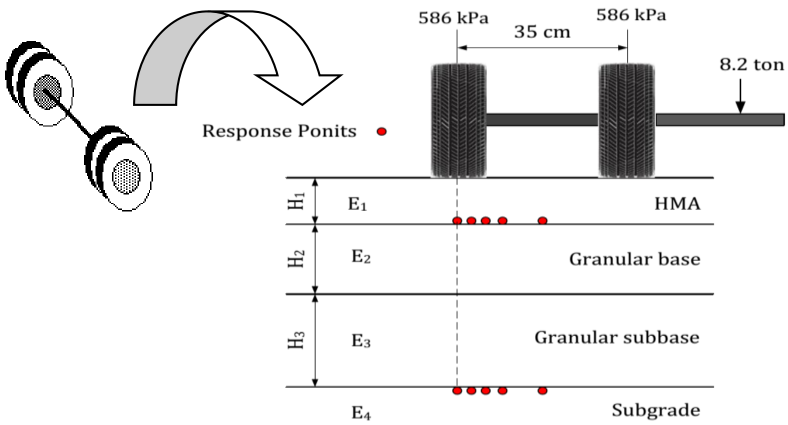


Figure 2. Specifications of standard axle, pavement section, and response points.

5. Performance of Support Vector Machine

Statistical parameters of SVM regression for both training and testing sets are given in Table 2 and 3. It can be seen that the coefficient of determination (R^2) in case of training set, testing set and overall data is greater than 0.998. It is also resulted that the developed SVM has an acceptable generalization because the coefficient of determination for both training and testing set is the same.

Performance of Support Vector Machine for predicting critical responses of flexible pavement in case of training and testing sets, are presented in Figures (3) to (6). These figures confirm that the trained SVM is capable of predicting critical responses of pavement with high accuracy.

Table 1. Statistical characteristics of the inputs and outputs used in database development

Statistical Parameter	H_1	H_2	H_3	E_1	E_2	E_3	E_4	ε_t	ε_c
Minimum	5.00	0.00	0.00	800.00	200.00	100.00	30.00	16.45	26.50
Maximum	45.00	50.00	60.00	10000.00	400.00	200.00	200.00	597.76	1473.29
Mean	22.69	25.98	32.73	5293.44	296.12	154.89	86.77	111.83	171.57
Standard Deviation	11.06	14.03	18.23	2800.68	64.18	29.50	38.73	82.11	150.13
Median	22.72	25.14	30.30	5056.67	300.00	153.06	82.35	86.54	126.13

H_i: Thickness of the i_{th} layer in cm.

E_i: Resilient modulus of the i_{th} layer in MPa.

ε_t: Maximum horizontal principal tensile strain at the bottom of asphalt layer in micro-strain.

ε_c: Maximum compressive strain on the top of subgrade in micro-strain

Table 2. Statistical parameters of SVM for predicting maximum tensile strain.

Data set	Training set	Testing set	Overall
RMSE	3.56	17.70	6.37
S.D ratio	0.023	0.056	0.031
R^2	0.999	0.998	0.999

Table 3. Statistical parameters of SVM for predicting maximum compressive strain.

Data set	Training set	Testing set	Overall
RMSE	21.64	18.95	21.11
S.D ratio	0.030	0.034	0.031
R^2	0.999	0.999	0.999

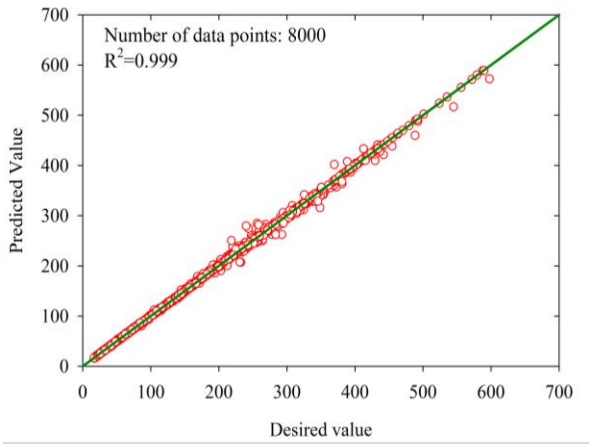


Figure 3. Performance of SVM for predicting maximum horizontal principle tensile strain based on training set.

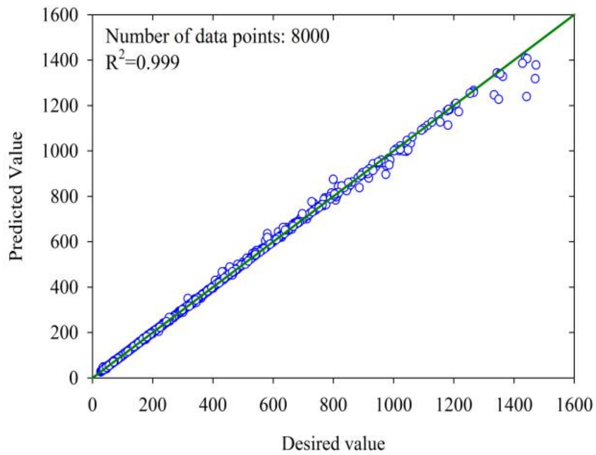


Figure 4. Performance of SVM for predicting maximum compressive strain on the top of subgrade based on training set.

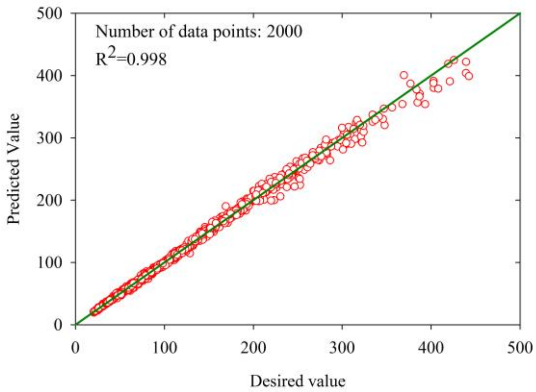


Figure 5. Performance of SVM for predicting maximum horizontal principle tensile strain based on testing set.

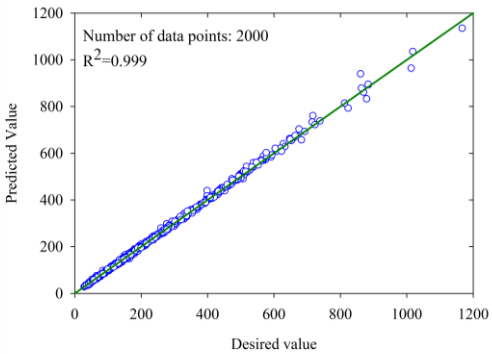


Figure 6. Performance of SVM for predicting maximum compressive strain on the top of subgrade based on testing set.

The frequency histograms for prediction error of critical strains using training and testing sets are presented in Figure (7) and Figure (8), respectively. As can be observed, prediction error of maximum horizontal principal tensile strain at the bottom of asphalt layer and maximum compressive strain on the top of subgrade in most cases is lower than 5 percent.

6. Validation of SVM Model using JULEA Program

The results of analysis obtained by means of JULEA program were used to validate the proposed SVM model in this study. JULEA program is among the most powerful and accurate pavement analysis programs that uses the multi-layered elastic theory for the analysis of pavements. This program has been developed by Dr. Jacob Uzan and has been employed as the analysis core for analysing flexible pavements in Mechanistic-Empirical Pavement Design Guide (MEPDG) software [NCHRP 2004]. For comparison of responses resulted by SVM and JULEA, eight different pavement sections were considered and the critical strains at the bottom of asphalt layer and on the top of subgrade were computed using SVM model and JULEA program. The specifications of these eight pavement sections as well as the obtained responses using SVM and JULEA program are given in Table (2). As evidence, critical responses computed using SVM model are very close to the critical responses computed using JULEA. The maximum prediction error is less than 10 percent.

7. Parametric Analysis

In order to investigate the effect of different parameters on critical responses of flexible pavements, the Cosine Amplitude Method (CAM), was employed. The express similarity relation between the target function and the input parameters is used to obtain by this method. In this method, all of data pairs are expressed in the common X-space. They would form a data array X defined as Eq. (13) [Yang and Zhang, 1997]:

$$X = \{x_1, x_2, x_3, \dots, x_n\} \tag{9}$$

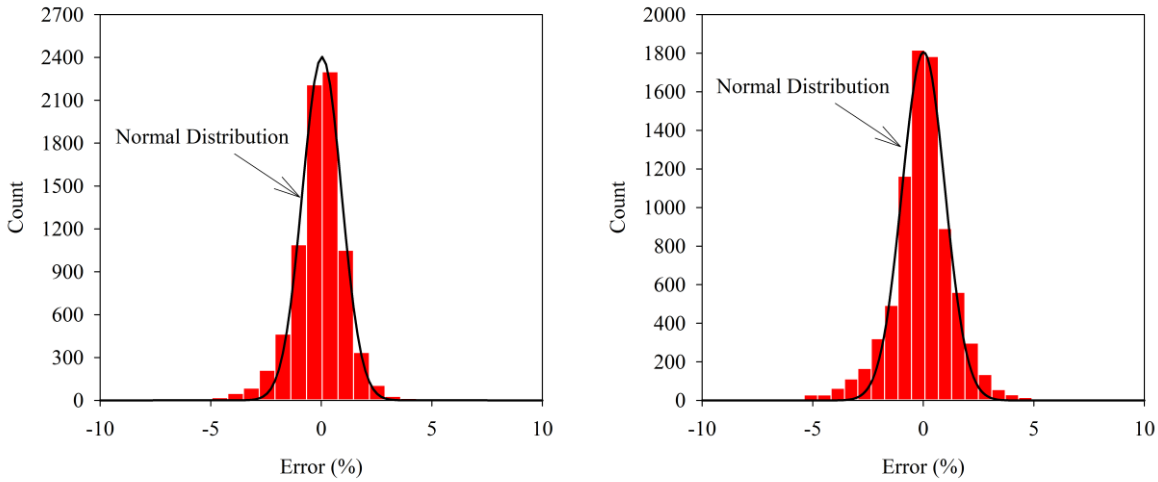
where x_i, is a vector of the length of m and is shown in the Eq. (14).

$$x_i = \{x_{i1}, x_{i2}, x_{i3}, \dots, x_{im}\} \tag{10}$$

Thus, each record of the dataset can be assumed as a point in the m-dimensional space and this point requires m-coordinates to be fully defined. Equation (15) can be used to compute the strength of the relationship between x_i and x_j:

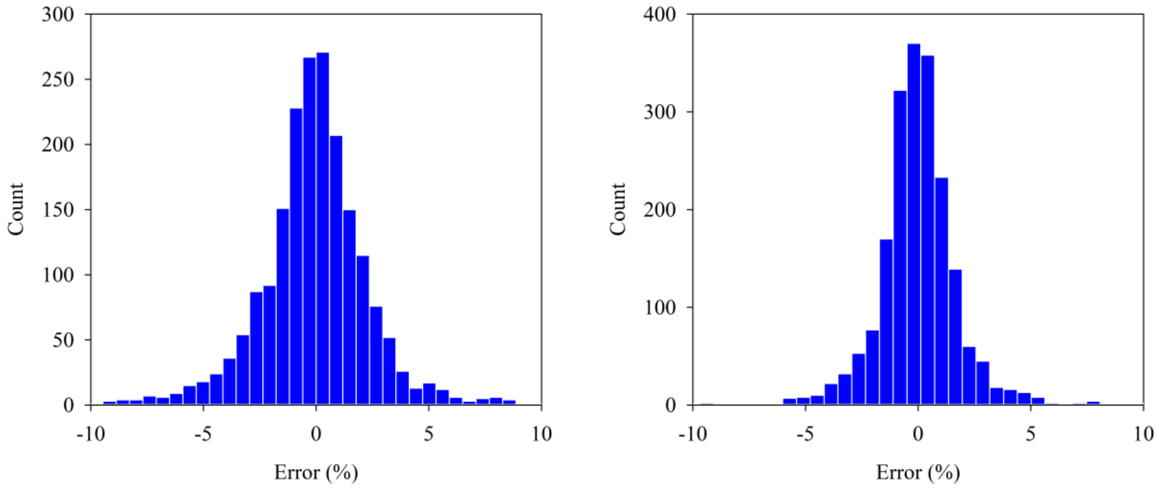
$$R_{ij} = \frac{\sum_{k=1}^{m} x_{ik} x_{jk}}{\sqrt{\sum_{k=1}^{m} x_{ik}^2 \sum_{k=1}^{m} x_{jk}^2}} \tag{11}$$

Regarding the CAM method, the strength of the relationship between maximum horizontal principal strain at the bottom of asphalt layer and input parameters, and also maximum compressive strain on the top of subgrade and input parameters were presented in Figures 9 and 10, respectively.



(B) (A)

Figure 7. Error percentage of predicted strain for test data: A) Maximum horizontal principle tensile strain (HPTS) at the bottom of asphalt layer and B) Maximum compressive strain on the top of subgrade



(B) (A)

Figure 8. Error percentage of predicted strain for test data: A) Maximum horizontal principle tensile strain (HPTS) at the bottom of asphalt layer and B) Maximum compressive strain on the top of subgrade

The results show that the thickness as well as resilient modulus of asphalt concrete layer are the most influencing factors on the maximum horizontal strain at the bottom of asphalt layer. Also, the thickness of subbase layer and the resilient modulus of subgrade soil are the parameters with the least effects on the maximum horizontal strain at the bottom of asphalt layer. Moreover, thickness of different pavement layers and resilient modulus of subgrade soil are the most sensitive parameters affecting the maximum vertical strain on the top of subgrade layer, and thickness of subbase layer is the least sensitive parameters. Similar results have been reported by Behiri (2012). He studied the effect of variation in modulus and thickness of different layers, on the fatigue and rutting life of pavement. He stated that fatigue life has no sensitivity with the variation of base thickness compared with rutting life, which is high sensitive. While both fatigue and rutting lives have a good sensitivity with the variation of surface thickness. He also concluded that increase of elastic modulus of asphalt or base layers has not obvious effect on the rutting life at base thickness thinner than 300 mm, thicker thickness lead to obvious increase in rutting life. With respect to fatigue life, it has no sensitivity with the variation of base thickness while has a good sensitivity with the variation of surface modulus or base modulus at all values of base thickness [Behiry, 2012].

Table 4. Comparison of the responses obtained using SVM and JULEA.

H_1 (cm)	H_2 (cm)	H_3 (cm)	E_1 (MPa)	E_2 (MPa)	E_3 (MPa)	E_4 (MPa)	SVM		JULEA		Error percentage	
							ε_t	ε_c	ε_t	ε_c	ε_t	ε_c
8	15	20	1500	220	110	40	377.50	939.58	383.53	861.13	-1.57	9.11
9	15	20	2000	240	120	45	320.76	760.28	320.09	717.22	0.21	6.00
10	15	20	2500	260	130	50	271.83	620.51	266.74	603.26	1.91	2.86
13	20	30	3000	280	140	55	193.46	315.49	187.13	320.59	3.38	-1.59
14	20	30	3500	300	150	60	163.09	273.85	157.79	277.49	3.36	-1.31
15	20	30	4000	320	160	65	137.26	239.12	134.30	241.80	2.20	-1.11
18	25	40	4500	340	170	70	98.45	146.12	98.23	147.54	0.22	-0.96
19	25	40	5000	360	180	75	84.15	129.18	85.08	131.30	-1.09	-1.61

ε_t: The maximum horizontal principal tensile strain at the bottom of asphalt layer, micro-strain

ε_c: The maximum compressive strain on the top of subgrade, micro-strain

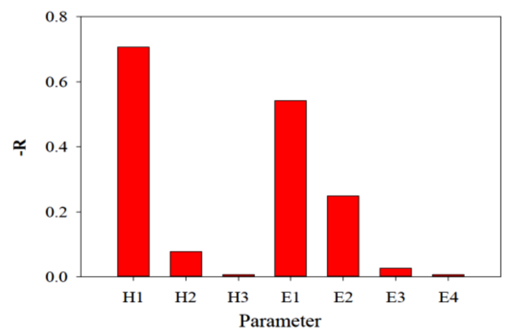


Figure 9. Strength of the relationship between different parameters and maximum horizontal strain at the bottom of asphalt layer.

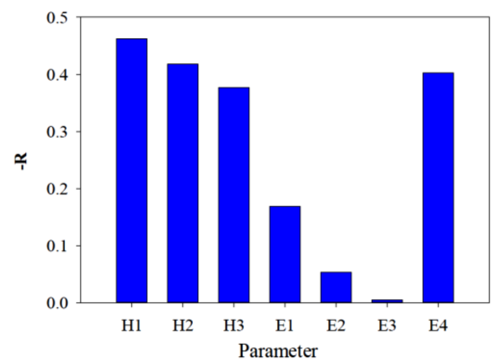


Figure 10. Strength of the relationship between different parameters and maximum vertical strain on the top of subgrade layer.

8. Conclusion

This study indicated that the critical responses computed using SVM model are very close to the critical responses computed using multi-layer elastic theory and the maximum prediction error is less than 10 percent. Due to the fast pavement analysis by means of SVM method in comparison with the finite element method and multi-layered elastic theory, it is possible to use the support vector machine regression without introducing the position of response points in order to quickly and accurately analyze the pavement structure. The fast analysis of pavements using this method allows the possibility of analyzing a very large number of pavement sections in order to optimal design of flexible pavements. Parametric analysis using Cosine Amplitude Method (CAM) shows that the thickness as well as resilient modulus of asphalt concrete layer are the most influencing factors on the maximum horizontal strain at the bottom of asphalt layer. Moreover, thickness of different pavement layers and resilient modulus of subgrade soil are the most sensitive parameters affecting the maximum vertical strain on the top of subgrade layer. Since in practice the loading spectra are commonly used for design of pavements using mechanistic-empirical methods, this research needs to be completed by developing other support vector machines in case of each loading axle.

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