# Assessment and Prioritizing the Risks of Urban Rail Transportation using Grey Analytical Hierarchy Process (GAHP)

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## Abstract

Some incidents in urban railway systems affect the function of the subway's company adversely and they could disorder the services. These events may inflict irreparable damage to passengers, employees and equipment. By recognizing the hazards existing in this type of transportation system and evaluating and prioritizing risks, we can perform appropriate actions to reduce the probability and severity of them. In this paper, thirteen risks of rail transport system in Tehran subway have been identified and nine evaluation criteria and sub criteria are specified. Analytical Hierarchy Process (AHP) approach based on the grey number scores (GAHP) has been developed in terms of complete uncertainty and incomplete information and risks are prioritized based on subway system expert's opinion. The use of grey or interval numbers in AHP, in addition to more accurate assessment beside crisp numbers has lower computational complexity than fuzzy numbers. To calculate local and global weights for pairwise comparison matrices, a non linear and two linear optimization models are used. Injuring the passengers because of closing the doors of train is the important risk in Tehran subway system.


**Keywords:** Risk assessment, multi attribute decision making (MADM), grey analytical hierarchy process (GAHP), Grey number, subway transportation system.

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## 1. Introduction

Given the development variety of transportation systems in the world and Iran, the importance of the risk management in these systems is fully revealed. Urban railway (subway) transport systems are constantly exposed to risks, because of their plurality of users and strategic conditions. The emergence of new biological, microbial and nuclear weapons threatens human communities even in peace time. The following cases and examples indicate the importance of planning, confronting threats and risk management in the subway systems.

Biological terrorists attack in subway due to the mass of passengers, cause the killing of thousands of innocent people (such as terrorist attack on Tokyo subway by nervous gas). Emergence of catastrophes with multi-consequences such as natural disasters cause secondary crisis in subway. Emergence of catastrophes with multi consequences such as natural disasters causes secondary crises in subway. Earthquake, bursting gas pipes and fire, entrance of dangerous gases and wastes of sewage to subway tunnels, leaking the subterranean and outbreak of the voluminous floods and derailing the trains, are all samples of these cases. Accidents resulting from negligence of passengers such as getting stuck at the train doors, from escalators, entering rail limits, suicide, sabotage operations, hostage taking, robbery, etc., could be threatening the life of this type of transport system users and staff, by some events such as lack of awareness of passengers concerning subway equipment, lack of observance safety regulations by passengers, hosting the passengers boarding trains, and sabotaging intentions.

Incidents resulting from negligence of employees in subway are rooted in reasons such as monotony and tiredness of working, inattention to the control and directing of passengers, and to safety rules or shortage of expertise in assigned working post. May cause incidents such as trains crashes, derailing, inattention to signal lights, crush the route needles and firing as well, because of improper usage of equipment. Also, equipment failure is another risk in subway, which causes accidents and threats to life of these system users. For instance, incomplete system of train automatic protection would lead collision of train, and train defects stopping the train and blocking the path.

Therefore according to the identification of subway risks and providing solutions to reduce the likelihood of their occurrence is necessary. But utilizing all solutions due to constraints such as budget and resources at the same time is not possible, but must be evaluated and prioritized based on criteria such as the possibility of occurrence, severity and effects etc. Process of evaluating and prioritizing risks is a Multi Attribute Decision Making (MADM) process, where the alternatives are evaluated and prioritized based on several criteria. Analytical Hierarchy Process (AHP) is a multiple criteria decision method which ranks the alternatives based on paired comparisons of alternatives and criteria. In this method which was offered by Saati at the first time, the relative importance of the two alternatives or two criteria was assessed based on crisp numbers from 1 to 9. Because of the inability of crisp numbers when Decision Maker's information is inadequate, and vagueness of criteria and alternatives researchers used the fuzzy numbers in Paired assessments [Chen and Tzeng, 2004, Kuo, Yang and Huang, 2008, Li Yamaguchi and Nagai, 2007, Mohammadi and Molaei, 2010, Shih et al. 2011, Tai et al. 2011, Wei, 2011].

However, using the fuzzy numbers increases the computational complexity of solving. But, the use of grey or interval numbers where there is information about alternatives or criteria however it is imperfect (but not sufficient). In addition to increasing the accuracy of assessment than crisp numbers it has less computational complexity rather than fuzzy numbers. On the other hand, in group decision making if experts give their opinions about criteria and alternatives as crisp numbers, the grey theory can integrate the crisp numbers into a grey number. Baradaran and Azarnia developed usage of grey numbers in paired alternatives and criteria assessment in AHP method [Baradaran and Azarnia, 2013].

Subway system in Tehran is one of the largest passenger transportation networks, in which displacement of more than two million passengers is an essential transportation element in this city. Failure to identify and prioritize risks in the system and lack of risks reduction activities is one of the challenges of this system. Incidents arising from the former risks imposed very heavy costs on passengers, employees and the system's conductor company. This article aims to study the incidents of the Tehran subway in the past and identify the risks. Identifying the Tehran subway's risks and determining the evaluating criteria and sub criteria in prioritizing subway's risks problem, extending Group AHP technique in the form of grey numbers, developing a mathematical programming model to extract grey weights form the pairwise matrices with interval numbers are the major contributions of this study. In this study, within the development of AHP method in grey conditions (data incomplete and uncertainty) entitled GAHP, in which the elements of pairwise matrices are grey numbers, the risks of railway transport system in Tehran have been identified. A questionnaire was used to gather the importance of risk criteria and the risks from group of experts in Tehran subway system as a crisp number. By a statistical data analysis, the crisp numbers changed to grey numbers in pairwise matrices in AHP. The identified risks in urban railway of Tehran have been prioritized using the GAHP and the grey numbers in pairwise matrices based on different risk assessment criteria.

The rest of the article is organized as follows: in Section 2, the concept of risk assessment, risk assessment methods, mathematical operations of grey numbers and algorithm of GAHP are discussed. In section 3, identified risks in Tehran subway are assessed and prioritized. The last section is dedicated to presenting the results and discussions of the paper.

## 2. Literature Review

In this section, the concept of risk assessment, the techniques of risk assessment and algorithm of GAHP are explained step by step. Also, the concepts and mathematical operations of grey numbers are expressed.

### 2.1 Risk Assessment

According to the definition of PMBOK guideline, the risk is an event or uncertain situation that in case of happing could have negative or positive impact on targets. Therefore, for increasing the positive impacts of opportunities and reducing the negative impacts of threats, the risks should be managed. The Project Management Institute (PMI), defines risk management as planning for identification, analysis, accountability and monitoring risk. Thus risk management has four phases: Identification, Analysis, Accountability and Monitoring.

In the phase of risk identification called risk diagnosis, the list of events which may happen and list of the reasons which may cause such incidents have been prepared. Redmill explained that the purpose of identifying the source of risk is to prevent the events that can go wrong and lead to breaches of safety [Redmill, 2002]. Shen argued that the purpose of risk identification is not only to identify a list of risk factors but also to identify the importance of those risk factors [Shen, 1997]. Chapman and Ward stated that risk identification is both important and difficult and called for creativity and imagination [Chapman and Ward, 2003]. They recommended the directed-thinking approach, which includes activities such as the interviewing of individuals or groups, brainstorming, and using checklists. Clark et al. suggested that an identified risk is not a risk unless it is a management problem [Clark, Pledger and Needler, 1990]. In addition to the mentioned risk identification methods, some other techniques based on group decision making methods could be focused remarkably in infrastructure projects. These methods include: Brainstorming, Pin card, Gallery, Battle–Belmuden–Brain writing (BBB), Collective Note Book (CNB) and Nominal Group Technique (NGT) [Makui, Mojtahedi and Mousavi, 2007, Mojtahedi, Mousavi and Makui, 2008].

Many different classifications of risk have been developed over the years. These classifications include: Project Risk, External Risk, Consortium Risk, Time Risk, Cost Risk, Incremental Risk, Catastrophic Risk, Technical Risk, Human Risk, Financial Risk, Political and Economical Risk, Predictable Risk, Unpredictable Risk [Ebrahimnejad, Mousavi and Seyrafianpour, 2009]. In case that basis of categorization of type be uncertainty, categorization includes Known Risk (risk and its impacts, both are known), Unknown Known Risk (recognized risk but impacts are unknown) and Unknown Unknowns Risk (risk and its impacts are unknown).

After identifying the risks, they should be analyzed qualitatively and quantitatively based on criteria such as repetition of occurrence and severity. In risk analysis phase, the risks are ranked and which are more important has higher priority. So, the possibility of comparing the risks with each other is provided and the subsequent phases of process of risk management, it is possible to make decisions on proper way of reaction to risks. It should be noted that in morphology of risk management, a set of two phases of identifying the risk and analyzing it are recognized as risk assessment which comprises the central case of risk management. Grassi et al. explained that the most important phase involved in risk management is risk assessment [Grassi et al. 2009]. Briefly, it consists of identifying all hazardous activities which are a potential cause of injury to companies and in estimating the risk level involved by them in such a way as to address prevention and intervention measures.

## 2.2 Risk Assessment Techniques

As it was mentioned, risk assessment ranks the risks according to their degree of importance. For ranking the risks we should define two parts: the criteria of assessment and the techniques ranking the risks based on the criteria.

### 2.2.1 Criteria of Risk Assessment

Most researchers only used two criteria in risk assessment process which are the possibility of occurrence and severity of risk. Nevertheless they are not sufficient for risk assessment. The priority of risks which has been ranked by mentioned criteria is not realistic. We should consider other criteria in risk assessment as well. For instance in some researches, criteria such as capability of organization in reaction to risk [Mikulak, McDermott and Beauregard, 1996] and lack of confidence of estimation [Klein and Cork, 1998] have also been introduced. Both said criteria could be used well in assessing the risks and their rankings qualitatively and quantitatively. Also, Lambert et al. have used three criteria including possibility of occurrence, potential impact on project and proficiency as well as the speed for countering the risk, and for ranking the risks [Lambert et al. 2001]. So the risks should be ranked from different perspectives through different criteria.

### 2.2.2 Risk Ranking Methods

One of the methods for ranking the risks is the Failure Modes and Effects Analysis (FMEA). First, this method was studied by NASA in 1963 [Hu et al. 2009]. In FMEA, a Risk Priority Number (RPN) is calculated for each risk which is achieved through multiplying the three variables including: severity (S), probability of occurrence (O), possibility of detection the error (D). At the end, the risks were ranked by RPN as descending [Hu et al. 2009]. Usually, the variables in FMAE are evaluated for each risk as crisp number. Because of lack of certainty and insufficiency of information during using crisp number, in this method, the fuzzy number [Kumrua, Vitae and Kumru, 2013, Kutlua and Ekmekcioglu, 2012, Liu et al. 2012, Liu and Tsai, 2012] and linear programming method were used to achieve RPN [Zhang and Chu, 2011]. Besides the FMEA technique, the Multi Attribute Decision Making (MADM) methods are used for risk assessment [Ebrahimnejad, Mousavi and Seyrafianpour, 2009, Ergu et al. 2014, Grassi et al., 2009, Lo and Chen, 2012, Mojtahedi, Mousavi and Makui, 2008, Wang, Liu and Elhag, 2008, Yang, Shieha and Tzeng, 2011]. In view of MADM, usually the decision maker faces the selection of an alternative among several alternatives to optimize several opposite attributes or criteria. The advantage of the MADM methods lies in their simplicity which is adapted to the behavior of decision maker and limitation of his information. The MADM techniques are divided by compensatory and non-compensatory model.

Non-compensatory model includes such methods by which trade off between criteria is not authorized. It means that existing weak point in an attribute (criterion) is not compensated by existing advantage of another attribute (criterion). In contrast, compensatory methods include these models by which the possibility of exchange among attributes is allowed. For instance, any change even the small ones in any attribute (criterion) can be compensated by opposite change in another attribute or attributes. The non-compensatory models could not be used for the issue of ranking the risk. Because, as mentioned before in such methods there should not be any trade off among attributes. While in the issue of ranking the risk, different attributes including the possibility for occurrence and impact have influence on each other and weak of any attributes could be compensated by advantage of another attributes. So, the non-compensatory methods such as linear allocation, LINMAP, TOPSIS, ELECTRE, MDS and AHP could be used in the issue of ranking the risks. Generally, the process for risk assessment is a multi-criteria decision making process by which risks are introduced as alternative and criteria of risk assessment entitled as decision making criteria.

Given the growing complexity of decision making, uncertainty in assessment is increased too. Under such conditions, decision makers cannot use crisp number for expressing their assessments. But they can use approximate range such as grey numbers or fuzzy numbers for expressing their judgments about criteria. In condition that decision maker has full information with regard to criteria, he/she can give a crisp number to any criteria, using crisp number as qualitative criteria. Given that major assessment, criteria are not quantitative and are expressed verbally, it is possible in such conditions that decision maker has no information or faces with high ambiguous information, he/she uses fuzzy numbers [Chen et al. 2012, Kuo and Lu, 2012, Lavasani et al. 2011, Wang and Elhag, 2006, Yucel et al. 2012]. But, when there is partial or limited information, the grey numbers be used in the format of an interval instead of crisp numbers or fuzzy numbers for determination of point of any criteria in order to both avoid complexities of modeling in fuzzy numbers, and fallible of precise models, and also, benefit the simplicity of grey numbers. Up until now, the grey numbers have not been used in the area of risk assessment.

## 2.3 The Grey Numbers

The Grey numbers are atoms and cellules of grey system. [David, 1994] White, grey and black numbers are three category of number, used for identifying the uncertainty level of information. As number $\otimes x$ is defined as follows:

$$[\underline{x},\overline{x}] = \{x \mid \underline{x} \le x \le \overline{x}, \underline{x} \text{ and } \overline{x} \in R\} \quad (1)$$

The relationship can be defined in three forms: If $\underline{x} \to \infty$, $\overline{x} \to \infty$, the $\otimes x$ is black or fuzzy number. Allocating this number to decision criterion means that in such condition, there is no significant information. If $\underline{x} = \overline{x}$, then $\otimes$ x is called a white or crisp value. Using crisp value in decision making means the DM is quite certain about his/her criterion (alternative). In case $\underline{x}, \overline{x} \in$ R, $\underline{x}, \overline{x} \neq \infty$, then $\otimes x$ is called a grey number. This means that, in such situation, the existing information is insufficient and unclear. Although it seems that grey numbers are similar to fuzzy numbers, the basic difference is that, in grey numbers, the exact value is unclear while intervals which encompass the amount of that value is clear, that is the exact value of lower and upper bound the number is clear. Despite, in a fuzzy number, although the number is defined as an interval, the exact amount of lower and upper bound is not clear, meanwhile it follows membership function [Mohammadi and Molaei, 2010]. Such a tiny difference between grey and fuzzy numbers makes the calculation with grey numbers much simpler than that of fuzzy numbers. Since determining membership function for lower and upper bound of a fuzzy number has own complexities in calculating operations. Therefore, using concept and calculations of grey numbers is useful for confronting unclear information.

The grey number is a part of grey theory [Deng, 1988] used for modeling systems with insufficient and incomplete information. Hence, using the theory of grey number in indispensable in real world and decision making [Chen and Tzeng, 2004, Li, Yamaguchi and Nagai, 2007, Mohammadi and Molaei, 2010, Shih et al. 2011, Tai et al. 2011, Wei, 2011].

### 2.3.1 Operations on Grey Number

Let $+, -, \times$ and $\div$ denote the operations of addition, subtraction, multiplication and division, respectively. Each arithmetic operation is given for grey numbers $\otimes x$ and $\otimes y$ as follows [Lin, Lee and Ting, 2008]

$$\otimes x + \otimes y = \left[\underline{x} + \underline{y}, \overline{x} + \overline{y}\right] \quad (2)$$

$$\otimes x - \otimes y = \left[\underline{x} - \underline{y}, \overline{x} - \overline{y}\right] \quad (3)$$

$$\otimes x \times \otimes y = \left[min\left(\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\,\overline{y}\right), max\left(\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\,\overline{y}\right)\right] \quad (4)$$

$$\otimes x \div \otimes y = \left[min\left(\frac{\underline{x}}{\underline{y}}, \frac{\underline{x}}{\overline{y}}, \frac{\overline{x}}{\underline{y}}, \frac{\overline{x}}{\overline{y}}\right), max\left(\frac{\underline{x}}{\underline{y}}, \frac{\underline{x}}{\overline{y}}, \frac{\overline{x}}{\underline{y}}, \frac{\overline{x}}{\overline{y}}\right)\right] \quad (5)$$

### 2.3.2 Grey Matrix and its Related Operation

When a huge amount of values and variables are involved in a study, one will have to make use of the concepts of matrices and related results [Liu and Lin, 2006]. An $n$-dimensional vector with grey component(s) is called a grey $n$-dimensional vector. An $n$-dimensional grey vector is denoted as:

A matrix with grey elements is called grey matrix and is displayed as $\mathbf{D}(\otimes) = [\otimes_{ij}]_{m\times n}$ .

$X(\otimes) = (\otimes_1, \otimes_2, \dots, \otimes_n)$ (6)

And, the grey element at the location of the $i^{th}$row and the $j^{th}$ column is denoted as $\otimes_{ij}$. Considering independent matrix $\boldsymbol{C}(\otimes) = [\otimes'_{ij}]_{m\times n}$, the mathematical relationship between matrices $\boldsymbol{C}(\otimes), \boldsymbol{D}(\otimes)$ is defined as bellow:

The equality of two grey matrices $\boldsymbol{C}(\otimes)$ and $\boldsymbol{D}(\otimes)$:

If all the corresponding entries of $\boldsymbol{C}(\otimes)$ and $\boldsymbol{D}(\otimes)$ are identical (that is, $\otimes_{ij}=\otimes'_{ij}$ for $i$ = 1, 2, ..., $m$; $j$ = 1, 2, ...,$n$), then the $\boldsymbol{C}(\otimes)$ and $\boldsymbol{D}(\otimes)$ are said to be equal.

The definition of sum of $\boldsymbol{C}(\otimes)$ and $\boldsymbol{D}(\otimes)$:

$\boldsymbol{C}(\otimes) + \boldsymbol{D}(\otimes) = [\otimes_{ij}+\otimes'_{ij}]$ (7)

The definition of difference of $\boldsymbol{C}(\otimes)$ and $\boldsymbol{D}(\otimes)$:

$\boldsymbol{D}(\otimes) - \boldsymbol{C}(\otimes) = [\otimes_{ij}-\otimes'_{ij}]$ (8)

The definition of the additive inverse of $\boldsymbol{D}(\otimes)$:

$-\boldsymbol{D}(\otimes) = [-\otimes_{ij}]$ (9)

The definition of the scalar multiplication of the grey number $\otimes$ and the grey matrix $D(\otimes)$:

$\otimes \times \boldsymbol{D}(\otimes) = [\otimes \times \otimes_{ij}]$ (10)

The definition of multiple two grey matrices $\mathbf{C}(\otimes)$ and $\boldsymbol{D}(\otimes)$ with scale $s \times n, m \times s$:

$\boldsymbol{D}(\otimes) \times \boldsymbol{C}(\otimes) = [\otimes''_{ij}]_{m\times n}$ (11)

Where $\otimes''_{ij}=\otimes_{i1}\otimes'_{1j}+\otimes_{i2}\otimes'_{2j}+\cdots+\otimes_{is}\otimes'_{sj}=\sum_{k=1}^{s}\otimes_{ik}\otimes'_{kj}$

### 2.4 The Grey AHP Model

In this section, the developed AHP model in grey conditions (GAHP) has been presented.

Step 1: Modeling the problem in the form of a hierarchical tree. After identifying the problem, determining the purpose, criteria and decision alternatives, in this step, a hierarchical tree is formed according to figure 1.

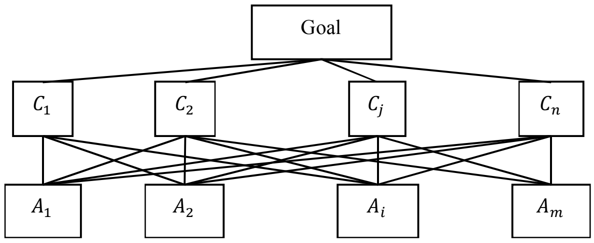


**Figure 1. The decision tree with tree levels**

Step 2: Forming Grey Pairwise Comparison Matrices (GPCM). By applying the opinion of experts either individually or in group the GPCMs are formed. This step is the most important part of the GAHP technique. To this end, some questionnaire are designed and delivered to the experts. In the questionnaires, the questions are designed in such a way that each person could express, his/her own opinion in the relation to the amount of priority of each criterion compared to other criteria, and also, express the amount of priority for alternative compared to other alternatives in the form of interval of two by two in the range of 1 to 9.

Step 3**:** Investigating consistency condition of the GPCMs. One of the most pre-conditions for the GAHP is investigating consistency condition of the GPCMs. The presence of inconsistent GPCM in the GAHP method leads to presenting illogical and far from reality result. The consistency condition in the grey matrix $n \times n$, where $\otimes x_{ij} = [\underline{x}_{ij}, \overline{x}_{ij}]$ is $i, j^{th}$ of the $\boldsymbol{D}(\otimes)$ is defined as follows:

$$\underline{x}_{ij} \times \overline{x}_{ij} = (\underline{x}_{ik} \times \underline{x}_{kj}) \times (\overline{x}_{ik} \times \overline{x}_{kj}) \quad (12)$$

Step 4: Calculating consistency ratio for the GPCMs. Having defined the conditions for consistency of the GPCMs, this step is finding a method to check for judgmental consistency of the DMs based on these conditions. Similar to Saaty's model, Gogus and Boucher [Gogus and Boucher, 1998], generated random indices ($RI$) for matrices (with triangular number) of order 1-15 using a sample size of 400 that are given in table 1. RI is the consistency index of a randomly generated reciprocal matrix from the scale 1-9, with forced reciprocity. For a given matrix size, they computed two RIs; one for matrix includes mean values ($RI^{m}$) and another one for the matrix which formed by geometric means of lower and upper bounds ($RI^{g}$). With regard to the fact that grey number in the GPCM posses lower and upper bound in interval [1/9,9] and Gogus and Boucher have generated $RI^{g}$ lower and upper bound of fuzzy numbers, the above table can be used to calculate the consistency ratio of grey matrix. Saaty's random indices are not usable for determining the degree of consistency's the GPCMs because all elements of matrix that were obtained by taking the geometric means of lower and upper bounds, $[\underline{x}_{ij}, \overline{x}_{ij}]$, may be non integer while in Saaty's approach the PCMs includes some integer numbers between 1 to 9 [Gogus and Boucher, 1998]. In order to check for consistency, the matrix $\boldsymbol{D}(\otimes)$ is formed by taking the geometric means of lower and upper bounds, $\boldsymbol{D}^{g} = \sqrt{\underline{x}_{ij} \times \overline{x}_{ij}}$. Since each matrix consists of precise data, Saaty's method of finding the weight vectors can be used. To find the consistency ratio, the weight vectors of each matrix have to be estimated. Weight vectors, $w^{g}$, can be found by (13) [Gogus and Boucher, 1998].

Following Saaty's rule, a consistency ratio of 0.1 or less is considered acceptable for each matrix type. If $CR^{g}$ of a given GPCM are greater than 0.1, then the DM should be encouraged to reassess her/his preferences. According to Saaty's approach, the $CI^{g}$, is given by:

$$CI^{g} = \frac{\lambda^{g}_{max} - n}{n-1} \quad (15)$$

The ratio of consistency can be calculated by:

$$CR^{g} = \frac{CI^{g}}{RI^{g}} \quad (16)$$

Wang et al. [Wang, Elhag and Hu, 2006] found out some defects in LLSM which this model presented by Laarhoven and Pedrycz, 1983 and revised by Boender, Graan and Lootsma, 1989. Such defects include: the inaccurate of normalizing model used for calculating local fuzzy weights, impossibility of determining normalized weights from incomplete the PCM and generating unrealistic final fuzzy weights. To resolve such deficiencies, Wang et al. developed a non linear model based on LLSM to calculate the weights. Nonlinear objective function of their model is minimizing logarithmic difference between the proportion of weights and matrix elements. The linear constraints of such correct

Table 1. Gogus and Boucher's RI

| N | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $RI^{g}$ | 0 | 0 | 0.1796 | 0.2627 | 0.3597 | 0.3818 | 0.4090 | 0.4164 | 0.4348 | 0.4455 | 0.4536 | 0.4776 | 0.4691 | 0.4804 | 0.4880 |

$$w^{g}_{i} = \frac{1}{n}\sum_{j=1}^{n}\frac{\sqrt{\underline{x}_{ij}\times\overline{x}_{ij}}}{\sum_{i=1}^{n}\sqrt{\underline{x}_{ij}\times\overline{x}_{ij}}}, \; w^{g} = [w^{g}_{i}], i = 1, \dots, n \quad (13)$$

Then, the largest eigen value for each matrix is given by: [Gogus and Boucher, 1998]

$$\lambda^{g}_{max} = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\sqrt{\underline{x}_{ij} \times \overline{x}_{ij}} \times \left(\frac{w^{g}_{j}}{w^{g}_{i}}\right) \quad (14)$$

model guarantee normalized and realistic final fuzzy weights [Wang, Elhag and Hu, 2006]. The LLSM by using grey numbers in the present paper is the developed model of Wang, Elhag and Hu which is used for fuzzy number.

For the GPCM, $\boldsymbol{D}(\otimes)$, there should exist a normalized grey weight vector, $W = ([\underline{w}_1, \overline{w}_1], \dots, [\underline{w}_n, \overline{w}_n])^{\mathrm{T}}$, which is close to $\boldsymbol{D}(\otimes)$ in the sense that $\otimes x_{ij} = [\underline{x}_{ij}, \overline{x}_{ij}] = [\underline{w}_i, \overline{w}_i]/[\underline{w}_j, \overline{w}_j]$. The grey weight vector $\otimes W$ is said to be normalized if and only if:

$$\underline{w}_j + \sum_{k=1,k\neq j}^{n} \overline{w}_k \geq 1 \quad ,j = 1, \dots, n \ , \tag{17}$$

$$\overline{w}_j + \sum_{k=1,k\neq j}^{n} \underline{w}_k \leq 1 \quad ,j = 1, \dots, n \, . \tag{18}$$

As was mentioned before, in a GPCM which is perfectly consistent such as $\boldsymbol{D}(\otimes)$, there exists the equation $\underline{x}_{ij} = \underline{w}_i / \overline{w}_j$ and $\overline{x}_{ij} = \overline{w}_i / \underline{w}_j$. If takes logarithm from both sides of the equations:

$$ln\, \underline{x}_{ij} = ln\, \underline{w}_i - ln\, \overline{w}_j \tag{19}$$

$$ln\, \overline{x}_{ij} = ln\, \overline{w}_i - ln\, \underline{w}_j \tag{20}$$

If matrix $\boldsymbol{D}(\otimes)$ are inconsistent, (19) and (20) not run:

$$ln\, \underline{x}_{ij} - ln\, \underline{w}_i + ln\, \overline{w}_j \neq 0 \tag{21}$$

$$\ln \overline{x}_{ij} - \ln \overline{w}_i + \ln \underline{w}_j \neq 0 \tag{22}$$

Using the second power of the two sides of (21) and (22), the following inequalities result:

$$(ln\, \underline{x}_{ij} - ln\, \underline{w}_i + ln\, \overline{w}_j)^2 \neq 0 \tag{23}$$

$$(ln\, \overline{x}_{ij} - ln\, \overline{w}_i + ln\, \underline{w}_j)^2 \neq 0 \tag{24}$$

For obtaining correct and logical weights, summation of (23) and (24) must be minimized. To this end, in the present paper, the non linear programming model of LLSM is developed as follows. Using the model presented, the grey weights obtain as $\otimes W_i = [\underline{w}_i, \overline{w}_i]$. Step 6: Calculating global weight of alternative. Bryson and Mobolurin proposed pair linear programming model for calculating lower and upper bounds of global weights of each alternatives [Bryson and Mobolurin, 1997]. First their LP models are used to calculate lower bounds of weights as follow:

$$\begin{aligned} & Minimize\ \underline{w}_{A_i} = \sum_{j=1}^{m} \underline{w}_{ij} \times w_j \\ & s.t. \\ & \underline{w}_j \leq w_j \leq \overline{w}_j \ ,j = 1, \dots, m \\ & \sum_{j=1}^{m} w_j = 1. \end{aligned} \tag{26}$$

The following LP model is for generating upper bounds.

$$\begin{aligned} & Maximize\ \overline{w}_{A_i} = \sum_{j=1}^{m} \overline{w}_{ij} \times w_j \\ & s.t. \\ & \underline{w}_j \leq w_j \leq \overline{w}_j \ ,j = 1, \dots, m \\ & \sum_{j=1}^{m} w_j = 1 \, . \end{aligned} \tag{27}$$

:

$$\begin{aligned} & Minimize \sum_{i=1}^{n} \sum_{j=1,j\neq i}^{n} (ln\, \underline{x}_{ij} - ln\, \underline{w}_i + ln\, \overline{w}_j)^2 + \sum_{i=1}^{n} \sum_{j=1,j\neq i}^{n} (ln\, \overline{x}_{ij} - ln\, \overline{w}_i + ln\, \underline{w}_j)^2 \\ & s.t. \\ & \underline{w}_i + \sum_{j=1,j\neq i}^{n} \overline{w}_j \geq 1 \quad ,j = 1, \dots, n \ , \\ & \overline{w}_i + \sum_{j=1 j\neq i}^{n} \underline{w}_j \leq 1 \quad ,j = 1, \dots, n \ , \\ & \sum_{i=1}^{n} (\underline{w}_i + \overline{w}_i) = 2 \, , \\ & 0 \leq \underline{w}_j \leq \overline{w}_j \ , \ i, \dots, n. \end{aligned} \tag{25}$$

$$P\{\otimes_1 \le \otimes_2\} = \frac{max(0, L^* - max(0, b - c))}{L^*} \quad (28)$$
$$L^* = L(\otimes_1) + L(\otimes_2)$$
$$L(\otimes_1) = \overline{\otimes}_1 - \underline{\otimes}_1$$

The global weight obtained for alternative by using (26) and (27) are always normal [Wang and Elhag, 2007].

Step 7**:** Ranking the alternative. Prioritizing alternatives is based on global weights calculated for the alternatives. Regarding the fact that global weight calculated for alternatives in the form of grey number, the concept of degree of possibility is used for their comparison [Li, Yamaguchi and Nagai, 2007]. If $\otimes_1 = [a, b]$ and $\otimes_2 = [c, d]$, is two grey numbers which $a < b$ and $c < d$ degree of grey possibility $\otimes_1 \le \otimes_2$ is defined as follows:

There are exists four relations between situation of two grey numbers $\otimes_1$ and $\otimes_2$ .

1- If $a = c$ and $b = d$, two grey number $\otimes_1$ and $\otimes_2$ are equal and one can write $\otimes_1 = \otimes_2$ , $P\{\otimes_1 \le \otimes_2\} = 0.5$ .

2- If $d > a$, can write $\otimes_2 > \otimes_1 =$ , $P\{\otimes_1 \le \otimes_2\} = 1$.

3- If $d < a$, can write $\otimes_1 > \otimes_2 =$ , $P\{\otimes_1 \le \otimes_2\} = 0$.

If there is common part between two grey numbers, in such case if $P\{\otimes_1 \le \otimes_2\} < 0.5$ then $\otimes_1$ is bigger than $\otimes_2$ and if $P\{\otimes_1 \le \otimes_2\} > 0.5$ then $\otimes_1$ is smaller than $\otimes_2$.

Using (28) and network graphs we can show the relationship between priorities of global weights in relation to each other. If global weight of alternative $A_i$ is $\otimes W_i$ and global weight of $A_j$ is $\otimes W_j$ and $P\{\otimes W_j \le \otimes W_i\} = p_{ij} > 0.5$, then according to figure 3, directional arrow is drown from $A_j$ to $A_i$ and number $p_{ij}$ locates on it.

Alternatives are arranged in order from less to more probability.

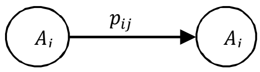


Figure 2. Preference representation for alternative $A_j$ to $A_i$.

## 3. Risk Assessment in Subway System

In this section, the risks of subway transportation system would be identified and ranked by approach which explained in section 2.4 step by step. The GAHP steps to solve the research problem are as follows:

Step 1: Modeling the problem in the form of a hierarchical tree

In this step, the criteria, which evaluate the risk's subway transportation system, have been recognized and the related risks have been identified. The risks and criteria are structured in the form of the decision hierarchy tree as mentioned in the GAHP model. The aim of research problem (evaluation and prioritization the subway's risks) is located at the top of the tree and the identified risks are located at the lowest level of the tree. The main criteria and sub criteria are in the middle level.

According to the literature review of management and assessment risk and interview to transportation experts, we recognized five main criteria and 4 sub criteria to evaluate and rank the subway's risks. These criteria include:

1) Criterion of possibility of occurrence the risk ($C_1$) [Ebrahimnejad, Mousavi and Seyrafianpour, 2009]: This criterion has been expressed in percentage and show the expectance of estimator from the occurrence of risk event. In cases that past data have been used for estimation the criterion of possibility the occurrence, the amount of such criterion is equal to the repetition of its occurrence in span of time.

2) Criterion of rate of severity ($C_2$): This criterion indicates negative or positive impact which has a risk on the objectives of an organization. Such criterion can be estimated based on time or money amount. In the issue of assessment and prioritizing the risks of subway transportation system, such criterion could be divided into 4 sub criteria as: The rate of damage on passengers ($C_{21}$), the rate of public dissemination ($C_{22}$), the rate of inflicted damage on equipment, buildings or symbols ($C_{23}$), economic effects ($C_{24}$). Sub criterion $C_{21}$, is determined based on the rate of inflicted damage on passengers and employees. Sub criterion $C_{22}$ is determined based on the range of dissemination the consequences and their influenced population. Also, Sub criterion $C_{23}$, can be considered as subordinate of rate of news media coverage. Considerable dissemination at the level of population groups could draw attention at national level. Because the rate of sub criterion $C_{23}$ given the importing equipment which are used in wagons and stations, higher cost of their purchase, long time of their orders and receiving the equipment or repairing and demolished buildings are in extra ordinary sensitivity, since they may suspend a line for several days or in a month. Sub criterion $C_{24}$, is determined based on inflicted damages on subway facilities or equipment or cost resulted from reduction of operational potential.

3) Criterion of detection possibility ($C_3$) [Grassi et al., 2009]: This criterion represents that to what extent the organization is able to identify and detect the possible existing risks in subway station.

4) Criterion of organization potential in reaction to risk ($C_4$) [Mikulak, McDermott and Beauregard, 1996] : this criterion shows the ability of organization in prediction the occurrence of risk and proportion for countering it.

5) Criterion of lack of confidence of estimation ($C_5$) [Klein and Cork, 1998]: This criterion also is expressed in percentage shape and is the rate of confidence the analyst from the results of estimation the amount of risk assessment.

After recognizing the evaluation's criteria, by study the historical accidents and hazardous events in Tehran subway transportation system and interview of experts in this system, a list include thirteen risks in subway transportation systems are identified. The identified risks as considered the alternatives in the GAHP approach include:

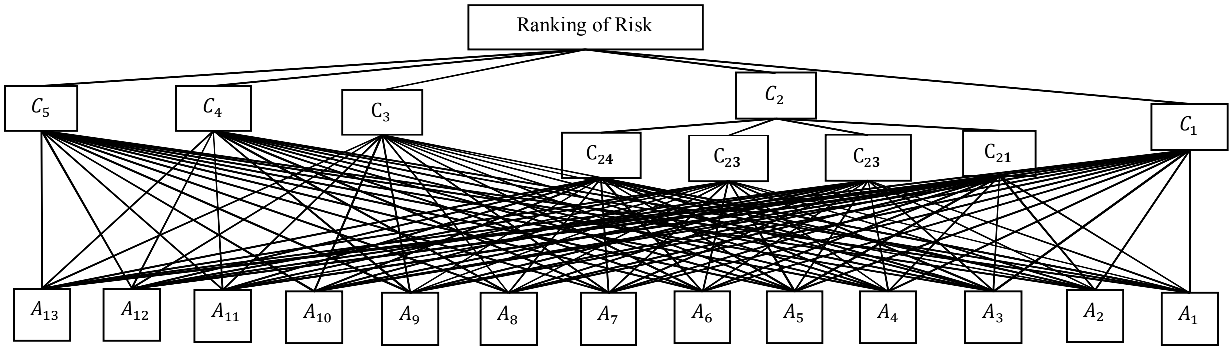


**Figure 3. Display of hierarchical structure for assessment risk problem of subway system**

**Table 2. The GPCM for the five criteria with respect to goal**

| Goal | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| $C_1$ | [1,1] | [1/6,1] | [1,3] | [1,3] | [4,6] |
| $C_2$ | [1,6] | [1,1] | [3,4] | [5,7] | [8,9] |
| $C_3$ | [1/3,1] | [1/4,1/3] | [1,1] | [1/3,1] | [1,3] |
| $C_4$ | [1/3,1] | [1/7,1/5] | [1,3] | [1,1] | [1,2] |
| $C_5$ | [1/6,1/4] | [1/9,1/8] | [1/3,1] | [1/2,1] | [1,1] |

**Table 3. The GPCM for the four sub criteria with respect to $C_2$**

| $C_2$ | $C_{21}$ | $C_{22}$ | $C_{23}$ | $C_{24}$ |
|---|---|---|---|---|
| $C_{21}$ | [1,1] | [2,3] | [1,3] | [8,9] |
| $C_{22}$ | [1/3,1/2] | [1,1] | [1/2,1] | [7,9] |
| $C_{23}$ | [1/3,1] | [1,2] | [1,1] | [5,7] |
| $C_{24}$ | [1/9,1/8] | [1/9,1/7] | [1/7,1/5] | [1,1] |

**Table 4. Consistency test**

| Name of matrix | Consistency condition | Example |
|---|---|---|
| GPCM for the five criteria with respect to goal | Not exist | $\underline{x}_{23}\times\overline{x}_{23}\neq(\underline{x}_{24}\times\underline{x}_{43})\times(\overline{x}_{24}\times\overline{x}_{43})$ <br> $3\times4\neq(5\times1)\times(7\times3)$ |
| GPCM for the four sub criteria with respect to the criterion $C_2$ | Not exist | $\underline{x}_{23}\times\overline{x}_{23}\neq(\underline{x}_{24}\times\underline{x}_{43})\times(\overline{x}_{24}\times\overline{x}_{43})$ <br> $\frac{1}{2}\times1\neq\left(7\times\frac{1}{7}\right)\times(9\times\frac{1}{5})$ |
| GPCM for the thirteen alternatives with respect to the criterion $C_1$ | Not exist | $\underline{x}_{23}\times\overline{x}_{23}\neq(\underline{x}_{24}\times\underline{x}_{43})\times(\overline{x}_{24}\times\overline{x}_{43})$ <br> $2\times4\neq(1\times1)\times(3\times5)$ |
| GPCM for the thirteen alternatives with respect to the criterion $C_{21}$ | Not exist | $\underline{x}_{53}\times\overline{x}_{53}\neq(\underline{x}_{52}\times\underline{x}_{23})\times(\overline{x}_{52}\times\overline{x}_{23})$ <br> $(4\times5)\neq(1\times2)\times(2\times4)$ |
| GPCM for the thirteen alternatives with respect to the criterion $C_{22}$ | Not exist | $\underline{x}_{45}\times\overline{x}_{45}\neq(\underline{x}_{43}\times\underline{x}_{35})\times(\overline{x}_{43}\times\overline{x}_{35})$ <br> $\frac{1}{3}\times1\neq\left(1\times\frac{1}{5}\right)\times(5\times\frac{1}{4})$ |
| GPCM for the thirteen alternatives with respect to the criterion $C_{23}$ | Not exist | $\underline{x}_{63}\times\overline{x}_{63}\neq(\underline{x}_{64}\times\underline{x}_{43})\times(\overline{x}_{64}\times\overline{x}_{43})$ <br> $1\times3\neq\left(\frac{1}{3}\times1\right)\times(1\times5)$ |
| GPCM for the thirteen alternatives with respect to the criterion $C_{24}$ | Not exist | $\underline{x}_{78}\times\overline{x}_{78}\neq(\underline{x}_{75}\times\underline{x}_{58})\times(\overline{x}_{75}\times\overline{x}_{58})$ <br> $2\times3\neq\left(\frac{1}{7}\times4\right)\times(\frac{1}{6}\times5)$ |
| GPCM for the thirteen alternatives with respect to the criterion $C_3$ | Not exist | $(\underline{x}_{89}\times\overline{x}_{89})\neq(\underline{x}_{83}\times\underline{x}_{39})\times(\overline{x}_{83}\times\overline{x}_{39})$ <br> $\frac{1}{6}\times\frac{1}{4}\neq\left(\frac{1}{3}\times\frac{1}{3}\right)\times(1\times\frac{1}{2})$ |
| GPCM for the thirteen alternatives with respect to the criterion $C_4$ | Not exist | $\underline{x}_{76}\times\overline{x}_{76}\neq(\underline{x}_{74}\times\underline{x}_{46})\times(\overline{x}_{74}\times\overline{x}_{46})$ <br> $\frac{1}{4}\times1\neq\left(\frac{1}{5}\times1\right)\times(\frac{1}{3}\times3)$ |
| GPCM for the thirteen alternatives with respect to the criterion $C_5$ | Not exist | $\underline{x}_{95}\times\overline{x}_{95}\neq(\underline{x}_{96}\times\underline{x}_{65})\times(\overline{x}_{96}\times\overline{x}_{65})$ <br> $\frac{1}{3}\times1\neq\left(1\times\frac{1}{4}\right)\times(3\times\frac{1}{3})$ |

Casualty during using escalator ($A_1$), pushing the people on the edge of rail which has electricity current ($A_2$), passing from the area of rail with aim at going and coming between two stations ($A_3$), colliding the train to the passengers ($A_4$), injury because of closing the doors of train ($A_5$), opening the opposite door for movement ($A_6$), suicide ($A_7$), electric shock and general fire ($A_8$), transferring epidemic and infection ailments ($A_9$), bad quality of air resulting from polluters emanated from friction of wheels ($A_{10}$), terrorist, biologic and bomb attacks ($A_{11}$), conflicts among passengers ($A_{12}$), flood, snow and earthquake ($A_{13}$). After determination the objective of issue, criteria and alternatives of decision, a hierarchy decision tree as figure 3 is created.

Step 2: Forming the Grey Pairwise Comparison Matrices (GPCM)

In this step, by using the opinions of subway transportation system experts, the preferences of any criterion (sub criterion) in comparison with another criterion (sub criterion) given the aim of issue and also preferences of any alternative in comparison with another alternative given the criteria, in decision tree (figure3) were determined in the format of grey numbers. In a meeting, we asked ten experts in Tehran subway system to fill the ten GPCMs. Two of which are shown in tables 2 and 3 with grey numbers.

Therefore, A GPCM with grey elements for comparison criteria with each other terms of aim (table 2), A GPCM with grey elements for comparison sub criteria of rate the severity the event to each other (table 3) and 8 matrices of pairwise comparison with grey elements for comparison the alternatives with each other in terms of criteria and sub criteria of issue of decision are formed.

Step 3: Investigating consistency condition of GPCM

To study the consistency condition for each the formed GPCMs, the Eq. (12) should be examined for each of 10 matrices. The results of this examination for all GPCMs have been shown in table 4. The condition of consistency is not exists in all formed matrices. So, the consistency ratio must be calculated in them.

Step 4: Calculating consistency ratio in GPCM

In this step, by using the Eq. (13) to (16), the rate of consistency for each of 10 matrices of pairwise comparison is calculated. The results of this study are shown in table 5. The CR value achieved for all matrices in less than 0.1, so one can disregard inconsistency.

Step 5: Local grey weighting of decision criteria and alternative

After studying the consistency conditions in GPCMs and confidence on authenticity the judgment of decision maker, the local weights are calculated for alternatives and criteria of problem. For this purpose, the Eq. (25) is used. Achieved local weights have been shown in table 6 and 7.

Step 6: Calculating global weight of alternative

To calculate the various weights of hierarchy, it is necessary in this step to move toward the goal with beginning from the last level (alternative), for achieving the global weights of alternatives. First by using Eq. (26) and (27), the weight of alternatives are calculated based on sub criteria of $C_2$. The result of this calculation is shown in table 6. Then, the global weights for alternatives are achieved based on the criteria of $C_1, C_2, C_3, C_4, C_5$ by using Eq. (26) and (27). The result of this calculation is shown in table 7.

Step 7: Ranking the alternative

In this step, to compare global grey weights which obtained from previous step and ranking alternatives, Eq. (28) was used

Figure 4 shows the preference of alternative $A_5$ to other alternatives. As it shown in figure 4, $A_5$ (injury because of closing the doors of train) has the highest priority among the identified risks. Tehran subway administrators have to consider appropriate programs to deal with this risk. $A_{12}$, $A_1$ and $A_2$ are ranked as other risks respectively and the schemers should plan suitable responses to manage them. Similar figure 4, other figures are drawn for other alternatives and ranked them as:

$$A_5 \overset{52\%}{>} A_{12} \overset{61\%}{>} A_1 \overset{85\%}{>} A_2 \overset{52\%}{>} A_9 \overset{64\%}{>} A_4 \overset{72\%}{>} A_6 \overset{95\%}{>} A_3 \overset{77\%}{>} A_7 \overset{54\%}{>} A_{10} \overset{100\%}{>} A_8 \overset{75\%}{>} A_{11} \overset{100\%}{>} A_{13}$$

## 4. Conclusion

In this paper, a model has been introduced for assessment and prioritizing the risks, by using decision making method of AHP in grey condition (uncertainty and insufficiency information). Regarding of the complexity of decision making issues and the subjective nature of DM judgments, using grey pairwise comparison matrices could be a reliable framework to account for such uncertainty. In this paper for studying the consistency condition in the GPCM in AHP method, the fuzzy method of Gogus and Bucher in grey condition has been used. Also, for achieving the weights of criteria and alternatives, a non linear programming method has been developed for matrices that could be consistent or inconsistent. Based on studying Tehran urban transportation system, 5 criteria, 4 sub criteria and 13 alternatives (risks) are identified and the steps of GAHP model are implemented step by step. By using a non linear and two linear programming models, local and global weights of risks were calculated and their comparisons were carried out by using the degree of possibility and network graphs. By using GAHP model, 13 identified risks in subway were prioritized and casualty resulting from closing the train door was the highest rate among the other alternatives.

**Table 5.** $\lambda_{max}$, **CI and CR in for GPCMs**

| Name of matrix | $\lambda_{max}$ | $CI$ | $RI$ | $CR$ | Consistency status |
|---|---|---|---|---|---|
| GPCM for the five criteria with respect to goal | 5.12988 | 0.03247 | .0.3597 | 0.0903<0.1 | Consistent |
| GPCM for the four sub criteria with respect to the criterion $C_2$ | 4.072444 | 0.024148 | 0.2627 | 0.0919<0.1 | Consistent |
| GPCM for the thirteen alternatives with respect to the criterion $C_1$ | 13.5506 | 0.04588 | 0.4691 | 0.0978<0.1 | Consistent |
| GPCM for the thirteen alternatives with respect to the criterion $C_{21}$ | 13.5433 | 0.04527 | 0.4691 | 0.0965<0.1 | Consistent |
| GPCM for the thirteen alternatives with respect to the criterion $C_{22}$ | 13.5614 | 0.04678 | 0.4691 | 0.0997<0.1 | Consistent |
| GPCM for the thirteen alternatives with respect to the criterion $C_{23}$ | 13.5433 | 0.04527 | 0.4691 | 0.0965<0.1 | Consistent |
| GPCM for the thirteen alternatives with respect to the criterion $C_{24}$ | 13.4556 | 0.03797 | 0.4691 | 0.0809<0.1 | Consistent |
| GPCM for the thirteen alternatives with respect to the criterion $C_3$ | 13.5378 | 0.04482 | 0.4691 | 0.0956<0.1 | Consistent |
| GPCM for the thirteen alternatives with respect to the criterion $C_4$ | 13.5618 | 0.04682 | 0.4691 | 0.0998<0.1 | Consistent |
| GPCM for the thirteen alternatives with respect to the criterion $C_5$ | 13.5308 | 0.04423 | 0.4691 | 0.0943<0.1 | Consistent |

**Table 6. The Grey weights obtained for sub criteria of $C_2$**

| $C_{ij}$ | Criteria weight | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ | $A_8$ | $A_9$ | $A_{10}$ | $A_{11}$ | $A_{12}$ | $A_{13}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_{21}$ | [0.3848 , 0.52] | [0.1254, 0.1617] | [0.0904, 0.1431] | [0.0339, 0.0526] | [0.0713, 0.1173] | [0.131, 0.1733] | [0.0510, 0.1004] | [0.0269, 0.0418] | [0.0192, 0.0241] | [0.0944, 0.1247] | [0.0206, 0.0297] | [0.0167, 0.0209] | [0.1373, 0.1661] | [0.0122, 0.0140] |
| $C_{22}$ | [0.2069, 0.2440] | [0.1318, 0.1643] | [0.0925, 0.1460] | [0.0328, 0.0516] | [0.0707, 0.1129 ] | [0.1316, 0.1734] | [0.0495, 0.0919] | [0.0258, 0.0428] | [0.0191, 0.0239] | [0.0899, 0.1268] | [0.0203, 0.0297] | [0.0181, 0.0219] | [0.1361, 0.1688] | [0.0125, 0.0151] |
| $C_{23}$ | [0.2052, 0.3572] | [0.1294, 0.1601] | [0.0922, 0.1441] | [0.03161, 0.048] | [0.0668, 0.11557] | [0.1331, 0.1728] | [0.0501, 0.1004] | [0.0251, 0.0428] | [0.0199, 0.0248] | [0.0909, 0.1264] | [0.2049, 0.0309] | [0.0181, 0.0219] | [0.1361, 0.1689] | [0.0131, 0.0156] |
| $C_{24}$ | [0.0408, 0.0407] | [0.1304, 0.1595] | [0.0877, 0.1348] | [0.0329, 0.0501] | [0.0663, 0.1112] | [0.1516, 0.1786] | [0.0484, 0.0908] | [0.0251, 0.0399] | [0.0185, 0.0229] | [0.0897, 0.1234] | [0.0202, 0.0304] | [0.0158, 0.0207] | [0.1435, 0.1815] | [0.0124, 0.0144] |
| Grey weight | | [0.1279, 0.1619] | [0.0911, 0.1438] | [0.0328, 0.0513] | [0.0694, 0.1157] | [0.1324, 0.1735] | [0.0502, 0.0982] | [0.0259, 0.0423] | [0.0192, 0.0243] | [0.0919, 0.1257] | [0.0302, 0.0583] | [0.0172, 0.0215] | [0.1369, 0.1683] | [0.0125, 0.0148] |

**Table 7. Global weights of Subway system risks**

| $C_j$ | Criteria weight | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ | $A_8$ | $A_9$ | $A_{10}$ | $A_{11}$ | $A_{12}$ | $A_{13}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_1$ | [0.1413, 0.3131] | [0.1348, 0.1666] | [0.0931, 0.1417] | [0.0342, 0.0522] | [0.0728, 0.1161] | [0.1260, 0.1726] | [0.0488, 0.10001] | [0.0252, 0.0407] | [0.0190, 0.0235] | [0.0952, 0.1236] | [0.0212, 0.0302] | [0.0165, 0.0205] | [0.131, 0.1701] | [0.0116, 0.0128] |
| $C_2$ | [0.4519, 0.5387] | [0.1279, 0.1619] | [0.0911, 0.1438] | [0.0328, 0.0513] | [0.0694, 0.1157] | [0.1324, 0.1735] | [0.0502, 0.0982] | [0.0259, 0.0423] | [0.0192, 0.0243] | [0.0919, 0.1257] | [0.0302, 0.0583] | [0.0172, 0.0215] | [0.1369, 0.1683] | [0.0125, 0.0148] |
| $C_3$ | [0.0789, 0.1336] | [0.1280, 0.1671] | [0.0847, 0.1325] | [0.033, 0.0536] | [0.0672, 0.1142] | [0.1549, 0.1784] | [0.0482, 0.0859] | [0.0244, 0.0404] | [0.0194, 0.0237] | [0.0865, 0.1224] | [0.0192, 0.0304] | [0.0165, 0.0209] | [0.1391, 0.1784] | [0.0135, 0.0175] |
| $C_4$ | [0.0900, 0.1353] | [0.1261, 0.1668] | [0.0826, 0.1384] | [0.0309, 0.0529] | [0.068, 0.1139] | [0.1527, 0.1846] | [0.0467, 0.0859] | [0.0244, 0.0396] | [0.0197, 0.0242] | [0.0871, 0.1217] | [0.0194, 0.0303] | [0.0169, 0.0215] | [0.1372, 0.1745] | [0.0147, 0.0192] |
| $C_5$ | [0.05539 , 0.06138] | [0.1148, 0.1612] | [0.0875, 0.1348] | [0.0306, 0.0492] | [0.0658, 0.1153] | [0.1472, 0.1899] | [0.0479, 0.0811] | [0.0264, 0.0397] | [0.0169, 0.0235] | [0.0845, 0.1266] | [0.0203, 0.0297] | [0.0166, 0.0227] | [0.1455, 0.1943] | [0.0125, 0.0154] |
| Grey global weights | | [0.1278, 0.1643] | [0.0892, 0.1414] | [0.0326, 0.0519] | [0.0692, 0.1155] | [0.1348, 0.1765] | [0.049, 0.0957] | [0.0254, 0.0404] | [0.0191, 0.0241] | [0.0905, 0.1246] | [0.0302, 0.0374] | [0.0169, 0.0214] | [0.1357, 0.1724] | [0.0125, 0.0155] |

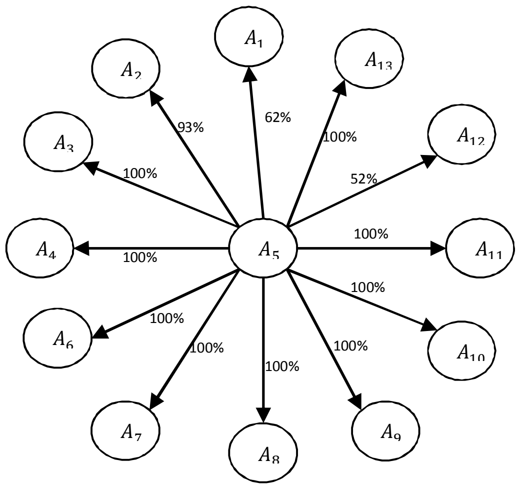


**Figure 4. Preference representations for alternative $A_5$ to other alternatives**

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