A Systematic Method to Analyze Transport Networks: Considering Traffic Shifts

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Abstract:
Current network modeling practices usually assess the network performance at specified time interval, i.e. every 5 or 10 years time horizon. Furthermore, they are usually based on partially predictable data, which are being generated through various stochastic procedures. In this research, a new quantitative based methodology which combines combinatorial optimization modeling and transportation network engineering has been implemented to identify the network performance over time horizon. This method incorporates both uniform traffic demand growth and demand shifts towards more attractive zones (demand uncertainty) in the network. The proposed combinatorial programming approach defines a quantitative measure of growth and shift in the traffic load that a network can sustain. This method can assess the various potential growth topology of a transport network and investigate if a specified topology can sustain more traffic demand or if any specific topology can handle traffic shifts without significant need to amend the network infrastructure. It is believed that this quantitative measure is useful both in transport network design and in the performance analysis of the existing networks. In this paper, the application of this method is demonstrated by applying the method to a part of the Melbourne's transportation network.

Keywords: Transportation engineering, network design and planning, transport network optimization

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1. Introduction

A typical urban traffic profile has a stochastic behavior which is generally of multi-hour nature during a day time (for the typical weekdays and weekends). By gradual increase in the intensity of the traffic, and also due to unpredicted traffic shifts in the Metropolitan areas as the result of traffic attraction increase/decrease for different zones of the network, the transportation authorities are confronting very challenging network design problems.

Existing modeling practices on transportation network design, consider the current traffic matrix as a benchmark and evaluate the future transport traffic demand forecasts dynamically but discretely over an specified time horizon for given time intervals, e.g. for years 2015, 2020, 2030, etc… [Papacostas, 2001, Ortuzar, 2005]. This implies that some predictable and evaluative events, which are usually captured by traditional four-step transport modeling are taken into account. An example is the linear growth of the population and the associated traffic demand. However some complex classes of events like "traffic shifts towards more attractive zones in the network" are not studied well in the literature.

Defining the growth associated with shift, namely Unexpected Traffic Growth (UTG) in the network is more difficult as there are many ways that the load on a network can change. It is easy to verify that existing practices do not provide the necessary means to systematically accommodate various traffic shifts in the transport network design (a representative of uncertainty and error in the model). Furthermore, the lack of consideration of data uncertainty in these models (such as population and employments) is another vital aspect that should be taken into account. As noted, the current design practices do not cope well with these real life practice requirements.

1.1 Literature Review

Many studies have been carried out to assess transportation network performance. According to the literature review, these can be generally classified into the following classes: reliability analysis (which consists of end-to-end connectivity, OD travel times reliability and link capacities based models), trip cost modeling, demand uncertainty analysis and reserve capacity modeling.

It should be noted that the concept of connectivity which is a deterministic concept is somehow related to stochastic measures like the probability of an OD-path to be active or not. Here the term "active" is defined in analogy to non-congested or its equivalent terms in transportation engineering. The following is a brief literature review on the above mentioned reliability concept analysis.

OD travel time reliability is defined as the probability that a trip between a given OD pair can be successfully completed within a specified time period. To assess the network performance, this measure is useful for normal daily flow variations [Asakura et al., 1994].

Link capacity reliability is defined as the probability that a given network can accommodate a certain traffic demand at a specified level of service while drivers' route choice behavior is accounted [Chen et al., 1999]. Chen et al (1999) proposed a new reliability measure based on the probability that a network can sustain a certain travel demand at a particular level of service (capacity reliability). This method accounts for drivers' route choice behavior. The authors proposed a mathematical model based on the reserve capacity for transportation networks. In their approach, the probability that a network can accommodate a certain traffic demand was calculated. This method adopts the concept of reserve capacity based on the largest multiplier, which can be applied to the individual demand matrix elements without violating any link capacity in the network. A common multiplier for each OD pair in the network is then calculated by the proposed method without identifying the critical links in the network. Chen et al (2002) extended their previous study by providing a new methodology, which was a combination of reliability (travel time and connectivity) and uncertainty analysis (using sensitivity analysis and equilibrium network flow methodologies). This measure assesses whether the current network capacity is sufficient for required level of service. The outcome of this method is to providing useful information to obtain better flow control via extending capacity, which helps to enhance the reliability of the network.

End-to-End connectivity reliability is the probability that the nodes of a given network remain connected.

For each node pair in the network there should be at least one operational path to consider the network reliable [Iida et al., 1989].

Scott et al (2006) introduced a network robustness index as a measure to evaluate network performance by identifying critical links. They also showed that the proposed new index to measure network performance is a better indicator compared to the traditional v/c ratio (volume over capacity). Furthermore, they argued that ratio is a localized measure, suitable to evaluate the performance of a specified link in the network and is unable to identify the performance of the entire network. To assess the network performance, considering the network topology, a new indicator as the ratio of actual number of links (e) to the maximum number of links in a given network (e_{max}) was defined (e / e_{max}) Where e_{max} is calculated by:

$$e_{max} = 3 (n - 2) \quad (1)$$

Here n is the number of nodes in the network. It is assumed that all nodes in the network are completely connected.

The indicator value (e / e_{max}) only accounts for the connectivity of the links in the network (connectivity reliability) and does not assess the traffic between each OD pair or link capacities. Combining these values, they introduced the Network Robustness Index (NRI) to assess the network performance by rerouting excess traffic to the alternative routes with spare capacities (Scott et al., 2006).

Trip cost reliability modeling: a network is considered to be reliable with respect to its ODs trip costs if the trip cost of any given OD node pair is within the acceptable range [Bell, 2000].

Using a game theory approach Bell (2000) offered a definition for network reliability based on expectation of trip cost. He studied the network reliability from different point of view. According to Bell's definition, a network is reliable if the desired cost of trip is acceptable for all OD node pairs. In other words, the cost of trip from a given origin to any destination is associated to the network performance. With the Bell's method, links with the highest impact on the network (in terms of network topology and link properties) can be identified.

Demand uncertainty analysis is usually defined based on traffic engineering variables such as traffic congestion during peak hour periods. It should be mentioned that uncertainty is usually referred to functions and/or parameters such as link capacity variation, travel demand and travel time (Chen et al., 2002).

Reserve capacity modeling is based on the maximum multiplier ($\alpha > 1$) which is applied to the original OD demand matrix where there is no capacity violence in any link in the network. This concept of reserve capacity was introduced [Webster and Cobbe, 1966] to analyze a signal-controlled intersection. Further enhancement of this concept, to evaluate the maximum network capacity of general signal-controlled network, under route choice model, was performed by [Wong and Yang, 1997].

In another work [Sumalee et al., 2009], authors introduced a new elastic assessment and design model for transportation network capacity analysis under variable demand conditions. This concept of reserved capacity methodology evaluates the performance of the network when the demand is not fixed. They extended the reserve capacity concept with stochastic OD demand definition. This model shows the degree of flexibility of the network in terms of OD demand variation. By increasing the mean (reserved capacity) and the Standard Deviation (SD) (index of flexibility) of OD demand matrix, this model can determine the optimal network design for various uncertain demand patterns. In this stochastic model it is assumed that the demand matrix follows a normal distribution function.

In the literature, we observe that the main focus of researchers is on the network performance from reliability point of view where they evaluate the performance of the network for a static OD demand matrix. In other words, these methods linearly update the current demand matrix (considering a series of multipliers) to estimate the demand matrix for a set of discrete times in future (i.e. at specific time intervals). Nevertheless, the continuity of time dimension and its effects on the traffic demand variation is not properly addressed in these studies. This implies that the assessment of the network performance continuously over time horizon is not considered by the previous researchers [Ouveysi and Sarvi, 2011]. Furthermore, the relative variation of OD demands, which could occur in irregular pattern

Table 1. Components Included In the Transportation Network Performance.

Components	Reliability					
Sources	Connectivity	Travel Time	Capacity	Trip Cost	Uncertainty	Reserve Capacity
Chen et al (1999)	✓	✓	✓			✓
Bell (2000)				✓		
Chen et al (2002)	✓	✓	✓		✓	✓
Scott et al (2006)	✓					
Sumalee (2009)	✓	✓	✓		✓	✓

(changing the demand in some zones over time compared to other zones), has not been considered in previous studies either.

In this paper, a new methodology called transport network lifetime analysis has been implemented [Ouveysi and Sarvi, 2011]. The lifetime analysis, addresses the shortcomings of previous studies by assessing the transportation network performance over its lifetime. Utilizing this methodology, the performance of a network under various traffic patterns can be analyzed. In this methodology the typical linear or uniform traffic growth together with various traffic shifts are applied to the network over the lifetime of the network. Generally, the linear traffic growth is due to increase in the population which can be estimated statistically. On the other hand, the traffic shift, which accounts for the likelihood of the population movement and/or business relocation, can significantly affect the transportation network performance. The main difference between the proposed methodology and the earlier approaches is the capability to measure the network performance continuously over time horizon and assessing the behavior of each individual traffic zone in association with the whole network.

The linear traffic growth is calculated by increasing the traffic demand of all ODs in the network with the same proportion, gradually, until the network cannot tolerate any more traffic with respect to a given criterion. This means that traffic matrix between each pair of zones increases by the same multiplier and once the first event of link failure is observed in the network, the correspondent multiplier is assumed to be the maximum sustainable linear traffic growth. Although in this paper only linear traffic growth has been considered for the justification of natural traffic growth, however, the developed methodology can simply handle any non-linear pattern as well.

In conjunction to linear traffic growth, this methodology considers various traffic shifts in the network by shifting traffic from less attractive zones towards more attractive ones (based on scenarios defined with respect to the data on job opportunities, the place attractiveness and the future demographic forecasts). It should be noted that in the rapid developing countries and cities, the transport traffic shift is the most significant and unpredictable factor contributing to traffic congestion.

The remaining of this paper is as follows. In the next section, a brief introduction to the proposed methodology is explained. Then its application to analyze the lifetime of the Melbourne transportation network is discussed. The last section is devoted to the results of the analysis and discussions.

2. Methodology

This method is a combination of a sophisticated mathematical algorithm and transportation network modeling.

Consider a transportation network denoted by graph G consisting of z zones, n nodes and L links. It is assumed $z, n, L \in N$. Assume the end-to-end travel matrix of the network G is given as T(i,j) Initially, it is considered that the network G is subject to the linear growth and the associated maximum linear growth of the network is calculated. In the next step, it is suggested that the network is subject to traffic shifts and maximum Unexpected Traffic Growth (UTG) is derived. Finally, the lifetime curve of the network is depicted according to the achieved values of maximum linear growth versus the UTG. These are explained in the following sections.

2.1 Calculating of Maximum Linear Traffic Growth

The maximum linear growth is a multiplier greater than 1, denoted by ψ, such that ψ T(i,j) is still sustainable. Having enough capacity to support traffic load without violating any link capacity in the network is the criterion of sustainability/reliability for the given network G. In order to calculate the maximum linear growth, the demand matrix of a transportation network is increased gradually, simultaneously and indiscriminately for all ODs in the network until the network cannot tolerate any more traffic load. In other words, Traffic Congestion Index (TCI) of at least one link in the network, say link reaches to the predefined limit value, denoted ζ. Where

$$(v / c) > \zeta \tag{2}$$

In this constraint, V and C are volume and capacity of link l_{ij} respectively and ζ is a measure of congestion, which is defined for each link in the network. In this study, it is assumed that this measure is 1.15 and identical for all links. Therefore, each link can tolerate 15 percent more traffic load than predefined capacity without significant congestion.

As an example, consider a network with four zones; where attraction and production values of zones are known a priori. Assume that the current traffic matrix is known. For illustration purpose a few OD traffic values are demonstrated schematically in Figure 1a.

For the sake of simplicity only 4 ODs are considered here (from a total of 16 OD entries in the traffic matrix of 4x4). In order to calculate the maximum feasible multiplier ψ the traffic matrix values are increased linearly up to the point that at least one link in the network reaches to its predefined maximum limit (ζ).

Assume that Figure 1b shows highest relative traffic increment that this network can sustain. Then as described above, the ratio of the increased traffic value to the primary traffic value is the maximum linear growth multiplier ψ for the given network.

$$\psi = \frac{\Delta\mu}{\mu} = \frac{\Delta\beta}{\beta} = \cdots \tag{3}$$

2.2 Maximum UTG

Ouveysi et al have assumed a series of scenarios associated with unexpected traffic shifts in the network. An example of such event could be establishing a new business or educational institute in a zone. This would amplify the attraction value of the corresponding zone, leading to traffic flow growth between this zone and its immediate neighbors' zones.

Denoting the attraction and production values of zone i as A_i, and P_i respectively, the conservation of flow criterion dictates that the total attraction of all zones must

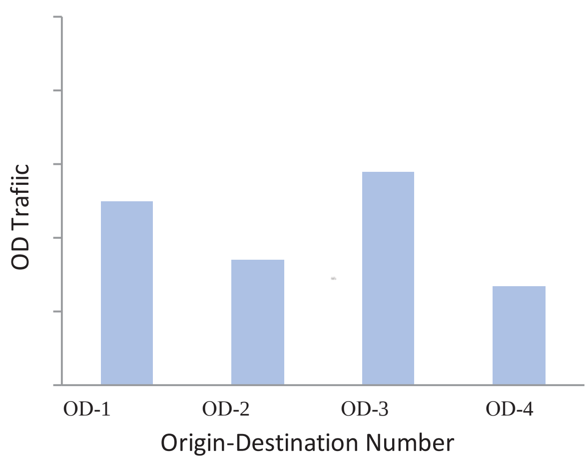

Figure 1a: OD traffic values.

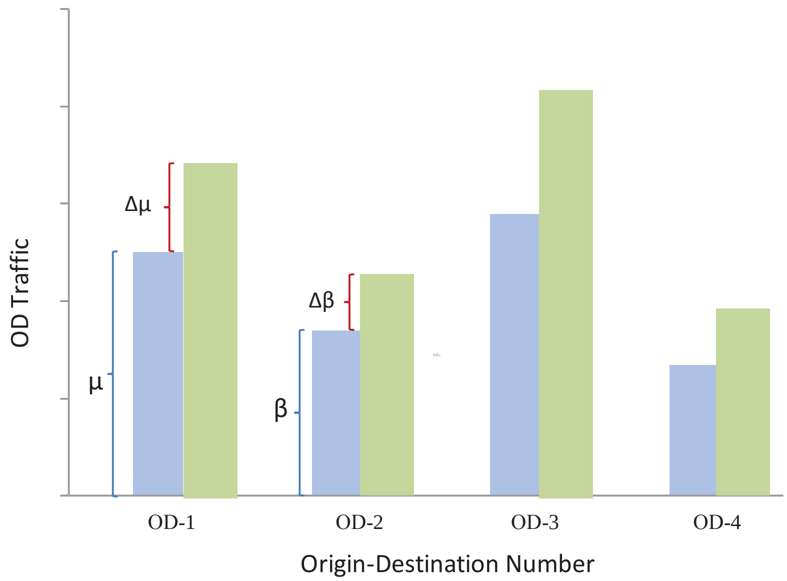

Figure 1b. Increased traffic values of ODs.

be equal to the total production of all zones. In other words, in a network with n zones:

$$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} P_i \quad (4)$$

Ouveysi et al acknowledged that when the attraction value of a zone increases (calling it a developing area), this would be compensated by reduction of attraction of some of other zones to satisfy Equation 4. For the purpose of calculating this reduction magnitude, the Gravity model has been used. In the proposed Gravity model, the travel time (or distance between zones) together with the attraction/production values constitute the main factors for trip distribution analysis. Simply the closer zones to the developing area have greater reduction factor compared to farther zones.

In terms of the number of traffic shifting scenarios, here it is assumed that all zones in the network are subject to UTG one by one in isolation. Figure 2a illustrates the procedure for calculating U_{max} schematically.

In this example, the attraction of Zone 1 is increased and the attractions of other zones are decreased. Using the Gravity model, we have:

$$T_{ij} = T_i \frac{A_j f_{ij}}{\sum_{i=1}^{n} A_j f_{ij}} \quad (5)$$

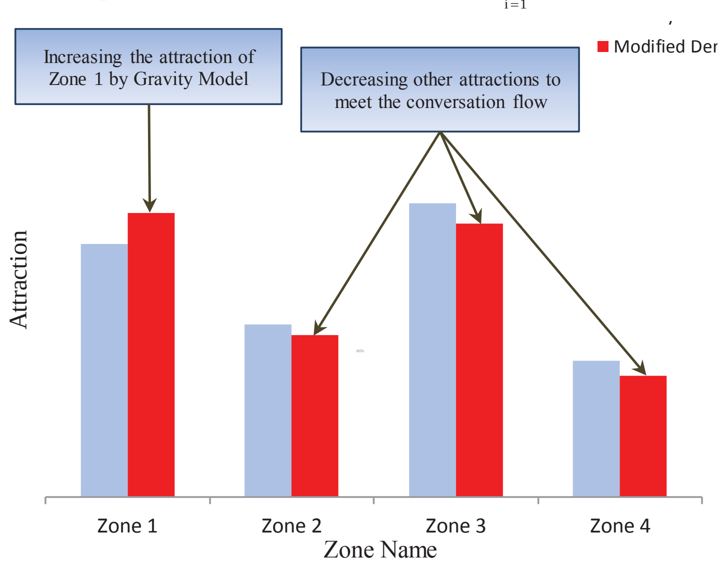

Figure 2a. Increment in Zone 1 attraction value must be compensated by other zones.

Where:
T_{ij} : Trips from i to j,
T_i : Trips from i, as per our generation analysis,
A_j : Trips attracted to j, as per our generation analysis,
f_{ij} : Travel friction factor (function of travel cost, travel time ...).

Figure 2b shows the process of increasing the attraction value of Zone 1 and the reduction of the attractions of the remaining zones. The modified attraction value of zone i, denoted as $\bar{A}_i$, can be shown in the form of $A_i (1 + U)$. For scenario associated with zone i, U^i_{max} is an upper bound for maximum traffic shift. For a network with zones, such scenarios could be defined, where each scenario is related to a specific zone. In order to obtain maximum traffic shift of a network, minimum U_{max} value of all scenarios should be selected.

$$U_{max} = \min (U^1_{max}, U^2_{max}, \ldots, U^n_{max}) \qquad (6)$$

In the next section, the implementation of the proposed lifetime analysis will be presented on a real network.

3. Numerical Example

Here, a part of the Melbourne transportation network is selected for the lifetime analysis. This network is located in eastern of Melbourne, consisting of one freeway and some arterials. It consists of 40 zones, 1080 links and 481 nodes. The topology, the capacity of links and the speed limits are extracted from the main Melbourne transportation network. This network is surrounded by Doncaster and Doncaster East suburbs in the North, Heathmont, Wantirna South in the East, Glen Waverly and Mt Waverly in the South, and Box Hill South and Box Hill North suburbs in the West. Figure 3 shows the layout of this area.

3.1 Five-Zone Scenario

It is assumed that out of the total of 40 zones, only 5 zones are subject to UTG. This implies that 5 scenarios should be considered into account where in each scenario only one of these 5 zones would be exposed to UTG. These five zones and input data required for analyzing of this network in the VISUM package are shown in Table 2.

3.1.1 Input Data

The data considered in this analysis are:
- The expected annual traffic increment: 0.03 (3% traffic increase per year)
- The number of zone(s) subject to UTG: 5
- Total number of zones: 40
- TCI value (v / c): 1.15
- Data file and VISUM file

In this table the term “Attraction” denotes the average number of weekdays’ person trips attracted to each transportation zone and the term “Production” is the average number of weekday’s person trips produced by households within each transportation zone.

It is assumed that the annual traffic growth in this transportation network is 3%. In other words, the

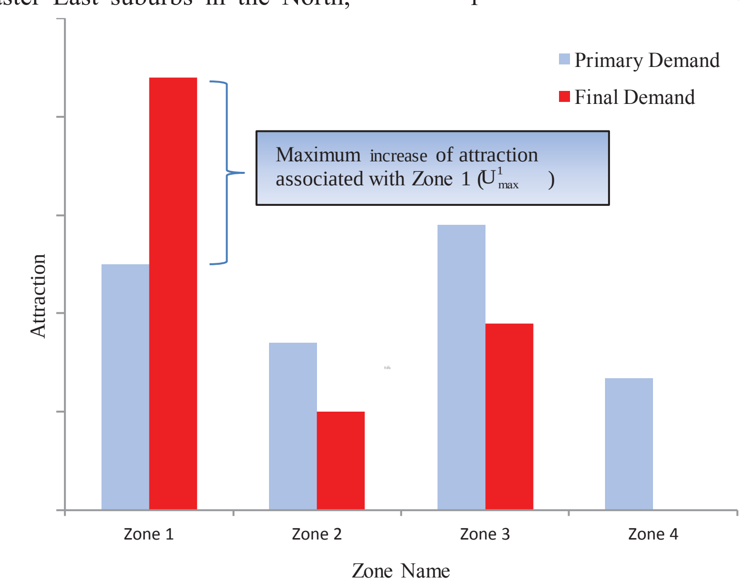

Figure 2b. The output of analyzing Zone 1: The value of U^1_{max} is calculated and Zone 4 is the most scarifying zone.

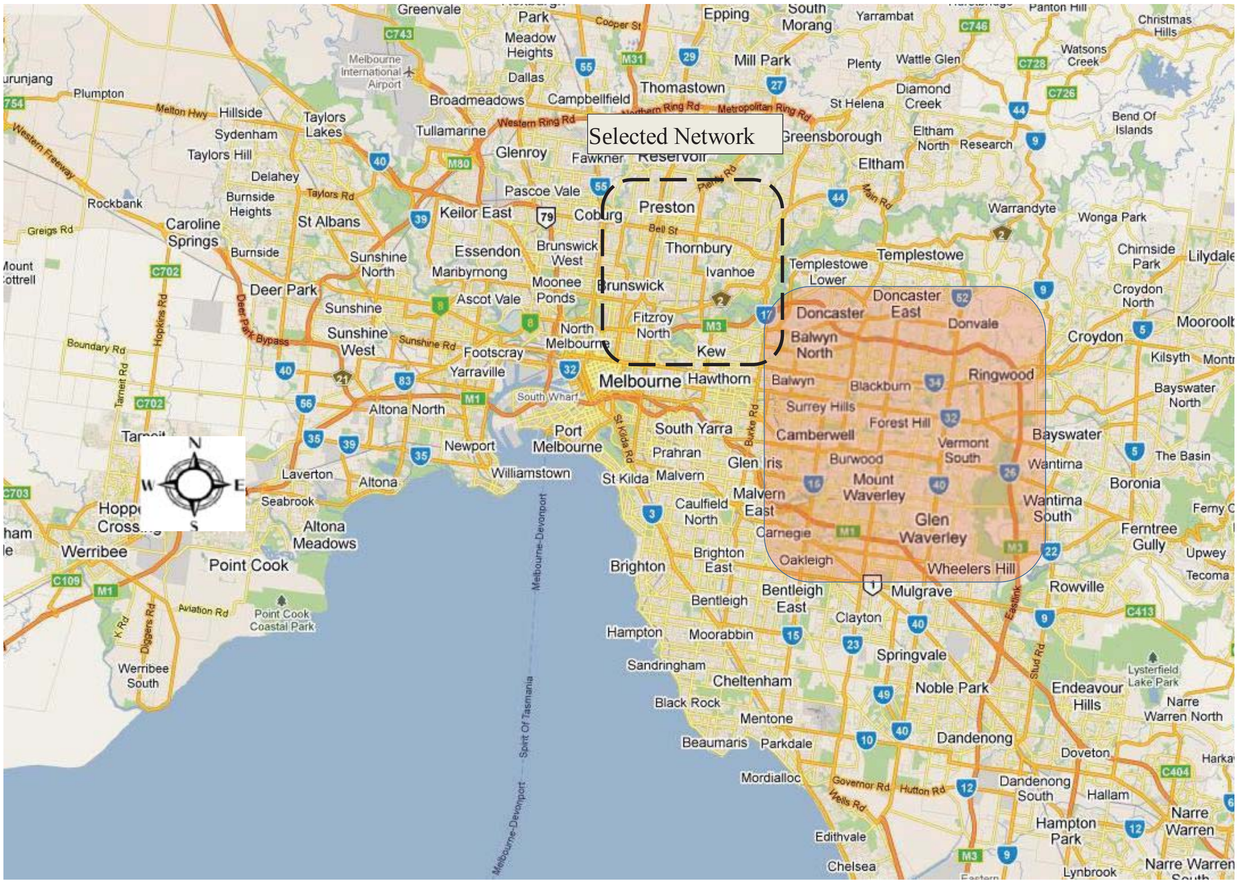

Figure 3. Layout of the selected area of Melbourne's transport network selected for implementation and analysis purposes.

Table 2. Input Data for 5-Zone Scenario.

Zone Number	Zone Name	Attraction (trip/day)	Production (trip/day)
Zone 601	Box Hill North	1560	1222
Zone 609	Burwood	1408	680
Zone 619	Vermont	1426	1162
Zone 1177	Heathmont	259	188
Zone 1603	Donvale	682	559

zonal production values are increased by three percent each year. The stopping criterion for the analysis is the TCI (once the flow in any link exceeds its 115% of capacity value).

3.2 Implementation Results

The results obtained from this implementation are given in the following table. Please note that U=3.97 is the maximum UTG value that is found by the lifetime algorithm. In order to achieve a semi-continuous behavior of the network with respect to UTG values, it is considered that the amount of shift imposed on the zones gradually increases in ten steps from 0 to U_{max} (here U_{max} =3.97).

The output data is shown graphically in Figure 4. The depicted graphs which are obtained from the output table show the behavior of individual zones with respect to traffic shift.

The vertical axis of this graph ($\psi(U)$) represents the linear traffic growth and the horizontal axis (U) corresponds to the values of UTGs.

Results show that Zone 609 (Box Hill area) is the critical one in the network because it reaches the capacity limit very soon, and hence, it is more critical than others. In this network the maximum feasible linear growth is computed to be equal to 1.24 ($\psi(U)$) in the case of U=0 (where there is no traffic shift). It indicates that this network can no longer tolerate any traffic growth

Table 3. Output Result for Five-Zone Scenario.

U values as Percent of U_{max}	0	$\frac{U_{max}}{10}$	$\frac{2U_{max}}{10}$	$\frac{3U_{max}}{10}$	$\frac{4U_{max}}{10}$	$\frac{5U_{max}}{10}$	$\frac{6U_{max}}{10}$	$\frac{7U_{max}}{10}$	$\frac{8U_{max}}{10}$	$\frac{9U_{max}}{10}$	U_{max}
	0	0.397	0.794	1.191	1.588	1.985	2.382	2.779	3.176	3.573	3.97
Zone 601	1.24	1.24	1.24	1.24	1.09	1	1	1	1	1	1
Zone 609	1.24	1	1	1	1	1	1	1	1	1	1
Zone 619	1.24	1.27	1.27	1.09	1	1	1	1	1	1	1
Zone 1177	1.24	1.24	1.24	1.27	1.27	1.27	1.27	1.27	1.27	1.21	1.15
Zone 1603	1.24	1.24	1.24	1.21	1.18	1.21	1.15	1.15	1.12	1.09	1
Min	1.24	1	1	1	1	1	1	1	1	1	1

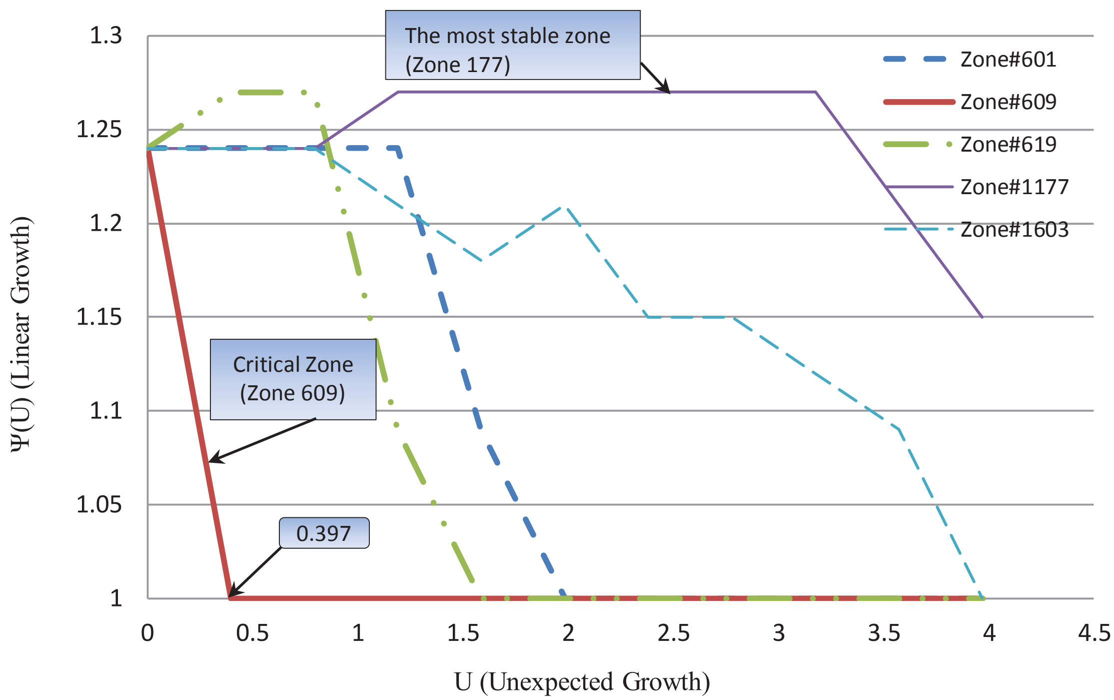

FIGURE 4. Individual zone behavior subject to UTG.

after 7 years, assuming 3 percent traffic growth per annum; even in the absence of any traffic shift, i.e.

$$\frac{\ln 1.24}{\ln(1+0.03)} \approx 7$$

It also points out that the maximum traffic shift relevant to critical zone is equal to U = 0.397, indicating that in the case of this traffic shift even in the absence of any linear traffic growth, this zone and consequently the whole network would experience failure.

It is observed that Zone 1177 [Heathmont area] has the most stable performance compared to others. This zone can carry significantly more traffic load over time without any traffic congestion, with the maximum feasible linear traffic growth of 1.27 in the case of U=1.191. An important observation for this zone is the linear growth of 1.15 corresponding to the traffic shift of U=3.97, indicating a lifetime of more than 4 years assuming a 3 percent of annual traffic growth.

Interestingly, Zone 1603 reaches to its ultimate capacity where U=3.97 and cannot tolerate any linear traffic growth beyond this point. Zones 601 and 619 reach to their capacity limit at U=1.985 and U=1.588 respectively.

Table 4 demonstrates the congested links in this analysis. Figure 5a shows the resulted traffic volume in the network, for U=3.97. The wider dark band means more v / c ratios in the links. Obtaining such schematic pattern for the network flow helps to identify critical links in the network.

Table 4. Congested Links with Associated TCI Values.

Link No.	TCI	Link No.	TCI
21663	4.596	39927	1.374
22876	2.77	22877	1.374
22884	1.835	9725	1.178
22882	1.835	22870	1.177
22880	1.835	8515	1.177
21620	1.813	22917	1.157
21648	1.809	9175	1.157
9681	1.425	9113	1.157
23298	1.41	9112	1.157
22874	1.397	8738	1.157
22863	1.397	7979	1.156

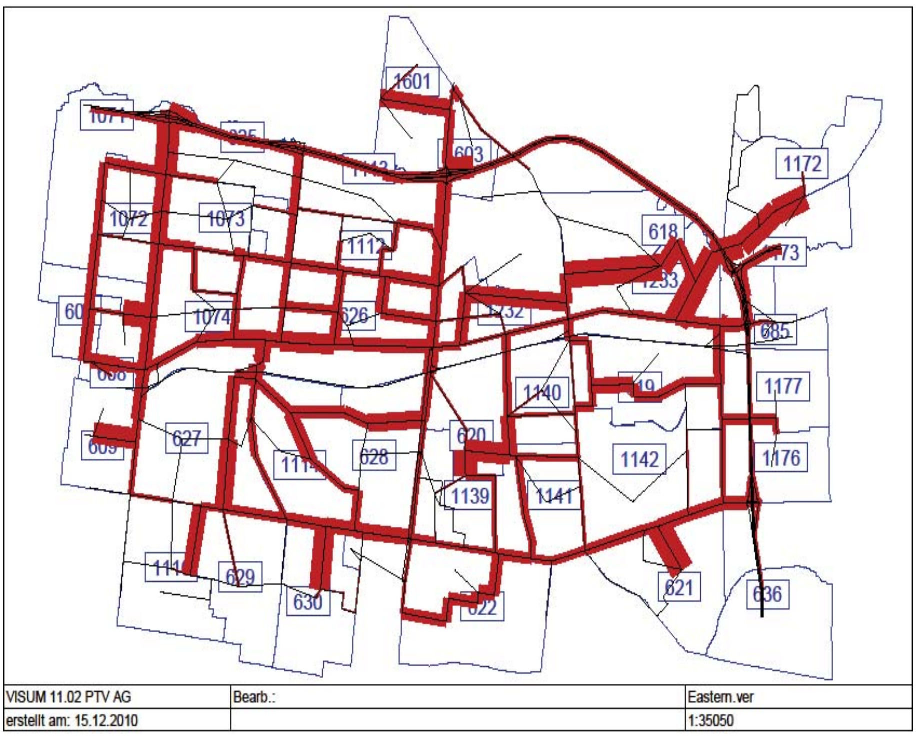

Figure 5a. Status of links in the network.

3.3 Sensitivity Analysis

After discovery of critical links, the capacity of 10 top most congested links (links with the top highest v/c ratio, links numbered 21663, 22876, 22884, 22882, 22880, 21620, 21648, 9681, 23298 and 22874) are increased by 20 percent for sensitivity analysis purposes. Figure 5b shows the lifetime curve after this capacity augmentation strategy.

As can be seen in Figure 5b, by increasing the capacity of 10 most congested links in the network (out of 1080 links in the network) by 20 percent, the congestion point of the most critical zone (Zone 609) shifts forward and reaches 0.794. It means that compared to the previous scenario, this zone and consequently the whole network, can tolerate twice more unexpected traffic growth. This observation underlines the benefit of the proposed methodology from two points of view.

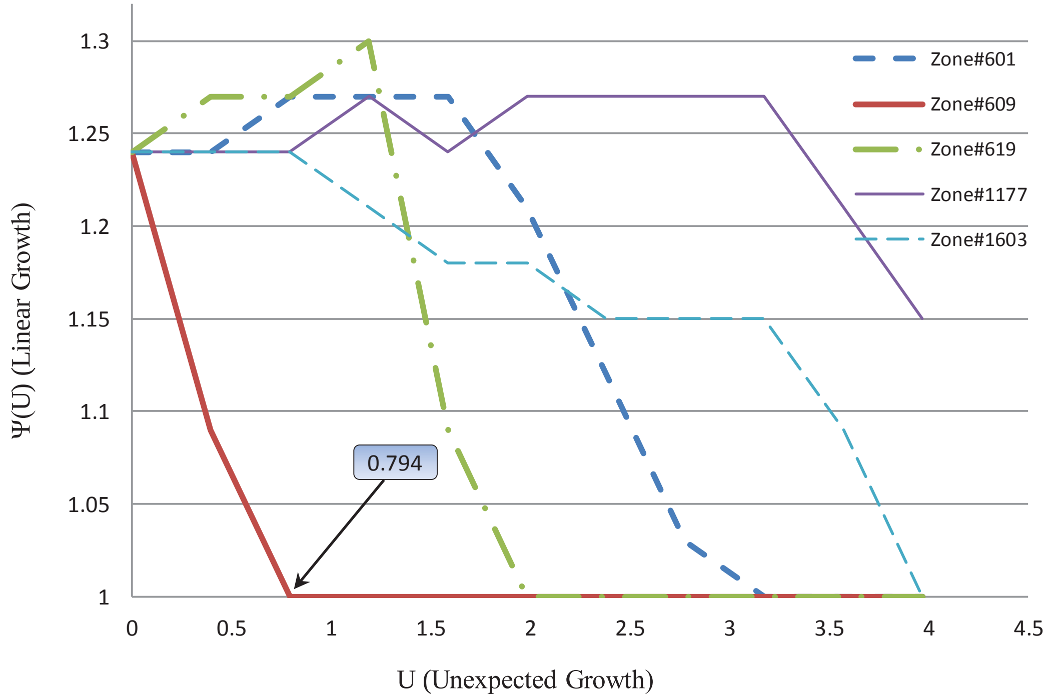

Figure 5b. Individual zone behavior subject to UTG after capacity increment of 10 most critical links by 20%.

Firstly, the lifetime curve is sufficient to the recognition of critical zone(s) and link(s) in the network. Secondly, by adopting a suitable network augmentation strategy the traffic stress in the network can be significantly alleviated leading to a more sustainable network.

An in-depth stress test of the network can be achieved if all of 5 zones are subject to UTG simultaneously. Figure 6 shows the result of such lifetime analysis. It is observed that U_{max} value is reduced from 0.793 to 0.372 in this case, predictable by common sense.

A final test is carried out when any two of 5 zones considered being subject to UTG at the same time. By choosing 2 of 5, ten further scenarios are introduced. The motivation for this test is twofold. Firstly, it would explore the effects of more complicated scenarios, to examine if the methodology is scalable up to a degree. The second reason was to study if the cardinality of multi-zone set has any linear effect on the network life-

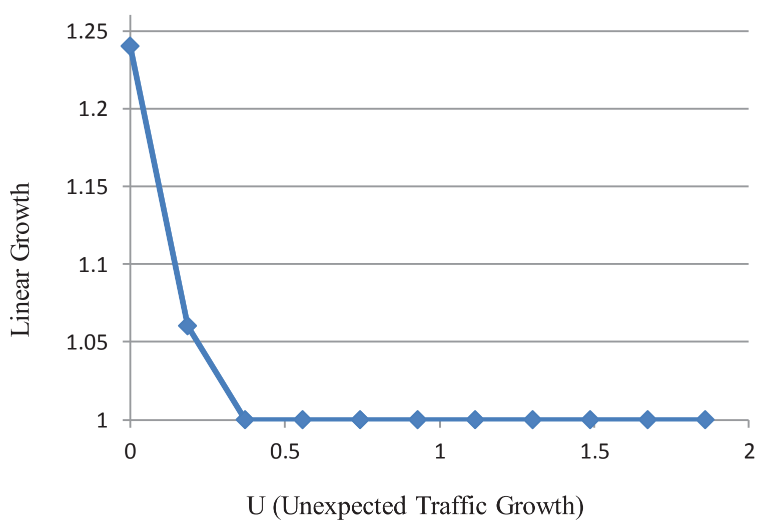

Figure 6. Network behavior when all 5 zones are subject to UTG at the same time.

time. The result of this test is shown in Figure 7. The darker line associated with the zone pair 609, 619 is actually the minimum of all these curves and represents the total network behavior as well (lifetime measure). It is seen that U_{max} value in this case is equal to 0.303. This value is smaller comparing to individual-zone-UTG scenarios (U_{max}=0.793), but interestingly, not larger comparing to all-zones-UTG case (U_{max}=0.372). This indicates that the cardinality of the multi-zone set does not have any linear effect on the network lifetime.

4. Conclusions

In this paper, the implementation of a new methodology, which comprehensively measures the transportation network performance, was presented. The contribution of this paper is to introduce a new methodology which considers the relative OD demands variation (traffic demand shift in an irregular manner). Additionally the methodology can continuously assess the performance of the network.

A real network, which is part of the Melbourne Metropolitan area, has been considered for implementation. The initial implementation considering 5 zones subject to UTG individually identified the weakest elements in the network. In order to increase the lifetime of the network, the capacity augmentation of 10 most congested links in the network (20% capacity increment), have boosted up the network lifetime by 100 percent. Considering that the total number of the links in this example is 1080, this demonstrates how intelligently the proposed lifetime analysis technique could identify the weakest elements in the network and consequently a suitable action plan could be arranged.

In this work, we initially assumed that at any given scenario only one of the 5 zones is subject to UTG. Further cases where 2 out of 5 zones are subject to simultaneous UTG were also assessed, with a further test scenario that all 5 zones are subject to UTG at the same time.

An important conclusion from these implementations was that the network lifetime behavior is not linearly dependent (neither directly nor inversely) on the number of the zones that are subject to UTG at the same time. This is an important finding, signifying the nonlinear behavior of the network in the case of traffic shifts. This also emphasizes the degree of efficiency and suitability of the lifetime measure methodology in investigating transport networks' behavior over time horizon.

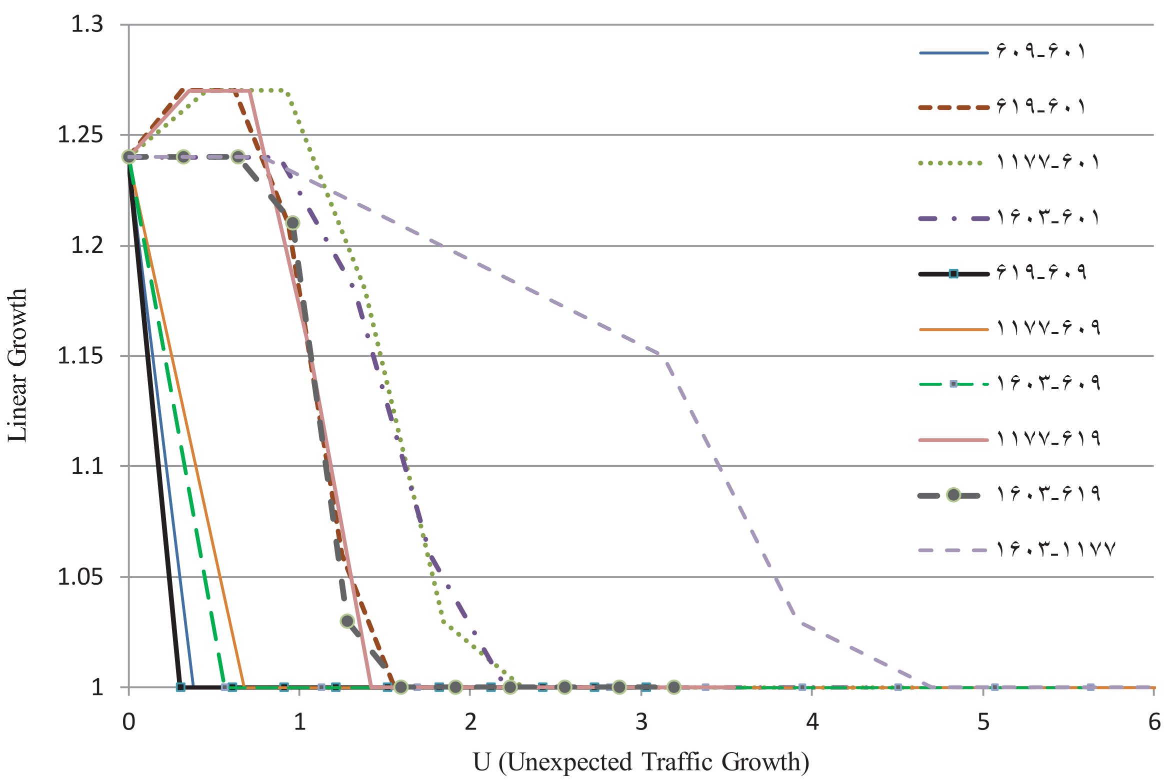

Figure 7. Network behavior when 2 zones out of 5 zones are subject to UTG at the same time.

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