Different Network Performance Measures Using a Multi-Objective Traffic Assignment Problem

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Abstract:

Traffic assignment algorithms are used to determine possible use of paths between origin-destination pairs and predict traffic flow in network links. One of the main deficiencies of ordinary traffic assignment methods is that in most of them one measure (mostly travel time) is usually included in objective function and other effective performance measures in traffic assignment are not considered. The current study is an endeavor to introduce a solution for this problem by applying a multi-objective optimization idea to traffic assignment models. To do this, first, a problem with three objective functions including travel time, total distance traveled, and the rates of cabin monoxide emissions is studied, and then problem with two objectives combining two well-known assignment approaches i.e. user equilibrium and system optimal is introduced. Using the weighting method to solve the multi-objective problem, and comparing the results, show that the analytical relationships resulted from weighting method is applicable to different networks. Furthermore, comparison of both multi-objective problems and single-objective one (travel time only) showed that the results of proposed model is more appropriate in terms of having a plenary view to this issue, and thus more useful.

Keywords: Multi-objective optimization, traffic assignment, Pareto optimal solution, user equilibrium, system optimization.

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1. Introduction

At present, different methods for traffic assignment, from the basic 'all-or-nothing' technique, to more complicated ones such as equilibrium models [Patriksson, 1994], use travel time as the 'impedance' measure while allocating the OD demand matrix (variable or fixed) to the network paths and links [Patriksson, 1994, Sheffi, 1984]. Considering only one measure in simulating travelers' decisions implies ignoring the other effective criteria, and this would be problematic in the precision of transportation demand modeling process. Furthermore, system's decision makers at times try to force travelers to choose a path in a way that satisfies other criteria (such as sustainability objectives) besides travel time.

To solve this problem, performance indicators other than travel time, which are decisive for network users and policy makers, should be considered in traffic assignment. Travel attributes such as distance, costs, safety, delay and the amount of pollutants emitted from the vehicles are some examples of measures that can be effective on travelers' (or system managers', depending on the problem definition) decision making pattern.

Recently, a number of researchers have paid attention to model, analyze, and solve multi-objective problems in traffic assignment. Tzeng and Chen studied the traffic assignment problem with fixed demand by three objective functions. In addition to travel time, they introduced travel distance and Carbon monoxide pollution measures to the assignment problem, and then presented a solution method for the problem using Frank-Wolfe algorithm [Tzeng and Chen, 1993]. Chen et al. have presented an algorithm to solve a bi-criteria traffic equilibrium problem with variable demand and nonlinear path costs. In this study, toll has been considered as the second attribute in addition to travel time [Chen et al. 2010]. Nagurney et al. modeled a fixed-demand, multi-criteria traffic equilibrium problem with an explicit environmental criterion. The network users in this model perceived their generalized cost on a route as a weighting of travel time, travel cost, and the emissions generated [Nagurney et al., 2002]. Chen and Hu focused on finding dynamic user equilibrium between signal setting and route assignment in a bi-level framework. The objective of the upper level is to minimize the total delay, while the objective of the lower level is to find the user equilibrium dynamic traffic assignment state [CHEN and HU, 2011]. Lu et al. developed a bi-criterion dynamic user equilibrium model which considered time-varying toll charges. [Lu et al., 2008]. Wang et al. developed heuristics to find bi-criteria user equilibrium solutions without missing efficient paths. They used travel time and toll cost as the criteria [Wang et al., 2010]. Zhang et al. presented a probit-based bi-criterion dynamic stochastic user equilibrium in which criteria are travel time and toll costs [Zhang et al. 2013].

In the current study, two different multi-objective optimization models are presented. The first model consists of three objectives that raise the most concerned problems in terms of network usage. In this model, in addition to travel time, total distance traveled and amount of carbon monoxide pollutant have been used as three objective functions of an optimization problem. In other words, we want to find an optimal equilibrium flow in network that minimizes these three criteria. Travel time criterion in this model is considered to be user equilibrium, because network users look for their minimum travel time. Government officials concerned with travel distance, because it is directly associated with required infrastructure that constructs the network. Moreover, community wants to reduce the emission rates as much as possible because of enormous damages emissions cause to human health. So, in short, travel time, travel distance, and air pollution are considered as decision-making criteria that reflect the needs for road users, the desire of governments, and the wishes of community, respectively.

The second model, on the other hand, considers both traditional assignment problems in a single one. User equilibrium reflects more realistic behavior on the network and is applied by users, while system optimal finds more appropriate behavior and is desired by decision-makers. This study challenges to find an intermediate situation, in which both above-said criteria may be satisfied. Further, a bi-objective optimization problem including both user equilibrium and system optimal is introduced and analytically solved.

The paper is organized as follows: After introduction, a brief review of multi-objective optimization techniques is presented. Then the multi-objective traffic assignment problem is presented as two different formulations. Then, as a numerical example, the second formulation is numerically analyzed. Finally, conclusions are presented.

2. A Brief Review of Multi-Objective Optimization

In a multi-objective optimization problem, objective functions are more than one and usually it is not possible to combine them and/or replace them with a single objective function. Because due to the (possible) opposite nature of the objectives, optimizing one of them may impose a lot of cost on the others. In multi-objective optimization, instead of single-objective optimization concepts, Pareto-optimization theory or Pareto Front are raised, in which the final solution must be chosen among Pareto-optimal set [Bui and Alam, 2008, Fu and Diwekar, 2004]. Occasionally, Pareto-optimal solution is called Efficient solution, Non-inferior solution, or Non-dominated solution [Obayashi et al., 2004].

Figure 1 schematically shows Pareto-optimal solutions in a bi-objective minimization problem, which minimizes two conflicting objectives f1 and f2. This multi-objective problem has numerous compromised Pareto-optimal solutions such as solutions A, B, and C. These solutions are optimal in the sense that there is no better solution in both objectives. Furthermore, as can be seen in the figure is not easily possible to label any point on the pareto front, (i.e. points A and, B, and C) as the best. Because there is no alternative on this line, which comparing to the others, has the lower values for both objective functions, and each point that has a better status in an objective function with respect to another, is worse in the other objective function with respect to that, so, there are two issues to consider in solving multi-objective optimization problems: finding Pareto-optimal results, and selecting the best result from the Pareto set.

There are many analytical methods for solving multi-objective optimization problems. In some of these methods like goal programming and lexicographic method, decision maker's preferences are considered in the problem and the solutions are obtained in a way that satisfies decision maker's preferences. In some other groups like weighting method and constraint method, the aim is finding a Pareto set exactly or approximately [Fu and Diwekar, 2004]. Some of the most important methods are briefly introduced hereafter in Table 1.

There are a limited number of methods for determining the best result among the Pareto set. One of the most frequent ones, which were presented by Eschenauer et al. is the $_L$p norm. As noted in Table 1, this method minimizes the distance between Pareto front and to an ideal solution, called 'utopia point', according to the following formula [Eschenauer et al., 1990]:

$$Min \quad d = \left(\sum_{i=1}^{m} \left(f_i(x) - f_i^* \right)^p \right)^{\frac{1}{p}} \quad p = 1, 2, \ldots, \infty \qquad (1)$$

where $f_i^* = \left(f_1^*, f_2^*, \ldots, f_m^* \right)$ is the coordinates of the

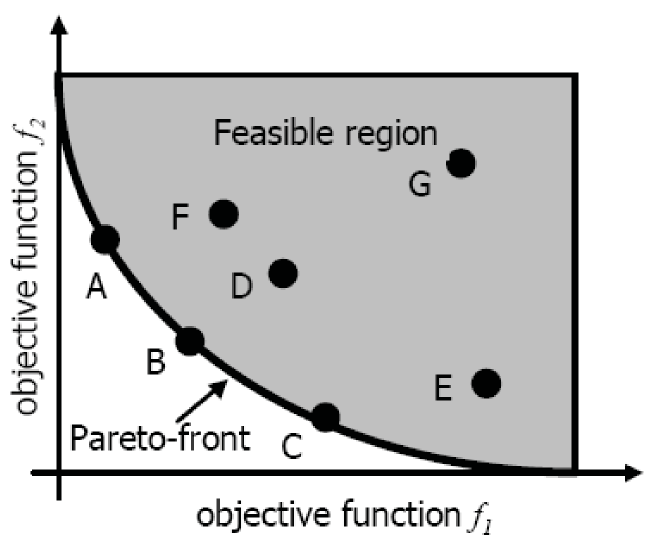


Figure 1. Pareto-optimal solutions in a bi-objective minimization problem [Obayashi et al., 2004]

Table 1. Some of the most important methods for solving multi-objective optimization problems

Methods	Description
Weighting method	By assigning weights to objective functions, multi-objective programming is transformed into a single-objective one. The optimal solutions gained from solving the single-objective problems for the different combinations of weights, would be Pareto optimal solutions of the multi-objective problem [Diwekar, 2008, Obayashi et al., 2004].
Constraint method	one of the objective functions is optimized while the other objective functions are added to the constraints of problem as inequality constraint with a parametric right hand side. After solving adequate number of single-objective problems, Pareto-optimal solutions to the multi-objective problem are obtained [Diwekar, 2008, Fu and Diwekar, 2004].
Lexicographic method	At first, the objective functions are ranked based on their importance (from 1 to k) and then optimization is started with the optimizing the most important function in a way that in optimizing the i-th objective, the optimal amount of (i-1) previous objectives is added to constraints. Optimizing the k-th objective will result in the final solution of the multi-objective problem [Coello *et al.*, 2007, Rao, 1996].
Hierarchical method	This method is very similar to lexicographic method, but while optimizing the i-th objective, the amount of previous (i-1) objectives is kept in a percentage of their optimal values [Bui and Alam, 2008.
Goal programming	The designer sets a series of goals for each objective that he or she wishes to achieve. The goals include objective functions, so all constraints and objective functions are put in the constraints. Optimal solution is the one that minimizes the deviations from these goals given the constraints [Coello *et al.*, 2007, Diwekar, 2008, Rao, 1996].
L_p norms	This method minimizes the distance between Pareto front and to an ideal solution, called 'utopia point' [Eschenauer et al., 1990].

utopia point. In fact, utopia point is the best point that can be achieved theoretically.

In this research, the L_2 norm (p=2) method is used to find the best result from the Pareto optimal set. According to L_2 norm method, the best result is the point form Pareto set which has the least geometric distance from the utopia point. In figure 2 the optimal point based on L_2 norm is shown in a bi-objective minimization problem. It is assumed that in this figure the origin is the utopia point.

3. Modeling Multi-Objective Traffic Assignment Problem

As mentioned before, in order for the travel distance and level of carbon monoxide emissions to be considered besides travel time, the three-objective optimization problem should be defined and solved. A few previous studies were devoted to multi-objective optimization in traffic assignment [Raith, 2009, Tzeng and Chen, 1993]. In the current study, the problem's objective

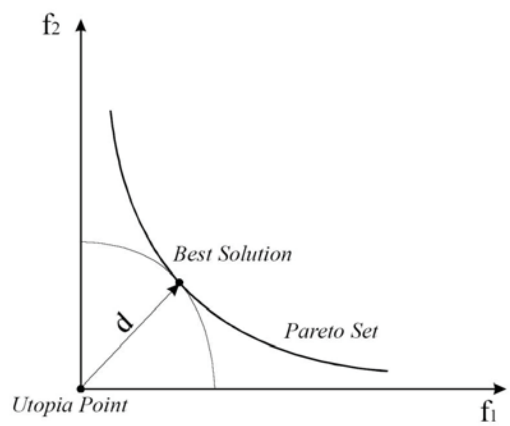


Figure 2. Determining the best solution using L2 norm, used with some alteration from [Kasprzak and Lewis, 2000]

functions are total travel time, total distance traveled, and level of carbon monoxide emissions in the network. The assignment problem in this study is characterized by two different formulations:

3.1 Three-objective Assignment Problem

In this formulation, we want to find an optimal equilibrium flow in network which minimizes three criteria named travel time, travel distance and emission rates. Network users seek minimal travel time, government officials concern with travel distance, and community want to reduce the emission rates as much as they can [Tzeng and Chen, 1993]. Therefore, the basic model consists of three objectives regarding travel time, travel distance, and air pollution. It is assumed that travel time function in three-objective assignment problem, follows Beckman's user-equilibrium formulation. It is also noteworthy that distances are obviously constant while travel times are nonlinear functions of traffic volume in the links. This difference makes the relevant two objective functions very different. Consider for instance a link with a short length whose travel time-volume relationship acts in such a way that it is rapidly congested. Generally, in urban networks that congestion happens frequently, distance and time are not interrelated. Mathematical form of three-objective assignment model in this state is as follows:

$$
\begin{aligned}
&Min \quad f_1 = \sum_{(i,j)} x_{ij} . d_{ij} \\
&Min \quad f_2 = \sum_{(i,j)} \int_0^{x_{ij}} t_{ij}(u) . d_u \\
&Min \quad f_3 = \sum_{(i,j)} x_{ij} . e_{ij}(x_{ij}) \\
&s.t. \\
&1) \sum_p x_p^{od} = x^{od} \qquad \forall (o,d) \in S \\
&2) \; x_{ij} = \sum_{(o,d)\in S} \sum_{p \in P_{ks}} x_p^{od} . \delta_{ij,p}^{od} \qquad \forall (i,j) \in A \\
&3) \; x_p^{od} \geq 0 \qquad \forall (o,d) \in S, p \in P_{od}
\end{aligned}
\tag{P1}
$$

where w_i is the weight of the i^{th} objective function and the followings have to hold.

$$w_1, w_2, w_3 \geq 0 \tag{2}$$

$$w_1 + w_2 + w_3 = 1 \tag{3}$$

and f_1 is the total distance traveled, f_2 is the total travel time, f_3 is the total carbon monoxide emissions in the network, d_{ij} is the length of link (i, j), e_{ij} is the level of carbon monoxide emissions in link (i, j), x_{ij} is the volume of traffic flow in link (i, j), t_{ij} is the travel time in link (i, j), x^{od}_p is the volume of traffic flow on path p from origin o to destination d, x^{od} is the traffic demand from origin o to destination d, A is the set of network links, S is the set of origin-destination, (o,d), pairs, p_{od} is the set of paths connecting origin o to destination d, and $\delta_{ij,p}^{od}$ is defined as follows:

$$
\delta_{ij,p}^{od} = \begin{cases} 1 \;; & if \quad (i,j) \in p \\ 0 \;; & otherwise \end{cases}
\tag{4}
$$

$$\forall (o,d) \in S, (i,j) \in p, p \in P_{od}$$

It should be noted that the weights (wi's) cannot be introduced to the model as variables, because in that case holding optimality conditions to find the optimum point results in the condition that says for nonzero weights, the relevant objective functions should be identical, and generally leads to assign 1 to the function with lower value. Therefore, the problem is solved for particular weights, and the resulting equation, because of the parametric presence of weights, is used to form the Pareto front for the problem.

In model P1, objective functions are nonlinear and constraints are linear. According to Pareto-optimal theory, this model does not have a unique solution, but it has a Pareto-optimal set of solutions. Using the methods of solving multi-objective optimization problems, Pareto-optimal solutions for this model can be found. Due to the nonlinearity of the model, uncertainty about the priority of the objectives, and properties of analytical methods of solving multi-objective optimization problems, weighting method was selected for finding Pareto-optimal solutions.

Using weighting method, three-objective problem P1 will change into the following single-objective problem:

$$
\begin{aligned}
&Min \quad Z = \{w_1 . F_1 + w_2 . F_2 + w_3 . F_3\} \\
&s.t. \\
&1-3) \; of \; P_1
\end{aligned}
\tag{P2}
$$

where F_i is the normalized objective function f_i, which is calculated by the following formula [Kim and de Weck, 2006]:

$$F_i = \frac{f_i - f_{i,utopia}}{f_{i,nadir} - f_{i,utopia}} \tag{5}$$

in which, $f_{i,\ nadir}$ and $f_{i,\ utopia}$ are the maximum and minimum values for the i-th objective function f_i, respectively. Like utopia point but in an opposite way, nadir point is an imaginary point with the maximum value for each objective function [Kim and de Weck, 2006]. Because F_i's are dimensionless, the weights in problem P2 don't take dimension either.

This Problem is a single-objective nonlinear optimization problem. To solve this nonlinear problem Kuhn-Tucker conditions to the Lagrangian transformation of the original problem, are used. The Kuhn-Tucker conditions are as follows [Sheffi, 1984]:

$$L(x,v,q) = Z + \sum_{(o,d)} v^{od} . \left(x^{od} - \sum_{p} x_p^{od} \right) \quad (6)$$

$$x_p^{od} . \frac{\partial}{\partial x_p^{od}} L = 0 \quad \forall (o,d) \in S,\ p \in P_{ks} \quad (7)$$

$$\frac{\partial}{\partial x_p^{od}} L \geq 0 \quad \forall (o,d) \in S,\ p \in P_{ks} \quad (8)$$

$$\frac{\partial}{\partial v^{od}} L = 0 \quad \forall (o,d) \in S \quad (9)$$

$$x_p^{od} \geq 0 \quad \forall (o,d) \in S,\ p \in P_{ks} \quad (10)$$

where v^{od} are dual variables (or Lagrange multipliers) of constraints 1 of P_1 and P_2. Calculating partial derivatives in Eqs. (6-10), and rewriting Kuhn-Tucker conditions leads to the followings:

$$x_p^{od} . \left(w_1 . \tilde{d}_p^{od} + w_2 . t_p^{od} + w_3 . \tilde{e}_p^{od} - v^{od} \right) = 0 \quad (11)$$
$$\forall (o,d) \in S,\ p \in P_{ks}$$

$$w_1 . \tilde{d}_p^{od} + w_2 . t_p^{od} + w_3 . \tilde{e}_p^{od} - v^{od} \geq 0 \quad (12)$$
$$\forall (o,d) \in S,\ p \in P_{ks}$$

$$x^{od} - \sum_p x_p^{od} = 0 \quad \forall (o,d) \in S \quad (13)$$

$$x_p^{od} \geq 0 \quad \forall (o,d) \in S,\ p \in P_{ks} \quad (14)$$

where:

$$\tilde{d}_p^{od} = \sum_{(i,j)} d_{ij} . \delta_{ij,p}^{od} \quad \forall (o,d) \in S,\ p \in P_{ks} \quad (15)$$

$$t_p^{od} = \sum_{(i,j)} t_{ij}(x_{ij}) . \delta_{ij,p}^{od} \quad \forall (o,d) \in S,\ p \in P_{ks} \quad (16)$$

$$\tilde{e}_p^{od} = \sum_{(i,j)} \left(e_{ij}(x_{ij}) + x_{ij} . \frac{de_{ij}}{dx_{ij}} \right) . \delta_{ij,p}^{od} \quad (17)$$
$$\forall (o,d) \in S,\ p \in P_{ks}$$

Using Eqs. (11) and (12) for any separate used ($x_p^{od} > 0$ and $x_r^{od} > 0$) paths p and r from origin o to destination d, the following equation is obtained from travel time, travel distance, and level of carbon monoxide emissions in parallel paths of p and r:

$$w_1 . \tilde{d}_p^{od} + w_2 . t_p^{od} + w_3 . \tilde{e}_p^{od} = w_1 . \tilde{d}_r^{od} + \quad (18)$$
$$w_2 . t_r^{od} + w_3 . \tilde{e}_r^{od} \quad \forall (o,d) \in S$$

Eq. (13) indicates the conservation of traffic flow in network nodes, and Eq.(14) points to the non- negativity of flow in paths. Therefore for solving the single-objective problem P_2, one possible way is to solve the equations system including Eq.(18), and flow conservation constraints in the network nodes, for different combinations of w_1, w_2 and w_3. By solving this system of equations, the optimal solutions of single-objective problem P_2, which constitutes the Pareto-optimal set of three-objective problem P_1, are obtained.

Kuhn-Tucker conditions are necessary for a point to be optimal in a single objective problem. Sufficient conditions for a point to be optimal, is that firstly, three objective functions must be convex, and secondly, the feasible region must be convex. If the two conditions are held, the optimal solution obtained through Kuhn-Tucker conditions (necessary conditions), are the optimal solutions for the problem P_2 and Pareto-optimal solutions for problem P_1.

3.2 An Assignment Problem Considering both User Equilibrium and System Optimal

Two well-known assignments formulation, i.e. user equilibrium and system optimal, which have been derived from Wardrop's principles [Patriksson, 1994], are two extreme points, reflecting different viewpoints towards the problem of loading origin destination demand volumes to the network. User equilibrium states that the generalized costs on all the roads actually used are equal and less than those experienced by a single vehicle on any unused route [Wang et al., 2010]. On the other hand, system optimal can result only from joint decisions by all motorists to act so as to minimize the total system travel time rather than their own. In other words, at the system optimal flow pattern, drivers may be able to decrease their travel time by unilaterally changing routes [Sheffi, 1984]. Such a situation is unlikely to sustain itself and consequently the system optimal flow pattern is not stable and cannot be considered as a model of actual behavior. Briefly, user equilibrium reflects more realistic behavior on the network, while system optimal finds more appropriate behavior. The first one is applied by users, but the latter is desired by decision-makers.

This following shows a challenge to find an intermediate situation, in which both above-said criteria may be, to some extent, satisfied, by utilizing the characteristics of multi-objective optimization. Obviously, the proposed problem has two objective functions, reported in Problem P_3:

$$\begin{aligned} & Min \quad SO = \sum_{(i,j)} x_{ij} \, . t_{ij} \\ & Min \quad UE = \sum_{(i,j)} \int_0^{x_{ij}} t_{ij}(u) \, . d_u \\ & \quad s.t. \\ & \quad 1-3) \; of \; P_1 \end{aligned} \qquad \text{(P3)}$$

Similarly, by using the weighting method two objective functions are transformed into a single, weighted objective function as follows:

$$\begin{aligned} & Min \quad w_1 . \sum_{(i,j)} \int_0^{x_{ij}} t_{ij}(u) \, . d_u + w_2 . \sum_{(i,j)} x_{ij} \, . t_{ij} \\ & \quad s.t. \\ & \quad 1-3) \; of \; P_1 \end{aligned} \qquad \text{(P4)}$$

Because two objective functions in this problem have the same measurement unit, there is no need to normalize the objective functions. Holding necessary conditions to find optimal solution for the problem P4, like the procedure used in previous section, the following equation is found with respect to w1 and w2 and for each pair of loaded paths p and r:

$$w_1 . t_p^{od} + w_2 . \tilde{t}_p^{od} = w_1 . t_r^{od} + w_2 . \tilde{t}_r^{od} \qquad (19)$$

where all variables and parameters have been previously introduced, and:

$$w_1 + w_2 = 1 \qquad (20)$$

Solving Eq.(19) for all paths and conservation of flow constraints, and having assumed particular values for weights will lead to Pareto front of problem P_3, on which the best solution exists. Like previous section, the idea of Utopia point is used to find the best solution. In Section 5, a simple numerical example is presented and the models introduced herein are solved for it.

4. Discussion

It should be noted that in both models analyzed in this study, it was assumed that the proposed strategy could be implemented to the network users by proper incentives and regulations. This assumption is very similar to that, made for system optimal formulation of traffic assignment problem. This study tries to find alternative solutions to the traditional problem of modeling network users' decisions while choosing their routes. Two points are of great importance in this regard: First, network users do not make their decisions based on only a single criterion, which is usually travel time. There may be other criteria such as aesthetic considerations, weather enforcements, safety, etc. that affect road users' choices. Second, assuming that the proposed strategies can be applied to the network users by proper motivations and other leverages, sometimes decision-makers are interested in finding patterns of flow, which, in addition to users' preferences (generally travel time), satisfies other criteria that are perfect for both users and non-users as a whole. This approach helps to find a good situation that to some extent satisfies all standards. Although, the focus of the paper is on the second point, the methodology presented herein is applicable to find models of the first point.

5. Numerical Example

In this section a multi-objective traffic assignment model of both formulations for a hypothetical network is solved using the results of weighting method and the results are compared with a single-objective model. The example network and the relevant functions for travel time, distance, and amount of carbon monoxide pollutant in the links has been presented in Figure 3.

The above network, has been taken from Sheffi [Sheffi, 1984] with some modification to match our case. The network consists of 4 nodes and 5 links, and it has two pairs of origin-destinations. The paths between origin 1 and destination 4 are considered as:

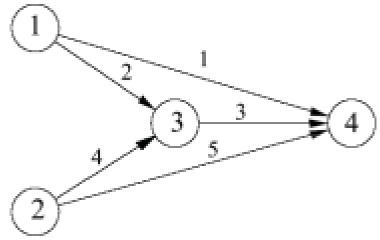
Path 1: consists of link1

Path 2: consists of links 2 and 3

And the paths between origin 2 and destination 4 are defined as:

Path 1: consists of link 5

Path 2: consists of links 3 and 4



$t_1 = 20 + 0.01\,x_1$	$d_1 = 30$	$e_1 = 5 + 2x_1$	
$t_2 = 10 + 0.005\,x_2$	$d_2 = 15$	$e_2 = 2 + x_2$	$x^{14} = 10000$
$t_3 = 6 + 0.005\,x_3$	$d_3 = 20$	$e_3 = 1 + 3x_3$	$x^{24} = 8000$
$t_4 = 12 + 0.005\,x_4$	$d_4 = 15$	$e_4 = 3 + 2x_4$	
$t_5 = 20 + 0.01\,x_5$	$d_5 = 30$	$e_5 = 6 + 3x_5$	

Figure 3. The example network and its performance functions

5.1 Three Objective Function Including User Equilibrium

As mentioned before, in order to find Pareto-optimal solutions of the three-objective traffic assignment model using weighting method, it is necessary to solve a system including Eq.(18) as well as flow conservation equations in network nodes for different amounts of w_1, w_2, and w_3 and then to examine the second-order optimization conditions. The following is the application of this method to the example network.

The solutions of the system of equations for different combinations of weights can be Pareto-optimal solutions if second-order conditions are satisfied. As previously mentioned, in order for second-order conditions to be met in weighting method, objective functions of travel time, traveled distance, and carbon monoxide emission as well as the feasible region must be convex. As Hessian matrices of the objective functions are positive definite, so the functions are convex. In addition, as the constraints for three-objective traffic assignment in the hypothetical network are convex, the feasible region will also be a convex set. Therefore, the second order conditions are met and the solutions of equation systems for different combinations of w_1, w_2, and w_3, are Pareto-optimal solutions of three-objective assignment problem.

Table 2 provides a summary of the solutions of the three-objective assignment model. In this table, w_1, w_2, w_3 are weight coefficients of travel distance, travel time and carbon-monoxide pollutant functions, respectively; x_i is the traffic volume of link i; d_i is the length of link i; p_i is the carbon-monoxide emission in link i of the example network; f_1 is the distance traveled in the network (Thousand PCU-km); f_2 is the total travel time (Thousand PCU-hr); and f_3 is the amount of carbon-monoxide pollutant emission in the network (Thousand PCU-ppm).

Comparing columns for the values of objective functions, shows that none of the 12 points in this table is superior to others, because a point with the least value of one of the objective functions, does not necessarily have the least values of other objective functions. It should be noted that because of the selected approach there is no need to balance the functions. Because of the proposed approach (weighting method), the range for selection of the weights is very broad, and the weights can be chosen such that they reduce or increase the impact of objective functions.

If the L_2 norm method, from Eq. (1), is used to determine the best solution from the Pareto-optimal set, Point 8 in Table 2 is found as the best solution for the three-objective traffic assignment. In fact, this point has the least distance from the utopia point. Coordinates of the utopia and nadir points as well as the best solution for the problem are as follows:

Best Solution (Point 12): $f_i = (f_1, f_2, f_3) = (556578, 935499, 302125563)$

Utopia Point: $f_{i,\,utopia} = (f_{1,\,utopia}, f_{2,\,utopia}, f_{3,\,utopia}) = (540000, 825973, 295268666)$

Nadir Point: $f_{i,\,nadir} = (f_{1,\,nadir}, f_{2,\,nadir}, f_{3,\,nadir}) = (577200,$

Table 2. Summary of some of the results of solving three-objective traffic assignment problem in the hypothetical network using weighting method

point	w_1	w_2	w_3	x_1	x_2	x_3	x_4	x_5	f_1	f_2	f_3
1	0	0	1	7743	2257	4410	2154	5846	562052	890314	295268666
2	1	0	0	10000	0	0	0	8000	540000	1180000	392098000
3	0.2	0.1	0.7	7826	2174	4201	2027	5973	561006	898064	295500769
4	0.3	0.1	0.6	7968	2032	3958	1926	6074	559792	908675	296291800
5	0.4	0.1	0.5	8162	1838	3625	1787	6213	558125	924447	298318263
6	0.5	0.1	0.4	8443	1557	3139	1581	6419	555693	949947	303227820
7	0.4	0.2	0.4	7998	2002	3754	1752	6248	558769	916443	297564807
8	0.5	0.2	0.3	8278	1722	3216	1494	6506	556082	943217	302577125
9	0.6	0.2	0.2	8753	1247	2281	1034	6966	551405	998217	318203138
10	0.2	0.3	0.5	7516	2484	4523	2039	5961	562613	883084	295527584
11	0.4	0.3	0.3	7799	2201	3918	1717	6283	559588	907146	296961520
12	***0.5***	***0.3***	***0.2***	***8072***	***1928***	***3316***	***1387***	***6613***	***556578***	***935499***	***302125563***
13	0.3	0.4	0.3	7413	2587	4504	1917	6083	562520	882223	295904215
14	0.4	0.4	0.2	7554	2446	4134	1687	6313	560669	896168	296694935
15	0.5	0.4	0.1	7806	2194	3447	1253	6747	557236	926650	302119921
16	0.2	0.5	0.3	7093	2907	5003	2096	5904	565013	864718	297605715
17	0.3	0.6	0.1	6808	3192	5203	2011	5989	566014	857559	299878519
18	0	1	0	5547	4453	7440	2987	5013	577200	825973	340748009
Utopia									540000	825973	295268666
Nadir									577200	1180000	392098000

1180000, 392098000)

If in assigning traffic to the above mentioned network, only travel time index (traditional user equilibrium function) is considered, then, the results is as follows:

$$\begin{cases} x_1 = 5547 \\ x_2 = 4453 \\ x_3 = 7440 \\ x_4 = 2987 \\ x_5 = 5013 \end{cases} \longrightarrow \begin{cases} f_1 = 577 \times 10^3 \\ f_2 = 826 \times 10^3 \\ f_3 = 341 \times 10^6 \end{cases} \quad (21)$$

Comparison of values of objective functions in Eq.(21) and Table 2 shows that the values of objective functions f_1 (the distance traveled) and f_3 (the amount of carbon monoxide emissions) in single-objective case are more than all Pareto-optimal solutions for three-objective case. While for objective function f_2 (travel time in the network) the situation is different, since the value of objective function f_2 in single case is less than all Pareto-optimal solutions for three-objective case. Although in single-objective case travel time in the network decreases as much as possible, the distance traveled and the amount of carbon monoxide emissions increases compared to three-objective case. As can be seen, the results for three-objective case are more ideal than the results of traditional case. Therefore, if one wants to decrease the 3 measures simultaneously, one should use the three-objective case for assignment problem.

5.2 Bi-objective Function

Solving problem P4 for the example network with different values of weights, (solution to the system of equations including Eq. (19) as well as constraints of conservation of flow,) results in finding the Pareto front. Table 3 shows the results of this solution for the selection of combination of weights.

Figure 4 depicts the Pareto front for the problem. In this figure, two extreme points are perfect user equilibrium and system optimal. The position of utopia point has been shown in this figure. It is an imaginary point having ideal values for both objective functions. Therefore, the best solution if the L2 norm method, from Eq. (1), is used to determine it from the Pareto-optimal set, would be point 7. This point emphasizes that best solution to the assignment problem is what with 60 percent user equilibrium and 40 percent system optimal.

Figure 4 and Table 3 show that the transition between

Table 3. Summary of the results of solving bi-objective traffic assignment problem in the hypothetical network using weighting method

point	w_1	w_2	x_1	x_2	x_3	x_4	x_5	UE	SO
1	0	1	5640	4360	7320	2960	5040	826080	1315520
2	0.1	0.9	5635	4365	7326	2961	5039	826069	1315521
3	0.2	0.8	5630	4370	7333	2963	5037	826058	1315523
4	0.3	0.7	5624	4376	7341	2965	5035	826046	1315527
5	0.4	0.6	5617	4383	7350	2967	5033	826033	1315533
6	0.5	0.5	5609	4391	7360	2969	5031	826021	1315544
7	***0.6***	***0.4***	***5600***	***4400***	***7371***	***2971***	***5029***	***826008***	***1315559***
8	0.7	0.3	5590	4410	7385	2974	5026	825996	1315582
9	0.8	0.2	5578	4422	7400	2978	5022	825985	1315615
10	0.9	0.1	5564	4436	7418	2982	5018	825977	1315663
11	1	0	5547	4453	7440	2987	5013	825973	1315733
utopia								825973	1315520

user equilibrium and system optimal occurs gradually and smoothly, and rates of the changes to functions are almost the same. The Pareto front of Figure 4 (coordinates) indicates that a “good” solution that has minimum values for both objectives (apart from the solution method) is located somewhere in the middle of the curve and intuitively speaking, the utopia point method has directed us to a proper solution. Such conclusion would be made if we sum up (or average) user and system function values (that due to their measurement units is possible) to reach a unique index.

6. Conclusions

In order to consider network performance measures in traffic assignment, in the current study first, a three-objective optimization problem with travel time, total distance traveled, and amount of carbon-monoxide pollutant emission, then a bi-objective one including both of user equilibrium and system optimal formulations were studied. Obviously, traditional single-objective ways of traffic assignment are a special case of traffic assignment in multi-objective mode. Therefore, it is expectable that multi-objective approaches present more comprehensive and therefore more sustainable results

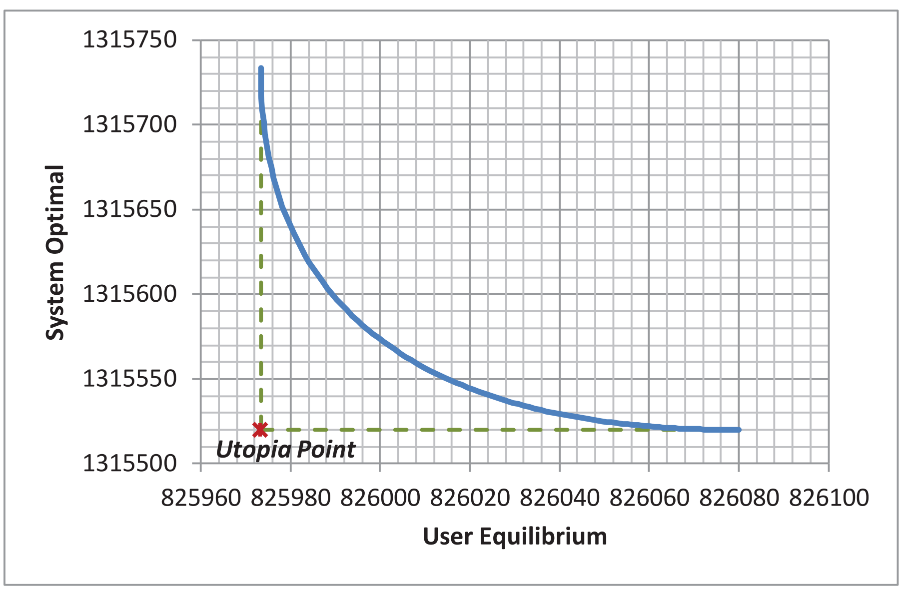


Figure 4. The Pareto front for the bi-objective assignment problem

for the study's problem. Furthermore, considering the nonlinearity of objective functions of the current study, lack of clarity in priority of the objectives, and specifications of methods of solving multi-objective problems, it seems that the best way to find Pareto-optimal solutions in these problems is weighting method. Comparing the values of objective functions in single-objective assignment (travel time function only) and three-objective assignment (travel time, distance traveled, and amount of carbon-monoxide pollutant emission functions) confirmed that the results for three-objective case are more tangible and more practical than the results for single-objective case. Among the Pareto-optimal results, which were obtained for this three-objective assignment problem, the best solution was determined by using the L¬2 norm method. In fact, this solution has the least distance from the utopia point.

The results showed that the values of total traveled distance, and total emissions in traditional user equilibrium problem are more than all Pareto-optimal cases in the three-objective problem. The situation for total travel time is different, since the value of objective function in traditional case is (obviously) less than all Pareto-optimal solutions of the three-objective case. It means that situation of this performance measure in three-objective case is worse than that in the ordinary case, because in a comprehensive managerial viewpoint the whole system has to pay more costs (time herein) in favor of improving more important features of the society (such as environment) and this is the nature of multi-objective thinking.

On the other hand, another multi-objective optimization problem was used in order to consider both traditional assignment models in a single one. User equilibrium reflects more realistic behavior on the network and is applied by users, while system optimal finds more appropriate behavior and is desired by decision-makers. This study challenges to find an intermediate situation, in which both above-said criteria may be, to some extent, satisfied. The results of the analyzed example network showed that the best solution, which is in the shortest distance from the utopia point has the lower values for the combination of both traditional assignment technics.

In the models analyzed in this study, it is assumed that the proposed strategy can be implemented to the network users by proper incentives and regulations. It is obvious that these users do not make their decisions based on only a single criterion, but there may be other criteria such as aesthetic considerations or temperature etc. besides travel times that affect road users' choices. Briefly, in the current study, three performance measures, i.e. travel time, traveled distance and amount of carbon-monoxide pollutant emission were considered. Furthermore, an imaginary situation was studied in which network users chose their routes according to two objective functions that satisfies both users and decision-makers. However, there are many other measures such as fuel consumption, HC and NO_x emissions, delay, and safety, which can be considered in a multi-objective environment. Moreover, the demand for travel between origin-destination pairs in the current study was considered constant. Further research on multi-objective problems with variable demand would be of great importance. The research is on its way to consider the above measures and assumptions in the analysis.

7. References

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