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Research Paper

Application of a Hill-Climbing Algorithm to Public Transportation Routes Design in Grid Networks

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Abstract

Transit Routes Design (TRD) problem deals with optimizing the configuration of transit routes to satisfy a given objective, such as maximizing network coverage, while holding the budget constraint. In its discrete form, TRD is recognized as a computationally interactive problem. A review of the literature reveals that, despite extensive research on this problem, the number of studies on specific urban network configurations has remained limited. Among these studies, many have applied simplifying assumptions such as continuous design variables which may not be applicable to real-world settings. The present study focuses on a discrete version of the TRD problem for an urban "grid" network and aims to maximize the service coverage through the network. To this end, a local search Hill-Climbing (HC) heuristic algorithm is proposed and evaluated. The proposed HC algorithm performs several replications in which it starts with a combination of randomly selected routes and iteratively improves them by moving to the "best neighbour" until it reaches a local optimum solution. Our results for a 6×10 grid network for three budget levels (i.e. low, medium, and high budget levels) indicate that, in much shorter run-times than the exact algorithm, the proposed HC algorithm can produce high-quality solutions with 0.12%, 4.16%, and 2.22% difference from global optimums.

Keywords: Transit Routes Design, Grid Network, Coverage, Hill Climbing

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1. Introduction

The process of sustainable urban development in modern cities is inextricably tied with challenges in transportation planning such as excessive car dependency, air pollution, and energy consumption. To deal with these issues, promoting public transportation (i.e. transit) ridership is considered as an integral component in urban planning. A variety of long-term strategic decisions to short-term tactical decisions must be taken into account in order to enhance a transit system, and thereby, shift the travel demand towards public transportation. The problem of Transit Routes Design (TRD) falls in the most basic levels of the chain of planning decisions in moving towards an efficient transit system.

The TRD problem aims at finding the optimal layout for transit routes in a transportation network. It is considered as a critical step in public transportation planning in a sense that the overall performance of a transit system is highly affected by the design of its routes [Kepaptsoglou and Karlaftis, 2009, Zarrinmehr et al., 2018]. Numerous studies can be found in the literature to study different aspects of this problem, e.g. computational complexity, multi-modality, socioeconomic impacts, etc. The interested reader may see Abedin et al. [2018], Saif et al. [2019], Iliopoulou et al. [2019], or Mauttone et al. [2021] for recent reviews on the TRD problem.

In spite of extensive research on how to design routes layout for a *general* network configuration, *certain* network configurations such as grid and radial networks have been studied to a lesser extent in the literature. Besides, the studies in this scope, to the best of our knowledge, adopt many simplifying assumptions such as uniform/centripetal demand matrices, continuous decision variables, uniform travel-times, or fixed routes' spacing [Miyagawa, 2018]. Such assumptions may contribute to obtaining analytical results and provide general insights into the cause-and-effect relations between design parameters [Fan et al., 2018]. Though, the corresponding results may not be applicable to realistic situations.

In this study we focus on a TRD problem over a grid transportation network. A discrete optimization framework is taken into account for this problem in which, within a budget limit, it is intended to select among a set of candidate transit routes. To solve the problem, a Hill-Climbing (HC) approach is proposed in which an initial random set of candidate routes is iteratively improved by moving to the best neighboring solution. Comparing the results against the exact solutions in a 6×10 grid network suggests that the proposed algorithm can achieve promising results in terms of both performance and quality of its solutions.

In the rest of this paper, a brief research background is presented in Section 2, followed by the problem statement in Section 3. The proposed HC solution algorithm is introduced in Section 4 and the performance and the results are reported in Section 5. Concluding remarks and potential research directions for future research are finally discussed in Section 6.

2. Research Background

Transportation planning in the first half of the twentieth century was predominantly influenced by the ideas which entailed developing highways and facilitating the transport of private vehicles without addressing the improvement of public transit systems. Accordingly, the promoted use of private vehicles reduced the demand for various modes of public transport. In recent decades, it was clearly noticed by urban authorities that use of private vehicles renders the traffic congestion out of control. In this light, public transport was raised as an essential solution for the issue of urban congestion, shifting the emphasis on public transportation in the aftermath of limitations of using private vehicles [Zarrinmehr et al., 2016; Merlin et al., 2021].

Designing transit networks that promote public transit ridership is one of the key steps in moving towards sustainable development of the cities. Transportation design problems, in general, and public transit network design problems, in particular, are complex, non-convex optimization problems, which, in computational terms, belong to the NP-hard complexity class. It is widely recognized that, for such problems, obtaining the exact solution is computationally prohibitive or even impossible [Li et al., 2020, Zarrinmehr and Shafahi, 2016]. Regarding transit network design problems, these problems are riddled with several challenges, including (1) nonlinearity, (2) non-convexity, (3) having multiple objective functions, (4) difficulty of defining decision variables and objective function, and (5) combinatorial explosion of the problem in large-scale, among others [Baaj and Mahmassani, 1991].

The extensive dimensions and intractability of public transport planning problems require them to be classified in several levels and treated specifically [Kepaptsoglou and Karlaftis, 2009]. Table 1 offers a systematic classification of transit planning problems according to the related literature [Guihaire and Hao, 2008]. As can be observed in this table, routes design belongs to the fundamental levels of transit planning and is of great importance to be addressed accurately due to its effects on other levels (i.e. medium and short-term levels) of the planning. Consequently, designing an efficient transit routes network can significantly contribute to improving the transit system and attracting passengers from private vehicles.

Table 1. Classification of transit planning problems [Guihaire and Hao, 2008]

Public Transportation Planning Activities	**Output**
Transit routes network design	Route changes, new routes, and operating strategies
Frequencies setting	Service frequencies
Timetable development	Trip Arrival and departure times
Vehicle scheduling	Bus operation schedules
Crew scheduling	Driver schedules

Regarding the importance of TRD problem and research challenges ahead of this problem, several studies, to date, have acknowledged the importance of TDR problems and addressed the challenges that lay ahead of it from a variety of perspectives [Ceder, 2016]. Various solution algorithms have been presented for this purpose in the literature [Ibarra-Rojas, Felipe Delgado and Juan, 2015]. In a public transport system, parameters such as travel time and comfort directly affect the level of transit service. The transit network quality may be evaluated and enhanced based on several objectives such as route directness, service coverage, operating cost, public transport cost for the users, and the average number of transfers [Guihaire and Hao, 2008, Mauttone et al., 2021]. In an ideal condition, it is best to address all the above objectives simultaneously. Though, the contribution of several objectives in a real-world problem results in an intractable complexity [Laporte et al., 2017]. Therefore, the problem with many objective functions is regularly split into well-defined sub-problems with one or two objective functions.

From a computational perspective, even after adopting simplifying assumptions and considering only a single objective function, the TRD problem poses challenges in various directions. It has been recognized, from early studies in the research background, that the intractable size of the solution space as well as local optimal solutions prevent optimization methods from finding the global optimal solution within a reasonable amount of time. As a result, meta-heuristic methods have been widely proposed to tackle these problems via random search and trial and error in the feasible region. These techniques allow for solving large-scale problems in a short time while adopting simplifying assumptions [Dokeroglu et al., 2019].

A variety of objective functions have been applied to the TRD problem among which the most frequently used objective functions include minimizing total travel time, maximizing the service coverage, and minimizing the number of line transfers between transit routes. The latter is important in a sense that travelers have a tendency to take the longer route with fewer line transfers. Interestingly, it has been found that a single line transfer in a route may reduce the travel demand by up to 58% in the US [Stern, 1996], which offers a variable demand scheme in competition with the private vehicles.

The variability of the travel demand in a multi-modal framework and in competition with the private vehicles has been another research line in the related literature of TRD [Ukkusuri and Patil, 2009, Zarrinmehr et al., 2016, Ranjbari et al., 2020]. It must be noted that in transport networks, demand for public transport is a variable itself that depends on the mutual human–system interactions, geographical and environmental settings, inputs, and the users' behavior. Predictions on land use, production, and attraction of commuters are not certain but based on various assumptions within the framework of scenarios of varying accuracy. Allowing for a variable demand approach or travel demand competing with the private mode of transportation, despite offering a reliable overview, might limit the design outlook in terms of social concerns and general policy-making. Accordingly, as the input to the problem, most studies in the literature have assumed a fixed travel demand matrix and tackled the design problem with a demand coverage approach.

A review of the literature shows that various lines of research have been extended to address different aspects of the TRD problem, e.g. computational intractability, multi-objective optimization, multi-modality, demand variability, etc. Nevertheless, to the best of our knowledge, not much attention has been paid so far to transportation network configurations with certain topologies.

Urban transportation networks have evolved over time into various configurations including amorphous, radial, grid, and hub-and-spoke. A grid-structured network provides a decentralized simplistic system of direct horizontal and vertical routes with several nodes or intersections [Walker, 2020]. Such structure is considered as a flexible and reliable pattern in urban planning theories and has been applied in the design of some modern cities such as Kyoto in Japan, Beijing in China, and many cities in North America [Miyagawa, 2018].

Within the limited research on TRD for grid networks, studies have mostly adopted simplifying assumptions, including uniform/centripetal demand functions and continuous design (spacing, headway, etc) variables. Such assumptions have been the recurring theme in the literature in a sense that they (1) require easier solution approaches, (2) allow for analytical discussions, and (3) provide insight about fundamental relationships between variables [Miyagawa, 2018, Wu, 2014]. Among these studies, Daganzo [2010] suggested a hybrid structure to design transit routes with different schemes for the network center and the peripheral area. The concept of hybrid structure proposed by Deganzo [2010] was further extended by Estrada et al. [2011], Badia et al. [2014] and Nourbakhsh and Ouyang [2012] to allow for rectangular network settings, radial structures and flexible transit systems.

The above-mentioned studies are useful in providing insight in the design and analysis of grid-structured settings. Though, as a result of their simplifying assumptions, they cannot be readily applied to real-word grid networks. The present study deals with the TRD problem as a "discrete" network design problem for grid networks. Also, the travel demand is not considered to be uniform nor centripetal, but can take a general (heterogeneous) form over the O-D pairs. To this end, a HC heuristic algorithm is proposed and investigated in this paper.

3. Problem Statement

Transit routes network design is an infrastructural problem in transportation planning. As usually defined, this problem aims to answer: "How should public transport routes be designed so as to ensure maximum demand coverage while adhering to budget constraints?" The present study relies on the HC algorithm (a local search heuristic technique) to address the problem of designing public transit routes, specifically, in a grid structured configuration. In this section, the grid network configuration is discussed first. Then, the objective function and the assumptions are explained, and a general formulation to describe the problem is presented in the end.

3.1. Grid Network Structure

The grid network is a simple and direct network that enables the distribution of traffic flow among network routes. In theory, this structure offers a flexible network that the multitude of nodes in intersecting routes enables traveling through different routes [Lieberman, 2008]. In grid networks with horizontal or vertical routes, a large share of the users covered by the public transit system can travel without any transfers or with only a single transfer in an L-shaped path (as from A to B in Fig. 1) between their origin-destinations [Walker, 2020]. In this paper, we focus on grid-structured networks and further assume that, due to the policies made by urban authorities, all candidate transit routes are either vertical or horizontal, as depicted in Fig. 1.

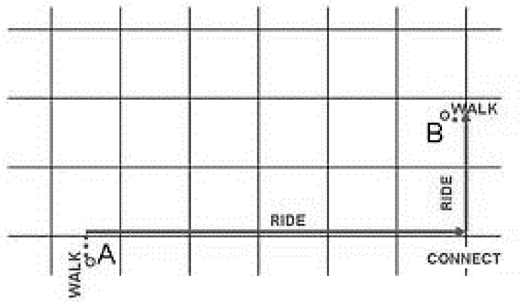


Figure 1. Transit trips with a single transfer in a grid network [Walker, 2020]

3.2. Objective Function

Users' satisfaction is a multi-faceted issue that can be discussed with several concurrent objectives [Rao et al., 2021]. As mentioned in the introduction, various design objectives are presented for this problem. Maximizing demand coverage over the network (considering different definitions of coverage) is one of the common objectives in this regard. In a general definition, measures of coverage account for the number of travelers that can potentially make their trips through the transit service. Section 4.3 will further elaborate on the definition of the coverage adopted in this study.

With respect to the number of transfers, costs, or demand coverage, it must be noted that focusing merely on one of these parameters can undermine the design quality, resulting in the loss of passenger satisfaction and an inefficient transit system [Stern, 1996]. Accordingly, it is often attempted to include the mentioned issues indirectly in the concept of travel coverage.

3.3. Problem Assumptions

We assume that:

- The travel demand matrix is not variable during the study;
- The underlying network is a grid network;
- The candidate transit routes, taken into account by urban authorities, are eiter horizontal or vertical routes;
- Each line-to-line transfer is modeled by artificially adding an extra 5-minute travel-time to the overall travel time, as a penalty standing for users' disutility.
- Reduction coefficients are considered for each transfer in calculating the total covered public demand. For zero, one or two transfers (i.e. transit trips consisting of one, two or three routes for the travel), these coefficients are considered as 1.0, 0.7 and 0.5, respectively. That is, for example, if a transit trip for 100 passengers entails one transfer, 0.7×100=70 passengers are considered to be covered in the transit network.
- For transit trips consisting of three or more transfers, the coverage reduction coefficient is assumed to be 0, implying that such trips are not covered by the transit system.

3.4. Mathematical Formulation

For a general description of the problem in this paper, let us define:

R: Set of K candidate transit routes to be constructed in the network;

b_k: Binary variable indicating if the kth transit route is selected to be constructed ($1 < k < K$);

B: Decision variables vector with the length K;

$R_{\mathbf{B}}$: The subset of transit routes selected based on the decision vector B.

l_k: The length of the kth transit route ($1 < k < K$).

L: The maximum length of new transit routes that can be constructed, given the budget constraints;

$N_{\mathbf{B}}$: The network obtained from the construction of transit routes $R_{\mathbf{B}}$, and

Cov ($N_{\mathbf{B}}$): Transit coverage associated with the network $N_{\mathbf{B}}$.

Based on the above definitions, the problem can be formulated in a general framework as follows:

$$\max_{\mathbf{B}} \quad \mathrm{Cov}(N_{\mathbf{B}}) \tag{1}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} b_k l_k \leq \mathrm{L} \tag{2}$$

In the above formulation, the problem's objective function in (1) seeks to find the decision vector B in such a way that it ensures maximum transit coverage. Meanwhile, constraint (2) limits the maximum transit route length to L.

With slight adjustments, this problem can be mathematically reduced to the knapsack problem and can be classified as an NP-Hard problem, a large-scale solution of which is not possible.

4. HC Algorithm

The HC algorithm, which is used in this study to solve the grid network routes design problem, is categorized within the class of local search heuristics [Lim et al., 2006]. The algorithm starts with a random initial solution, iteratively moves to the best "neighboring" solution, until it reaches a solution where no improvement is possible. As the progression resembles climbing a hill until reaching the brow, the algorithm is named "hill-climbing".

With that mentioned, a closer look must be taken at concepts such as the random initial solution and neighborhood, as used in the context of our network design problem, before the HC algorithm can be described.

4.1. Random Initial Solution

In the present study, the random initial solution was obtained by considering a solution where all candidate transit routes are constructed. Obviously, with budget constraints, this solution will be infeasible. To resolve this issue, in this solution, a candidate transit route is randomly selected in each iteration and removed from the set of transit routes. This process continues until we reach a solution that satisfies the budget constraints, in other words, a feasible solution.

4.2. Neighborhood

It is obvious that, overall, the definition of "neighborhood" in the network would affect the quality and efficiency of the search method in the feasible region. Accordingly, based on its definition, the solution's neighborhood wields considerable influence over the algorithm performance and the solution quality.

We already know that, in a grid network, the solutions comprise horizontal or vertical transit routes. Each horizontal or vertical route has two candidate transit routes (or at least one candidate transit route) within a one-block distance that may not have been selected in the current solution. For example, Fig. 2 depicts a vertical transit route and two neighboring ones on a grid-structured network.

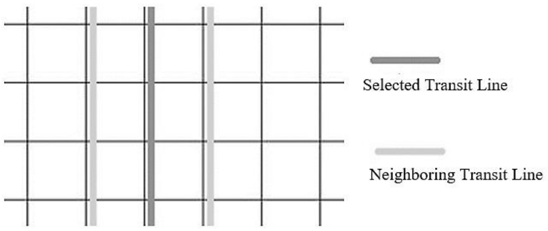


Figure 2. A selected transit route and two neighboring candidates on a grid network

To define a neighbor solution in this study, let us assume that there are n candidate transit routes which have been selected in the current solution. Then, a neighboring solution for the current solution is defined as a solution in which (n -1) transit routes are the same and only 1 transit route is changed to a neighboring route. Adopting such a definition allows us to narrow down the exponential search space of neighboring solutions to a polynomial search space (containing $2 \times n$ solutions at most).

4.3. Coverage

As mentioned earlier, in this study, we attempt to find a set of lines in this network, where the maximum travel demand coverage is achieved while holding the budget constraint. In a general definition, "coverage" shows the number of passengers that can potentially use transit routes when traveling between OD pairs. Previous studies have shown the passengers' tendency to opt the longer routes with fewer line transfers [Lieberman, 2008]. The point to be considered here is the increased travel disutility due to a line-to-line transfer and the passengers' reduced tendency to take such transit routes. To address such disutility for each line transfer, a 5-minute penalty is added to the travel time of an OD pair on transit routes (e.g. 10-minutes for two transfers). The resulting shortest path (after adding the penalties) is considered as the "revised" shortest path.

Fig. 3 illustrates the stepwise process of calculating the coverage for R_B, i.e. the subset of selected transit routes. According to this figure, the amount of coverage, denoted by $Cov(R_B)$, is initially set to zero. Then, each O-D pair (r,s) is iteratively selected and the corresponding *revised shortest path*, namely $p^{rev}(r,s)$, is calculated. The amount of coverage is increased by 100%, 70%, or 50% of the travel demand from r to s, namely d^{rs}, for the number of 0, 1, or 2 transfers in $p^{rev}(r,s)$, respectively. In other words, 100%, 70%, and 50% of the travelers are considered to be covered by the routes, for 0, 1, and 2 transfers, respectively.

$Cov(N_B) \leftarrow 0$;

For each O-D pair like (r,s)

$p^{rev}(r,s) \leftarrow$ the revised shortest path from r to s ;

$n^{tr} \leftarrow$ the number of transfers in $p^{rev}(r,s)$;

if $n^{tr} = 0$

$Cov(N_B) \leftarrow Cov(N_B) + d^{rs}$;

else if $n^{tr} = 1$

$Cov(N_B) \leftarrow Cov(N_B) + 0.7\, d^{rs}$;

else if $n^{tr} = 2$

$Cov(N_B) \leftarrow Cov(N_B) + 0.5\, d^{rs}$;

Figure 3. The process of calculating the coverage for a network with R_B as the set of transit routes

4.4. The HC Algorithm Used in This Study

Based on the points mentioned so far, Fig. 4 depicts a schematic of the HC algorithm adopted in this study.

According to Fig. 4, after reading the required input, the algorithm starts with building up a random initial solution in the second step (as described in section 4.1). This initial solution is then adopted as the current solution. In the third and fourth steps, in each iteration, the set of neighbors is formed for the current solution and the coverage is calculated for each of the neighbors to find the best neighbor (i.e. the neighbor with maximum coverage). Then, if the best neighbor's coverage exceeds that of the current solution, the current solution is updated and a new iteration is initiated. Otherwise, the current solution will be reported as the final solution of the HC algorithm.

Step 1: Read the problem information (network, demand matrix, available budget, etc.) from the input.

Step 2: While holding the budget constraint, generate a random initial solution as "current solution".

Step 3: For all neighbors of the current solution, evaluate the coverage and denote the neighbor with maximum coverage as the best neighbor.

Step 4: Is the coverage for the best neighbor more than that of the current solution? If yes, set the best solution as the current solution and go back to step 3. Otherwise, print the current solution as the best solution of the algorithm in the output.

Figure 4. The HC algorithm adopted in this study

It must be noted that Steps 2, 3, and 4 make up one iteration of the HC algorithm. The algorithm may stop at the end of the process on a local (but not necessarily the global) optimum. The algorithm is often executed with several replications with different arbitrary initial solutions and the best result of the replications is reported as the final solution of the HC algorithm. This strategy (using several replications) will help the algorithm to find solutions with higher qualities and reduce the chance of getting trapped in local optimums.

In this study, we use 20 replications of the HC (Fig. 3) to report the best solution. Denoting these 20 replications as a single "run" of the algorithm, and regarding the randomness inherent in each replication, it is obvious that the algorithm can end up with different solutions for different runs. To account for this randomness, our results are averaged over 30 independent runs and reported in terms of mean and standard deviation values.

4.5. Exact Solution through Enumeration of Dominant Solutions

Evaluating the efficiency and quality of the solutions obtained from the proposed HC algorithm requires a comparison of the results against exact (global optimum) solutions. For this purpose, the present study relies on an enumeration algorithm with emphasis on the dominant solutions of the problem, namely Enumeration of Dominant Solution (EDS) [Zarrinmehr and Shafahi, 2014]. Regarding the exhaustive search, enumerating all possible solutions and investigating the coverage for each solution would result in an exponential search spaces in the order of 2^K (K, here, denotes the number of transit routes).

However, in some combinatorial problems as in this paper, instead of searching the entire feasible search space, we can limit the search within particular solutions, referred to as the "dominant" solutions. Although limiting the search space may not significantly reduce the complexity order of the search space, in small-to-medium problems, it would lead to saving the time and resources.

In the case of the present study, a dominant solution refers to a subset of lines to which, while holding the budget constraint, a new line cannot be added. It is obvious that, based on the definition of travel coverage, the optimal solution in our problem can be found among these dominant solutions in a sense that adding new transit routes will never decrease the amount of network coverage.

A simple recursive algorithm was used to enumerate dominant solutions, in which, after finding each possible set of transit routes, the solution is evaluated if it is dominant. In the end, the best dominant solution (the one with the highest demand coverage) is reported as the exact (global optimum) solution of the problem [Zarrinmehr and Shafahi, 2014].

5. Results of the Algorithm

The proposed algorithm was executed on an illustrative example of the grid network shown in Fig. 5 [Khanzad et al., 2017]. This network comes with *60* nodes, in a 6×10 configuration, and the horizontal and vertical sides of the blocks measured *2.2* and *2 km*. The network comprises *16* proposed transit routes, among which routes 1–10 and 11–16 are vertical and horizontal, respectively.

Further, the entire travel demand throughout the network equals to 333,018 trips per hour. All these details may be found by the OD pairs in Khanzad el al. [2017] in the appendix.

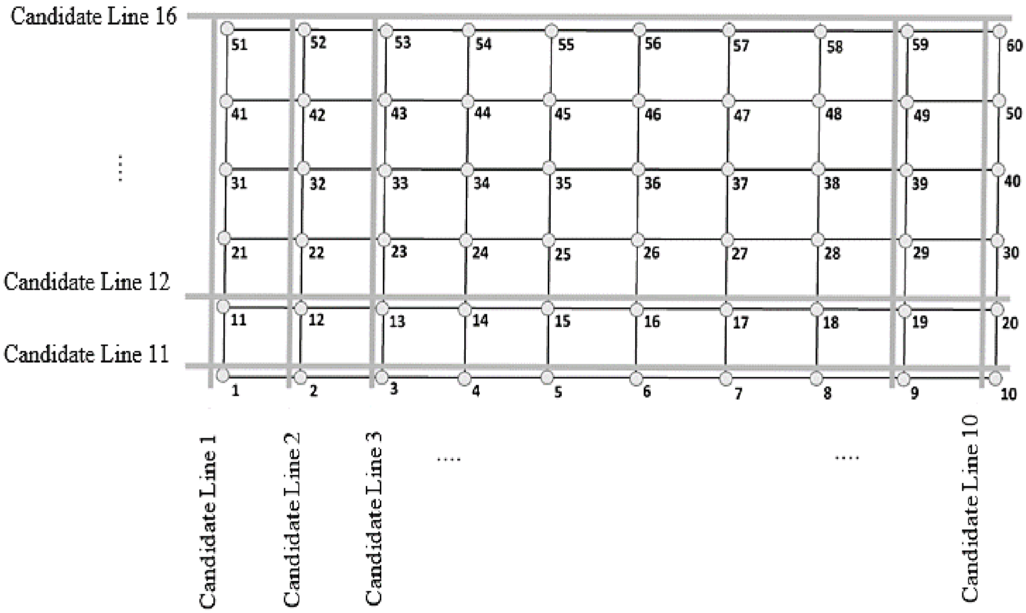


Figure 5. The 6×10 grid network used in this study [Khanzad et al., 2017]

Table 2. Algorithm performance in a single run on the 6×10 grid network

Current Solution			Neighboring Solution(s)			Best Neighbor		
Transit Line Combination	Required Budget	Coverage (trips (per hour	Transit Line Combination	Required Budget	Coverage (trips per hour)	Transit Line Combination	Required Budget	Coverage trips per) (hour
[3,9,10,15]	49.8	40693.8	[2,9,10,15]	49.8	42810	[3,9,10,14]	49.8	43099.8
			[4,9,10,15]	49.8	39933.6			
			[3,8,10,15]	49.8	40734.6			
			[3,9,11,15]	59.6	Infeasible			
			[3,9,10,14]	49.8	43099.8	Is the coverage for the best neighbor more than the coverage of the current solution?		
			[3,9,10,16]	49.8	42395.4	Yes		
[3,9,10,14]	49.8	43099.8	[3,9,10,13]	49.8	41158.8	[2,9,10,14]	49.8	43331.4
			[3,9,10,15]	49.8	Already met			
			[2,9,10,14]	49.8	43331.4			
			[4,9,10,14]	49.8	41200.8			
			[3,8,10,14]	49.8	42805.8	Is the coverage for the best neighbor more than the coverage of the current solution?		
			[3,9,11,14]	59.6	Infeasible	Yes		
[2,9,10,14]	49.8	43331.4	[1,9,10,14]	49.8	41618.4	[2,8,10,14]	49.8	42725.4
			[3,9,10,14]	49.8	Already met			
			[2,9,10,13]	49.8	40902			
			[2,9,10,15]	49.8	42810			
			[2,8,10,14]	49.8	42725.4	Is the coverage for the best neighbor more than the coverage of the current solution?		
			[2,9,11,14]	59.59	Infeasible	No		

5.1. Performance of the HC Algorithm in a Single Replication

As an illustrative example, Table 2 shows the performance of the proposed HC algorithm in a single short replication on the grid network of Fig. 5, assuming a *50 km* construction budget as the constraint. As evident, the program starts with an arbitrary initial solution that includes Lines 3, 9, 10, and 15, with a *49.8 km* budget constraint (which does not exceed *50 km*) and coverage of 40,693.8 trips per hour. The solution's neighbors are then evaluated, leading to the set of Lines 3, 9, 10, and 14 with coverage of 43,099.8. Since the coverage of the best neighbor is superior to that of the current one, the program replaces the current solution with the best neighbor. Then, the next iteration begins with the new solution, i.e. [3, 9, 10, 14]. This process continues until the program stops at the current solution [2, 9, 10, 14] with a 49.8 budget level and the coverage of 43,331.4. Termination of the algorithm, as discussed in Section 4.3, is due to the point that, based on the neighbors, no further improvements are available, implying that the algorithm has reached a local optimal solution.

5.2. Algorithm Results

The proposed HC algorithm was implemented in Python programming language and its performance was evaluated by comparing the results against exact solutions extracted from the enumeration algorithm. The results are reported for *safe mode* runs on a Windows 10 Core i7 1.80–1.99 GHz laptop with 8 GB RAM. Table 3 summarizes these results for three low (*50 km*), medium (*100 km*), and high (*150 km*) budget levels.

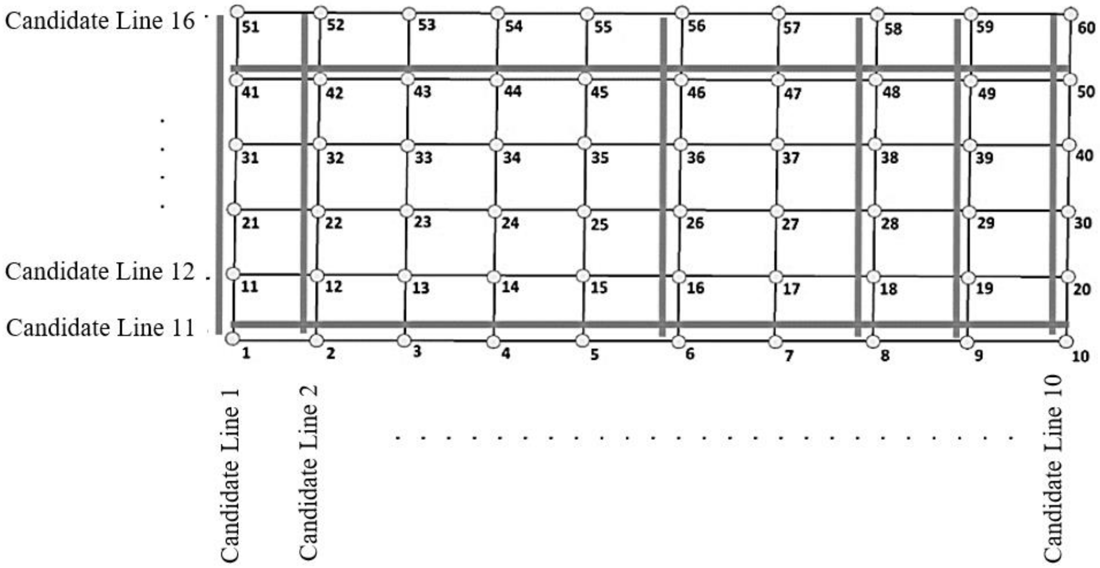
Comparing the results of the HC algorithm with the exact solution from the EDS solutions provides insight in evaluating the performance of the both algorithms at different budget levels. Given the stochastic nature of the HC algorithm (due to its random initial solution), the reported results of the algorithm are average values over 30 separate runs (each having 20 replications as mentioned in 3.4). Therefore, the coverage values for the HC algorithm are reported in this table in terms of the average and standard deviation over 30 independent runs. The reported solution of transit route combinations obtained from the HC algorithm is also the 15th out of the 30 solutions (sorted by coverage values).

It is evident from Table 3 that the HC algorithm can rapidly approach the exact solution at all three budget levels. On average, the quality differences between the algorithm solutions and the exact solutions at *50*, *100*, and *150 km* budget levels are 0.12, 4.16, and 2.22%, respectively. Meanwhile, the runtime of the HC algorithm is considerably shorter than enumerating dominant solutions. The runtime difference is particularly notable at the medium budget level i.e. *100 km* (4 hours and 10 minutes), where the combinatorial problem has larger dimensions.

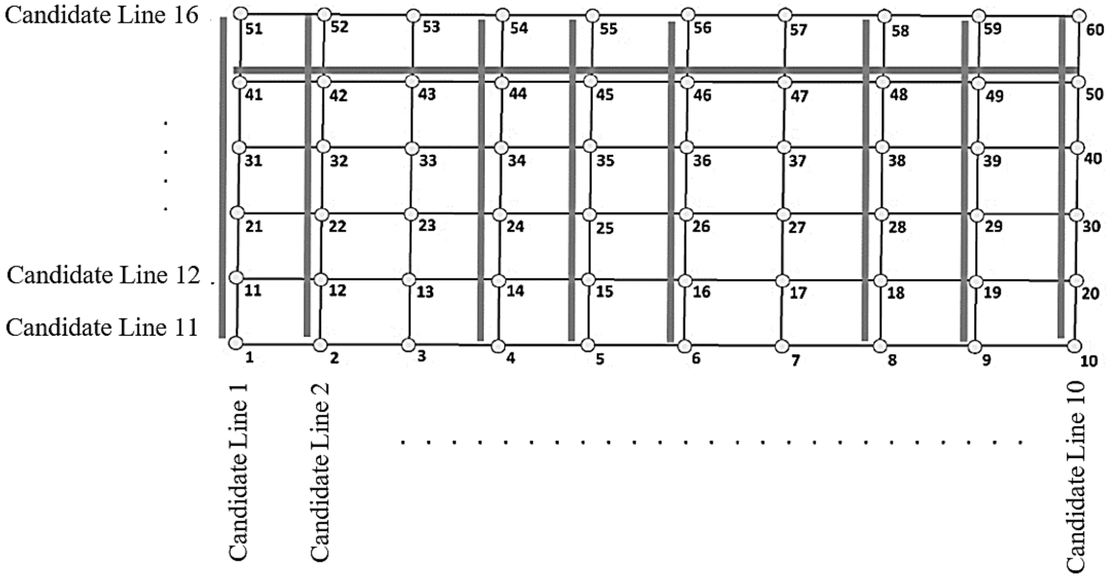
Table 3. The results for the performance of the HC and EDS algorithms over three budget levels

Budget Limit (km)	Algorithm	Transit routs configuration	Required Budget (km)	Coverage		Runtime (hr:min:sec)
				(Trip per hour)	(Percentage)	
50	HC	[5, 6, 9, 11]	49.8	44149 (± 397)	13.26% (± 0.12)	0:00:43
	EDS	[5, 6, 9, 11]	49.8	44546	13.38%	0:03:05
100	HC	[1, 2, 6, 8, 9, 10, 11, 15]	99.7	131804 (± 7494)	39.58% (± 2.25)	0:16:41
	EDS	[1, 2, 4, 5, 6, 8, 9, 10, 15]	99.8	145655	43.74%	4:27:25
150	HC	[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11,14]	145.4	216167 (± 5015)	64.91% (± 1.51)	0:34:30
	EDS	[2, 5, 10, 11, 12, 13, 14, 15, 16]	148.8	223566	67.13%	3:39:39

Finally, Fig. 6 shows the route combinations of the HC algorithm (15th solution out of 30 solutions) besides the exact solution for the *100 km* budget level. Both solutions suggest constructing Transit Routes 1, 2, 6, 8, 9, and 10 (vertical routes), and Route 15 (horizontal route), implying the remarkable role of these routes in improving the network coverage.



(a) HC algorithm



(b) EDS algorithm

Figure 6. Transit routes combination obtained for the 100 km budget limit

6. Concluding Remarks

In this paper we studied a discrete network design version of the TRD problem for a grid-structured network. A coverage measure was defined to evaluate the number of travelers that could potentially navigate through the transit routes. The objective was to find the best routes configuration that maximizes the network's travel demand coverage while holding the budget constraint. It was discussed that the complexity of this problem makes exact solution methods unable to find the solution in large scale. As a result, a local search heuristic algorithm, namely the HC algorithm, was proposed in this paper.

The HC algorithm was designed to start from an arbitrary initial solution and move toward the best neighbor until reaching a local optimum. The concept of the neighborhood was built upon changing a single route of the current solution in each iteration. The best local optimum solution after 20 replications was considered as the best solution of the algorithm. The proposed HC algorithm was finally implemented in Python and run over a 60-node grid network where the results were examined against the global optimum solutions obtained by an EDS exact algorithm.

Regarding the inherent randomness of the proposed HC algorithm, 30 independent runs of the algorithm (each including 20 replications) were performed to report the results. The results, averaged over these 30 runs, suggest that, in a notably shorter run-time than the EDS algorithm, the proposed algorithm can hit solutions close to the global optimums of the problem. For three budget limits of 50km, 100km, and 150km (i.e. low, medium, and high levels of budget) the differences between the coverage obtained by the proposed algorithm and the EDS algorithm, on average, were 0.12%, 4.16%, and 2.22%, respectively.

To extend the findings of this study, the following research directions may be considered in future research:

- The illustrative example used in this paper was an artificial 6×10 network. However, the algorithm can be applied to realistic grid network settings to investigate the results and the performance.
- Grid structures may be applied in urban areas as sub-networks rather than fully-grid networks. It will be interesting to extend the proposed algorithm to allow for such settings. To achieve this, application of analytical models in the literature may be insightful.
- To achieve more realistic models, not only the discrete decision variables or heterogeneous demand matrix, but many other practical features of the problem need to be incorporated like, for example, multi-objective functions or multi-modality [Zarrinmehr et al., 2019]. Future research may enhance the current model to allow for such features and more practical results.

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8. References

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