

# Application of Artificial Neural Networks for Analysis of Flexible Pavements under Static Loading of Standard Axle

Ali Reza Ghanizadeh<sup>1</sup>    Mohammad Reza Ahadi<sup>2</sup>  
*Received: 2015.04.22*                      *Accepted: 2016.01.20*

## Abstract

In this study, an artificial neural network was developed in order to analyze flexible pavement structure and determine its critical responses under the influence of standard axle loading. In doing so, more than 10000 four-layered flexible pavement sections composed of asphalt concrete layer, base layer, subbase layer, and subgrade soil were analyzed under the impact of standard axle loading. Pavement sections were analyzed by means of multilayered elastic analysis theory and critical responses of pavement including maximum horizontal principal tensile strain at the bottom of asphalt layer and maximum vertical compressive strain on the top of subgrade were computed in each case. Then, a Feed-Forward back propagation neural network was served to predict these responses. The results show that the artificial neural network can be used as a powerful and accurate tool to predict the critical response of flexible pavements. Application of artificial neural networks for pavement analysis reduces the analysis time and can be used as a quick tool for predicting fatigue and rutting lives of different pavement sections and so in optimum design of pavement structure.

**Keywords:** Pavement analysis, artificial neural network, critical responses, standard axle

---

Corresponding author E-mail: ghanizadeh@sirjantech.ac.ir

1. Assistant Professor, Department of Civil Engineering, Sirjan University of Technology, Sirjan, Iran

2. Assistant Professor, Transportation Research Institute, Tehran, Iran

### 1. Introduction

The first step in designing a pavement through the mechanistic-empirical method is the analysis of pavement and calculation of critical responses of the pavement under the influence of different loads. To date, different methods have been proposed for the analysis of flexible pavements. Boussinesq [1885] was the first person who obtained the responses of a semi-infinite system under the influence of point loading [Boussinesq, 1885]. Equations proposed by Boussinesq were developed for loads with uniform distribution in later years by other investigators [Newmark 1947; Sanborn and Yoder 1967]. Equivalent thickness method was presented by Odemark (1949). Here, it is assumed that the deflection in a multi-layered pavement is equal to that in a semi-infinite system with the equivalent thickness of  $H$  and the modulus of  $E$ . After converting the multi-layered system into a semi-infinite system, different responses including stresses, strains and deflections can be obtained using Boussinesq's equations [Odemark, 1949]. For the first time, Burmister presented stress and strain equations for two and three-layered systems under the impact of circular loading [Burmister, 1943 and 1945]. Schiffman provided a general solution for the analysis of stresses and strains in a multi-layered elastic system [Schiffman, 1962]. This method is known as the multi-layered elastic theory. Currently, most flexible pavement analysis programs use multi-layered elastic theory for analyzing pavement structure. For example, we can refer to the programs of BISAR [De Jong and Peutz, 1979], JULEA [Uzan 1994], LEAF [Hayhoe 2002], KENLAYER [Huang 2004], Mnlayer [Khazanovich and Wang, 2007], and NonPAS [Fakhri and Ghanizadeh, 2012]. Modeling of pavement structure using multi-layered elastic theory is simpler and requires less time for system solution by computer compared to finite element method. As well, for non-professional users, working with programs based on multilayered elastic theory is simpler than working with finite element programs [Huang, 2004].

Duncan et al. (1986) used finite element method for the analysis of flexible pavements for the first time. MICH-PAVE and ILLI-PAVE are the most common programs that use finite element method for the analysis of flexible pavements [Harichandran et al., 1990; Raad and Figueroa,

1980]. General finite element programs such as ANSYS and ABAQUS have also been used successfully for the analysis of pavement structure [Al-Hadidy and Tan 2009; Yang and Liu 2010; Huang et al. 2010; Ahmed et al. 2013].

In using finite element method, the selection of the correct form of elements has a major impact on the desired accuracy. Finite element method in the modeling of those systems with specific dimensions enjoys more capability because the layered method has been proposed with the assumption of the infinity of layers in the radial direction. Finite element method for the nonlinear analysis of pavement is advantageous to the programs based on layered system theory [Huang 2004]. However, in practical applications, it might not be possible to use finite element method due to the increase of analysis time; therefore, analysis through multilayered elastic theory may be preferred to finite element analysis method.

In order to design the pavement under the influence of standard axle load using a mechanistic – empirical method, we need to analyze the pavement structure under the influence of this loading and to determine maximum horizontal principal tensile strain at the bottom of asphalt layer and also maximum vertical compressive strain on the top of subgrade soil. For this purpose, it is necessary to determine pavement responses at 10 different points and then predict fatigue life and rutting according to critical strain values.

Artificial neural network (ANN) is a sort of mathematical tools, which establishes a mapping between a set of input numbers and output numbers. Generally, artificial neural network method has been used to analyze rigid pavements [Khazanovich et al, 2001; Ceylan 2002; Abu-Lebdeh and Ahmed, 2013]. Amery and Molayem (2006) used artificial neural network method to predict critical responses of flexible pavements under the influence of single wheel load. In their study, the neural network was trained using a database of 320 records and the neural network was usable only in a certain range of pavement sections due to the limited range of thickness and the modulus of pavement layers in developed synthetic database [Amery and Molayem 2006]. If we can determine the critical responses of pavement using neural network under the influence of standard axle loading, we will have the speed of pavement analysis several times faster than that of

analysis using software based on multi-layered elastic theory. This ratio is much higher compared to the ratio in the software based on finite element method; therefore, it is possible to analyze a higher number of pavements in a shorter period of time in order to select the optimal pavement structure.

In the present study, a Feed - Forward back propagation neural network has been proposed to predict critical responses of flexible pavements and the results obtained from this method have been compared with those of Kenlayer program.

## 2. Artificial Neural Network

The ANN modelling approach is a computer methodology that attempts to simulate some important features of the human nervous system; in other words, the ability to solve problems by applying the information gained from the past experiences to new problems or case scenarios. Analogous to a human brain, an ANN uses many simple computational elements, named artificial neurons, connected by variable weights. The ANN modelling consists of two steps: to train and to test the network. During the training stage, the network uses the inductive-learning principle to learn from a set of examples called the training set [Haykin, 1998].

Artificial Neural Network (ANN) was widely used in pavement engineering for finding the patterns between the input data and the output results [Attoh-Okine, 2005; Cylan et al. 2005; Pekan et al. 2008; Terzi, 2007; Saltan and Terzi, 2008; Tapkin et al. 2009; Ozgan 2011; Ghanizadeh and Fakhri, 2014; Ozturk and Kutay, 2014]. Compared to conventional methods such as linear and nonlinear regression, the accuracy of ANN method is much higher. For the regression problems, two famous types of neural networks commonly have been used namely Feed-Forward Neural Networks (FFNN) and General Regression Neural Networks (GRNN). The Feed-Forward Neural Networks which is known also as Multilayer Perceptron (MLP) was the first and most simple type of artificial neural network devised. In this network the information moves in only one direction from the input nodes data through the hidden nodes (if any) and to the output nodes [Hagan et al 1996]. The mathematical theory of neural networks states that every continuous function that maps intervals of real numbers to some output interval of real numbers can be

approximated arbitrarily closely by a Feed-Forward Neural Networks with just one hidden layer [Hornik, 1991].

In this study a Feed-Forward artificial neural network with backpropagation training algorithm was employed to establish the relationship between input data (thickness and modulus of pavement layers) and output results (critical strains of pavement).

The training of a feed forward neural network using a back-propagation (BP) algorithm involves two phases [Werbos, 1974; Rumelhart et al. 1986]:

- Forward Phase: During this phase, the free parameters of the network are fixed, and the input signal is propagated through the network layer by layer. The forward phase ends with the computation of an error signal.

$$e_i = d_i - y_i \quad (1)$$

where  $d_i$  is the desired response, and  $y_i$  is the actual output produced by the network in response to the input  $x_i$ .

- Backward Phase: During this phase, the error signal  $e$  is propagated through the network in the backward direction. It is during this phase that adjustments are applied to the free parameters of the network so as to minimize the error  $e$  in a statistical sense.

The performance of a neural network model mainly depends on the network architecture and parameter settings. One of the most difficult tasks in ANN studies is to find this optimal network architecture which is based on the determination of numbers of optimal layers and neurons in the hidden layers by trial and error approach. The assignment of initial weights and other related parameters furthermore influences the performance of the ANN in a great extent. There is, however, no well-defined rule or procedure to obtain optimal network architecture and parameter settings where trial and error method still remains valid [Tapkin et al. 2009]. RMSE, MSE, NMSE and R are the criteria that are usually used to evaluate the performance of a neural network. In this study, RMSE parameter has been used to evaluate the performance of the artificial neural network. This parameter is obtained using the following equation.

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (d_i - y_i)^2} \quad (2)$$

where  $N$  denotes the number of records to test the neural network,  $d_i$  denotes the desired value for the  $i$ th record,  $y_i$  denotes the predicted value by neural network for the  $i$ th record and RMSE is root mean square error.

### 3. Establishment of Synthetic Database

In this study, two important pavement responses, including the maximum horizontal principal tensile strain at the bottom of asphalt layer and maximum vertical compressive strain on the top of subgrade have been taken into consideration. The maximum horizontal principal tensile strain at the bottom of asphalt layer and maximum vertical compressive strain on the top of subgrade are two criteria that control the bottom-up fatigue cracking and rutting depth of pavement, respectively [NCHRP, 2004, Austroad 2010, IRC 2012].

In order to create a comprehensive synthetic database with the purpose of training and testing the neural network, 10000 different flexible pavement sections were analyzed under the influence of standard axle load and the maximum horizontal principal tensile strain at the bottom of asphalt layer and maximum compressive strain on the top of subgrade in each case were calculated. Maximum value for each of these two responses was determined with respect to the results obtained from the analysis of five different points at the bottom of asphalt layer and five different points on the top of subgrade. Specifications of the standard axle load (single axle with dual wheel with the weight of 8.2 tons), features of pavement sections, and also the position of the response points are shown in Figure (1). Statistical specifications of the analyzed pavements along with the critical

responses are shown in Table (1). In all the analyses, the Poisson's ratio of asphalt concrete, granular base, and granular subbase was assumed as 0.35 and the Poisson's ratio of subgrade was assumed as 0.4. These values are typical values of Poisson's ratio for Hot Mix Asphalt, untreated granular materials and fine-grained soils [Maher and Bennert, 2008]. Previous findings have also shown that the selection of Poisson's ratio has a small effect on pavement responses [Huang, 2004]. Analysis of pavement sections was completed using layered elastic analysis program, NonPAS, which enables the analysis of a pavement system consisting of a maximum of ten linear or nonlinear layers subjected to the maximum of ten circular contact loads [Fakhri and Ghanizadeh, 2012]. Detailed verification of NonPAS code using Kenlayer program proved that the developed code can accurately predict the pavement responses subjected to single and multiple loading [Fakhri and Ghanizadeh, 2012].

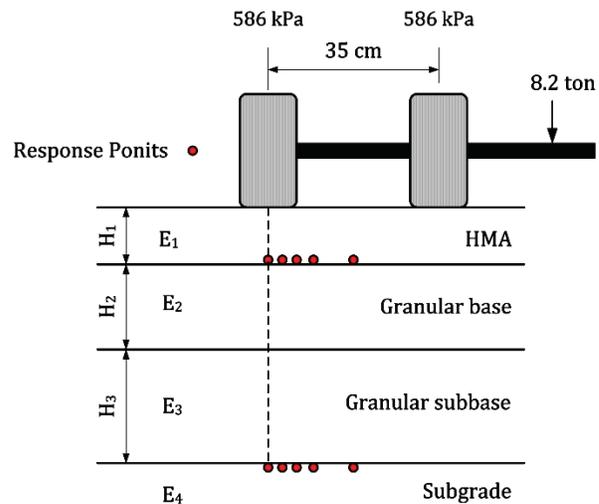


Figure 1. Specifications of standard axle, pavement section, and response points.

Table 1. Statistical characteristics of the inputs and outputs used in database development

Statistical Parameter	H <sub>1</sub>	H <sub>2</sub>	H <sub>3</sub>	E <sub>1</sub>	E <sub>2</sub>	E <sub>3</sub>	E <sub>4</sub>	ε <sub>t</sub>	ε <sub>c</sub>
Minimum	5.00	0.00	0.00	800.00	200.00	100.00	30.00	16.45	26.50
Maximum	45.00	50.00	60.00	10000.00	400.00	200.00	200.00	2046.59	7048.34
Mean	22.66	25.93	32.67	5289.15	295.96	154.81	86.72	113.14	176.04
Standard Deviation	11.08	14.06	18.28	2803.01	64.24	29.57	38.75	88.36	187.47
Median	22.69	25.09	30.22	5049.85	300.00	152.99	82.31	86.77	126.26

H<sub>i</sub>: Thickness of the  $i_{th}$  layer in cm.

E<sub>i</sub>: Resilient modulus of the  $i_{th}$  layer in MPa.

ε<sub>t</sub>: Maximum horizontal principal tensile strain at the bottom of asphalt layer in micro-strain.

ε<sub>c</sub>: Maximum compressive strain on the top of subgrade in micro-strain

#### 4. Optimal Architecture of the Artificial Neural Network

In this study, MATLAB ANN toolbox was used for ANN applications [Beale et al. 2011]. MATLAB ANN toolbox randomly assigns the initial weights for each run each time which considerably changes the performance of the trained ANN even if all the parameters and ANN architecture are kept constant. This leads to extra difficulties in the selection of the optimal network architecture and parameter settings. To overcome this difficulty, a program was developed in MATLAB which handled the trial and error process automatically. The program tried various numbers of the neurons in the hidden layer for several times and selected the best ANN architecture with the minimum RMSE (Root Mean Squared Error) of the testing set. The testing (30%), cross validating (10%) and training (60%) sets for ANN training procedure were selected randomly from the synthetic dataset.

Changes in root mean square error according to the number of neurons in the hidden layer has been shown in figure (2). As it is seen, increasing the number of neurons beyond 30 in the hidden layer does not have that much effect on increasing the accuracy of artificial neural network. Therefore, we can observe that the neural network with one hidden layer and a 7-30-2 architecture enjoys the sufficient accuracy in predicting critical responses of pavement. The selected architecture of neural network is shown in Figure (3).

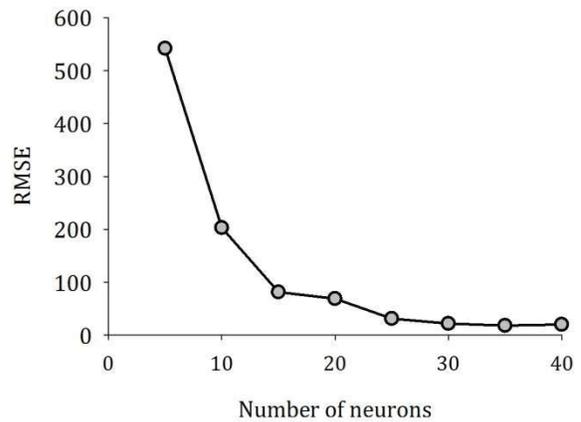


Figure 2. RMSE with respect to the number of neurons in the hidden layer.

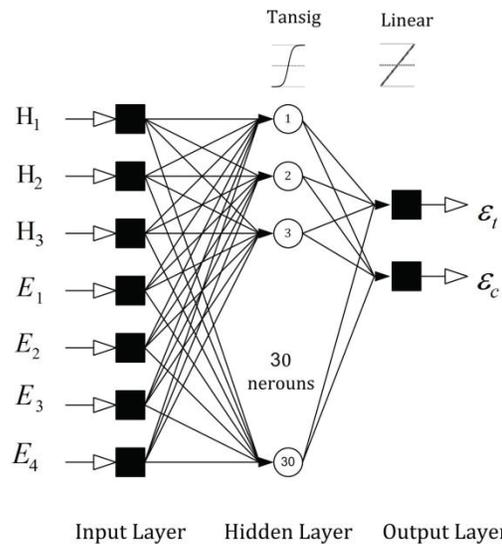


Figure 3. Optimal architecture of artificial neural network.

#### 5. The Evaluation of the Performance of Artificial Neural Network

The performance of artificial neural network in predicting critical responses of pavement using training and testing sets, are shown in Figures (4) and (5), respectively. As evidence, the proposed neural network is capable of predicting critical responses of pavement (maximum horizontal principal tensile strain at the bottom of asphalt layer and maximum compressive strain on the top of subgrade) with high accuracy with regard to the pavement structure (thickness and modulus of pavement layers). This neural network

## Application of Artificial Neural Networks for Analysis of Flexible Pavements under ...

provides the possibility of analyzing and determining the critical responses of pavement

and finally predicting fatigue life and rutting resistance of pavement sections.

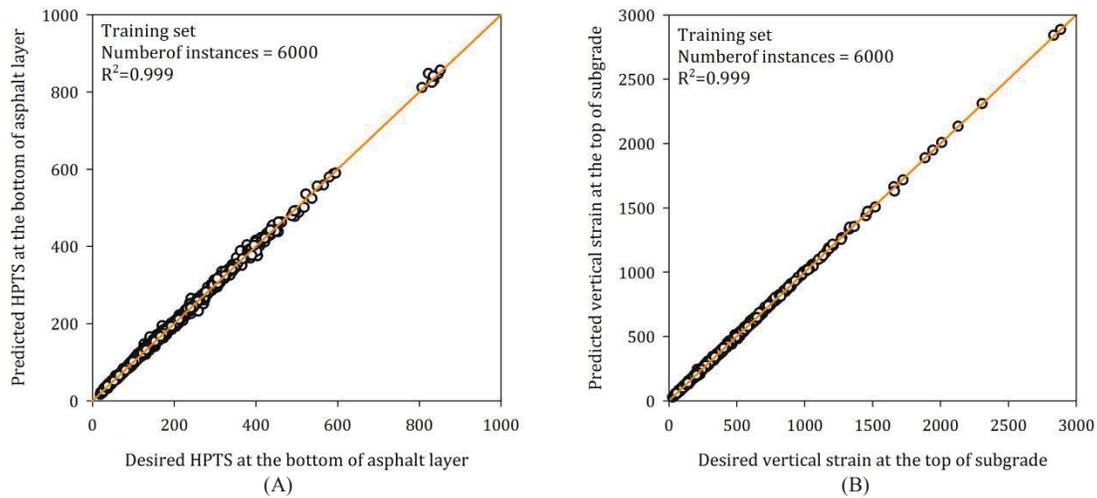


Figure 4. Results of artificial neural network training

: A) Maximum horizontal principle tensile strain (HPTS) at the bottom of asphalt layer and B) Maximum

compressive strain on the top of subgrade

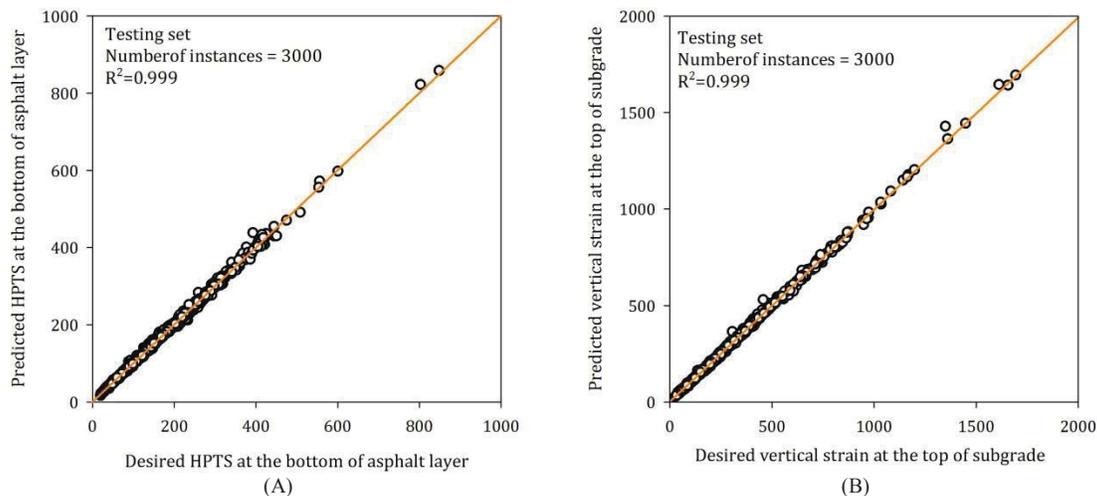


Figure 5. Results of artificial neural network testing: A) Maximum horizontal principle tensile strain (HPTS) at the bottom of asphalt layer and B) Maximum compressive strain on the top of subgrade

The frequency histogram for prediction error of critical strains using testing set is shown in Figure (6). As it is observed, prediction error of maximum horizontal principal tensile strain

at the bottom of asphalt layer and maximum compressive strain on the top of subgrade are 15 and 10 percent, respectively while in most cases prediction error is lower than 5 percent.

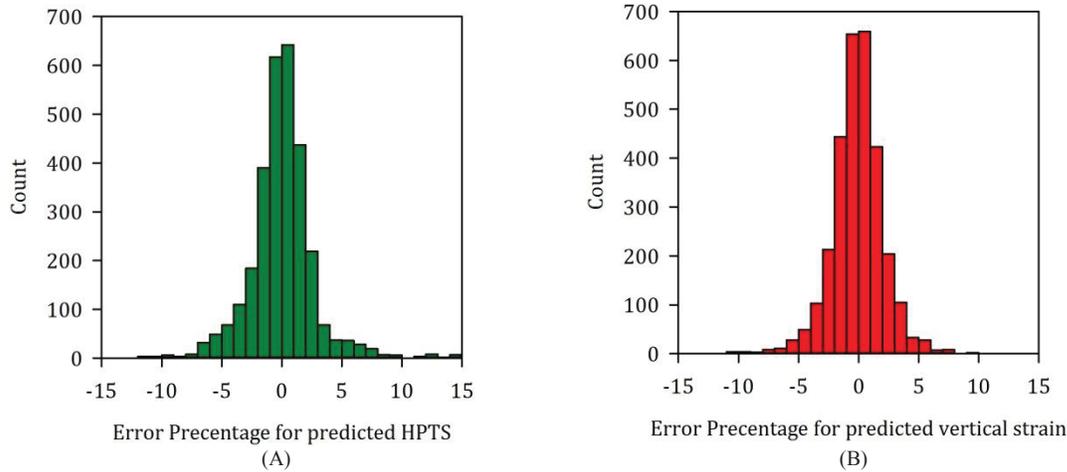


Figure 6. Error percentage of predicted strain for test data: A) Maximum horizontal principle tensile strain (HPTS) at the bottom of asphalt layer and B) Maximum compressive strain on the top of subgrade

**6. Validation of Artificial Neural Network Method using Kenlayer Program**

The results of analysis obtained through Kenlayer program were used to validate the developed artificial neural network method in this study. Kenlayer program is among the most powerful and accurate pavement analysis programs that uses the elastic multi-layered theory for the analysis of pavements [Huang 2004]. To this end, nine different pavement

sections were considered and the critical strains at the bottom of asphalt layer and on the top of subgrade were determined using artificial neural network method and Kenlayer program. The specifications of these nine pavement sections along with the obtained responses using artificial neural network and Kenlayer program are given in Table (2).

Table 2. Comparison of the responses obtained using artificial neural network and Kenlayer.

H <sub>1</sub> (cm)	H <sub>2</sub> (cm)	H <sub>3</sub> (cm)	E <sub>1</sub> (MPa)	E <sub>2</sub> (MPa)	E <sub>3</sub> (MPa)	E <sub>4</sub> (MPa)	ANN		Kenlayer		Error percentage	
							ε <sub>t</sub>	ε <sub>c</sub>	ε <sub>t</sub>	ε <sub>c</sub>	ε <sub>t</sub>	ε <sub>c</sub>
5	10	15	1000	220	120	30	366.86	1708.02	386.00	1706.00	-4.96	0.12
6	12	15	3000	250	125	40	329.92	1115.81	344.30	1112.00	-4.18	0.34
7	15	15	6000	300	130	50	225.55	721.07	226.30	723.10	-0.33	-0.28
10	20	25	1500	220	135	60	341.95	457.82	341.70	458.10	0.07	-0.06
15	25	25	3500	270	140	70	154.40	240.02	154.10	245.10	0.20	-2.07
20	35	25	6500	320	145	80	71.59	121.36	71.40	123.90	0.27	-2.04
25	25	55	2000	230	150	90	123.02	99.04	119.90	100.10	2.61	-1.06
30	40	55	4000	280	160	100	55.34	54.92	57.50	55.42	-3.74	-0.90
35	50	55	7000	330	165	110	28.63	33.61	30.06	34.42	-4.73	-2.34

ε<sub>t</sub>: The maximum horizontal principal tensile strain at the bottom of asphalt layer, micro-strain  
 ε<sub>c</sub>: The maximum compressive strain on the top of subgrade, micro-strain

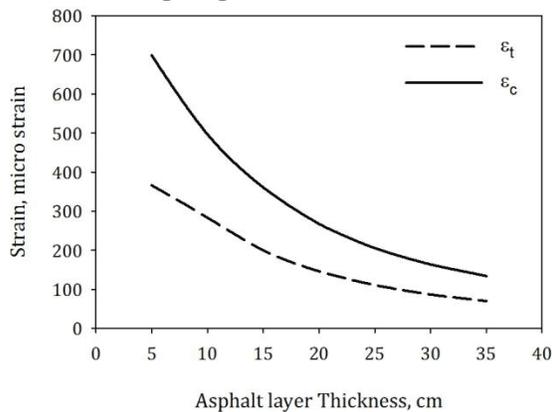
As it can be seen, the maximum prediction error of the critical tensile strain is less than 5 percent; therefore, the developed neural network can be applied to predict the maximum horizontal principal tensile strain at

the bottom of asphalt layer and the maximum compressive strain on the top of subgrade, accurately.

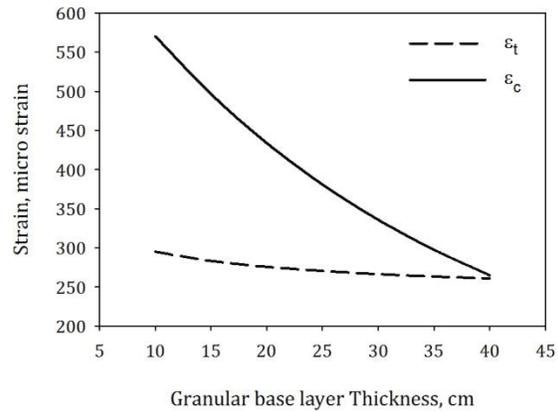
### 7. Parametric Analysis

A four-layered pavement section consisting of asphalt layer with the thickness of 10 cm, granular base layer with the thickness of 15 cm, granular subbase layer with the thickness of 20 cm, and subgrade soil was considered in order to perform sensitivity analysis and to investigate the effect of various parameters on the maximum horizontal principal tensile strain at the bottom of asphalt layer and on the maximum compressive strain on the top of subgrade under the influence of passing the 8.2-ton equivalent single axle load. Resilient modulus of each of these layers was assumed 3000, 200, 120, and 70 MPa, respectively. Then, under the constancy assumption of all the parameters and change of one parameter, the influence of the desired parameter on critical responses of pavement was assessed using developed ANN. Figure 7 shows the effect of each input parameter of the neural

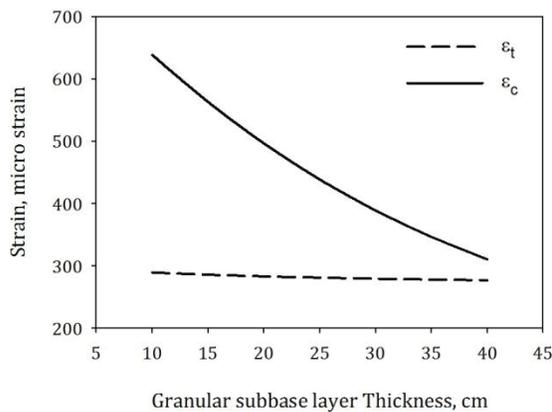
network on critical responses. As evidence, the increase of the thickness of asphalt layers, base, and subbase significantly reduces the maximum compressive strain on the top of subgrade. However, only the increase of the asphalt layer thickness causes the significant reduction of the maximum horizontal principal tensile strain at the bottom of asphalt layer. It is also observed that the increase of resilient modulus in asphalt layers reduces critical responses whereas the increase of resilient modulus of granular base and subbase within the acceptable range of this parameter does not have a great effect on the maximum horizontal principal tensile strain at the bottom of asphalt layer. It is also observed that the resilient modulus of the subgrade soil only affects the compressive strain while it has a very small effect on the maximum horizontal principal tensile strain at the bottom of asphalt layer.



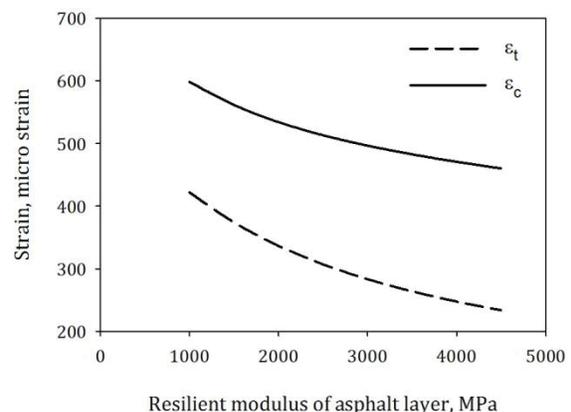
(A)



(B)



(C)



(D)

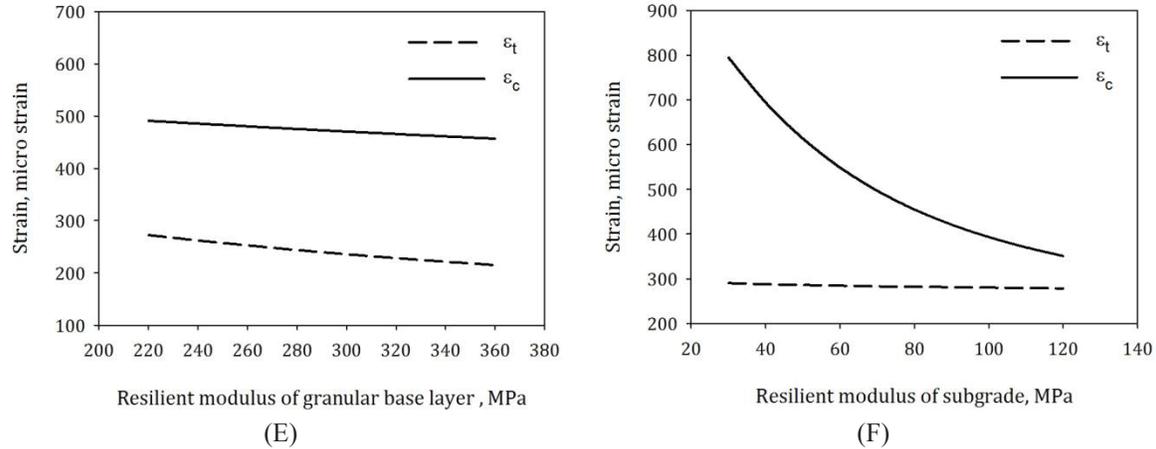


Figure 7. Sensitivity analysis of the neural network in order to evaluate the impact of network input parameters on network outputs.

### 8. The Development of Pavement Analysis Program Based on Artificial Neural Network Method

In order to facilitate ANN application in practice, a computer program was developed in the Visual Basic.Net environment.

The developed program entitled PAVANN makes it possible to quickly calculate the critical responses of pavement including the maximum horizontal principal tensile strain at the bottom of asphalt layer and the maximum compressive strain on the top of subgrade. It is met through obtaining the input parameters including the thickness and resilient modulus

of different layers. The graphical user interface (GUI) of this program is shown in figure (8).

### 9. Conclusion

In this study, a Feed-Forward backpropagation neural network with the optimal architecture of 7-30-10 was proposed to predict the critical responses (the maximum horizontal principal tensile strain at the bottom of asphalt layer and the maximum compressive strain on the top of subgrade) of flexible pavements under the influence of standard axle loading. Artificial neural network in this study was trained and

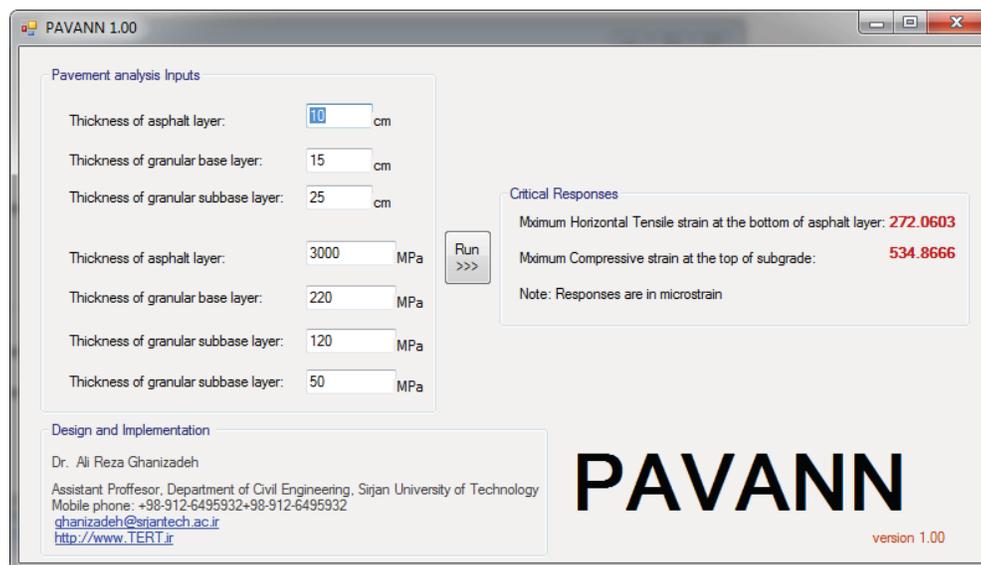


Figure 8. GUI of PAVANN program.

tested based on a synthetic database containing of 10000 records. Comparing the results of the analysis via artificial neural network and Kenlayer program is indicative of the high accuracy of the proposed neural network to determine the critical responses of pavement. Due to the fast pavement analysis by means of artificial neural network method in comparison with the finite element method and multi-layered elastic theory, it is possible to use the current developed method without introducing the position of response points in order to quickly and accurately analyze a pavement. It is also possible to predict the pavement life with respect to the passing traffic during pavement design and based on the criteria of fatigue cracking and rutting. However, the fast analysis of pavements via this method allows the possibility of analyzing a very large number of pavement sections to optimal design of flexible pavements. Since in practice the spectra of loading axles are commonly employed for design of pavements using mechanistic-empirical methods, this research needs to be completed by developing other artificial neural networks considering different loading axles.

## **10. References**

- Abu-Lebdeh, G. & Ahmed, K. (2013) "A Neural Network Approach for Mechanistic Analysis of Jointed Concrete Pavement", Proceedings of the Eastern Asia Society for Transportation Studies, Vol. 9.
- Ahmed, M., Tarefder, R. & Islam, M. (2013) "Effect of cross-anisotropy of hot-mix asphalt modulus on falling weight deflections and embedded sensor stress-strain". Transportation Research Record: Journal of the Transportation Research Board, Vol. 2369, pp.. 20-29.
- Al-Hadidy, A. I. & Tan, Y. Q. (2009) "Mechanistic analysis of st and sbs-modified flexible pavements". Construction and Building Materials, Vol. 23, No. 8, pp. 2941-2950.
- Ameri, M. & Molayem, M. (2006) "Application of Artificial Neural Networks for the Analysis of Flexible Pavements". International Journal of Engineering Science, Vol. 17, No. 5, pp. 60-54.
- Attoh-Okine, N. O. (2005) "modeling incremental pavement roughness using functional network". Canadian Journal of Civil Engineering, Vol. 32, No. 5, pp. 899-905.
- Austroads. (2010) "Guide to pavement technology (Apt-02/10) – Part 2: Pavement structural design". Sydney, Australia: Austroads.
- Beale, M. H., M.T., H. & Demuth, H. B. (2011)"Neural Network Toolbox. for Use with Matlab", Themathworks, Natick.
- Boussinesq, M. J. (1885) "Applications des potentiels à l'étude de l'équilibre et du mouvement des solides élastiques", Gauthier Villars, Paris.
- Burmister, D. M. (1945) "The general theory of stresses and displacements in layered systems". International Journal of Applied Physics, Vol. 16, No. 2, pp..89-94.
- Ceylan, H. (2002) "Analysis and design of concrete pavement systems using artificial neural networks", (Ph.D Dissertation), University of Illinois at Urbana-Champaign.
- Ceylan, H., Guclu, A., Tutumluer, E. & Thompson, M. R. (2005) "Backcalculation of full-depth asphalt pavement layer moduli considering nonlinear stress-dependent subgrade behavior". International Journal of Pavement Engineering, Vol. 6, No. 3, pp..171-182.
- Duncan, J. M., Monismith, C. L. & Wilson, E. L. (1968) "Finite element analyses of pavements". Highway Research Record, Vol. 228, Pp.18-33.
- Fakhri, M. & Ghanizadeh, A. (2012) "Program Development for the Nonlinear Analysis of Flexible Pavements. Quarterly Journal of Transportation Engineering", Vol. 3, No. 3, pp.257-245.
- Ghanizadeh, A. R. & Fakhri, M. (2014) "Prediction of frequency for simulation of asphalt mix fatigue tests using Mars and Ann", the Scientific World Journal, pp..1-16.
- Hagan, M. T., Demuth, H. B. & Beale, M. H. (1996) "Neural Network Design". PWS Publishing, Boston.
- Harichandran, R. S., Yeh, M.-S., & Baladi, G. Y.

- (1990). "MICH-PAVE: A nonlinear finite element program for analysis of flexible pavements". Transportation research record, Vol. 1286., pp. 123-131.
- Hayhoe, G. F. (2002) "LEAF: A new layered elastic computational program for FAA pavement design and evaluation procedures", Federal Aviation Administration.
  - Haykin, S. (2001) "Neural networks: a comprehensive foundation", New Jersey, Prentice Hall.
  - Hornik, K. (1991) "Approximation capabilities of multilayer feedforward networks", Neural Networks, Vol. 4, No. 2, pp. 251-257
  - Huang, Y. H. (2004) "Pavement analysis and design", New Jersey, Prentice Hall.
  - Huang, C. W., Abu Al-Rub, R. K., Masad, E. A., & Little, D. N. (2010) "Three-dimensional simulations of asphalt pavement permanent deformation using a nonlinear viscoelastic and Viscoplastic Model". Journal of Materials in Civil Engineering, Vol. 23, No. 1, pp. 56-68.
  - Indian Road Congress - IRC (2012) "Guidelines for the Design of Flexible Pavements", (3rd Ed.), Indian Road Congress.
  - Jong, D. D., Peutz, M., &Korswagen, A. (1979) "Computer program bisar, layered systems under normal and tangential surface loads". Koninklijke/Shell Laboratorium, Amsterdam, Shell Research BV.
  - Khazanovich, L., & Wang, Q. C. (2007) "Mnlayer: High-Performance Layered Elastic Analysis Program". Transportation Research Record, Vol. 2037, Pp.63-75.
  - Khazanovich, L., Selezneva, O. I., Thomas Yu, H. & Darter, M. I. (2001) "development of rapid solutions for prediction of critical continuously reinforced concrete pavement stresses", Transportation Research Record: Journal of the Transportation Research Board, Vol. 1778, Pp. 64-72.
  - Maher, A. & Bennert, T. A. (2008) "Evaluation of Poisson's ratio for use in the mechanistic empirical pavement design guide (MEPDG)", Final Report: FHWA-NJ-2008-004. Federal Highway Administration U.S. Department of Transportation Washington, D.C.
  - NCHRP (2004) "Guide for mechanistic-empirical design of new and rehabilitated pavement structures", Final Report for Project 1-37a. Washington, Dc: National Cooperative Research Program.
  - Newmark, N. M. (1947) "influence charts for computation of vertical displacements in elastic foundations", University of Illinois.
  - Odemark, N. (1949) "Investigations as to the elastic properties of soils and design of pavements according to the theory of elasticity", Meddelande, 77.
  - Ozgan, E. (2011) "Artificial neural network based modeling of the Marshall stability of asphalt concrete". Expert Systems with Applications, Vol. 38, No. 5, pp.6025-6030.
  - Ozturk, H. I., & Kutay, M. E. (2014) "An artificial neural network model for virtual superpave asphalt mixture design", International Journal of Pavement Engineering, Vol. 15, No. 2, pp.151-162.
  - Pekcan, O., Tutumluer, E. & Thompson, M. (2008) "Artificial neural network based backcalculation of conventional flexible pavements on lime stabilized soils". The 12<sup>th</sup>. International Conference Of International Association for Computer Methods And Advances in Geomechanics (IACMAG), Goa, India.
  - Raad, L. & Figueroa, J. L. (1980) "Load response of transportation support systems", Journal of Transportation Engineering, Vol. 106, No. 1, pp. 111-128.
  - Rumelhart, D. E., Hintont, G. E. & Williams, R. J. (1986). "Learning Representations by Back-Propagating Errors". Cambridge, MIT Press.
  - Saltan, M. (2008) "Modeling deflection basin using artificial neural networks with cross-validation technique in backcalculating flexible pavement layer moduli.advances in engineering software", Vol. 39, No. 7, Pp. 588-592.
  - Sanborn, J. L., & Yoder, E. J. (1967) "Stress and displacements in an elastic mass under semiellipsoidal loads". 2<sup>nd</sup> International Conference of Structural Design of Asphalt

## Application of Artificial Neural Networks for Analysis of Flexible Pavements under ...

Pavements, An Arbor, Michigan.

- Schiffman, R. L. (1962) "General Analysis of Stresses and Displacements in Layered Elastic Systems". 1<sup>st</sup> International Conference on the Structural Design of Asphalt Pavements, An Arbor, Michigan.
- Tapkin, S., Çevik, A. & Usar, Ü. (2009) "accumulated strain prediction of polypropylene modified marshall specimens in repeated creep test using artificial neural networks", Expert Systems with Applications, Vol. 36, No. 8, Pp. 11186-11197.
- Uzan, J. (1994) "Advanced backcalculation techniques", ASTM Special Technical Publication, 1198, 3-3.
- Werbos, P. (1974) "Beyond regression: new tools for prediction and analysis in the behavioral sciences". (PHD Dissertation), Harvard University, Cambridge.
- Yang, C. F. & Liu, L. (2010) "Dynamic response analysis of cement concrete pavement under different vehicle speed", Hebei Gongye Daxue Xuebao, Vol. 39, No. 3, pp. 112-115.

predictors in the input vector is as follows:

$$\{Inp\} = \{H_1, H_2, H_3, E_1, E_2, E_3, E_4\}_{1 \times 7} \quad (A.1)$$

The order of normalized output parameters in the output vector is as follows:

$$\{Out\} = \{\varepsilon_t, \varepsilon_c\}_{1 \times 2} \quad (A.2)$$

Before simulating of network, Inputs should be normalized based on the following relation:

$$Q_n = 2 \frac{(Max - Min)}{(Q - Min)} - 1 \quad (A.3)$$

Where:

$Q_n$  = Normalized value of parameter  $Q$

$Max$  = Maximum observed value for parameter  $Q$

$Min$  = Minimum observed value for parameter  $Q$

After the simulating of network, real outputs should be computed based on normalized outputs using Equation (A.3). Maximum and minimum value of input and output parameters were presented in Table (1).

Weight matrix for hidden and output layers are given in Table (A.1) and Table (A.2), respectively.

Bias vector for hidden and output layer are given in Table (A.3) and Table (A.4), respectively.

Equation (A.4) may be used for simulation of ANN and prediction of output vector based on given input vector.

(A.4)

$$\{Out\} = \tan \text{sig} \left( \{Inp\} \times [W_h]^T + \{\theta_h\}^T \right) \times [W_o]^T + \{\theta_o\}^T$$

Where  $\tan \text{sig}(x)$  can be obtained as follows:

$$\tan \text{sig}(x) = \frac{2}{1 + e^{-2x}} - 1 \quad (A.5)$$

### Appendix A. Weights and biases of Artificial Neural Network (ANN)

This Appendix is assigned to input vector, output vector, weight factors, and bias factors of the backpropagation network which was discussed in section 4. The optimum architecture of backpropagation network is 7-30-2 with sigmoid transfer function in hidden layer and linear transfer function in output layer. The order of normalized

Table A.1. Weight matrix of hidden layer ( $[W_h]_{30 \times 7}$ )

0.3187748343	-1.7975142985	-1.5552241011	-0.0116698727	-0.4959649170	2.7953770381	-2.1738908683
1.0411752905	-0.1388813769	-0.1758503575	0.2544911242	-0.0288563957	-0.0161134212	0.0193823365
-0.2521399258	-0.6011273450	-2.1269683586	-2.0728522684	0.0928789162	-0.0930170044	-0.9357762909
-1.2532301243	-1.1324713494	-0.9729009103	-1.0444137925	-0.0848026541	-0.0697138593	-0.7963291660
1.4190173686	-0.9863851472	-1.0303250822	-1.1335327469	-0.0706348528	0.0038986851	-0.8862271013
1.3009481056	2.1950466345	1.8407973169	0.4566581882	0.0754165369	0.1398547790	0.0686435889
1.4298030661	2.1391874176	1.8897031922	0.3230288554	0.0326176656	0.0624060146	0.4798895814
-0.7431677443	-0.3664314869	-0.7177667257	-0.0839108932	0.0150397200	-0.1362544389	-0.5239350920
0.6966455307	0.3126887001	0.7084978291	0.0811204760	-0.0440995513	0.1570313754	0.5410716134
0.3764243191	0.2210389323	0.2091185990	0.0882483305	0.0383749901	0.1148560962	0.7466900018
-0.3086775942	-0.0882280039	-0.0179228128	-0.8906059230	-0.0039002209	-0.0167258017	-0.0810555676
-0.4606435567	-0.3227494683	-0.3120265133	-0.0965858360	-0.0416339927	-0.1264076810	-0.6287525157
-0.0166007538	0.9107091449	-0.5264527308	-0.6412957865	0.6616437253	0.3926885025	-0.4425423354
0.9694524525	0.8059183558	0.4241154100	0.2624819799	-0.1729967357	0.1534312702	0.6915628183
-1.1697094718	-0.0262157319	0.0311658061	-0.2212536483	0.0176879639	0.0265969313	-0.2259023643
-0.7946115648	-2.9611517348	-1.3168181213	-2.5338716848	-1.2501906235	-0.3499172032	-0.9816788665
-0.4836556783	0.0205765960	0.0173920821	-1.8558822625	-0.3497556745	-0.009693582	0.0358210081

**Ali Reza Ghanizadeh, Mohammad Reza Ahadi**

-1.0444477437	0.0934066956	0.1364608696	-0.2620085683	0.0384120411	0.0229007845	-0.1156075270
1.1028457694	-0.0221895045	-0.0726258533	0.2406844854	-0.0294469280	-0.0256057865	0.1886424756
4.9753718076	0.0466253580	-0.0310239226	1.0868348922	0.0547939303	0.0309179665	0.0253874701
-0.0832286229	-0.6828200256	-0.6753244260	-2.9450524803	-0.0396612279	-0.0216022248	-0.5696588215
1.4101805009	2.1010673355	-0.0002288314	0.2288432339	0.3725711118	-0.0098059265	0.1558807287
-0.2218478451	-1.6179204042	-1.4870510836	0.3680886516	-0.3333070991	-0.1966763660	-0.3663895158
0.5927062564	0.5438481942	0.5452918720	0.0476845466	0.0785271541	0.1008472931	0.3725293665
-1.8604876381	-1.1781463287	-0.9054145744	-1.2474831141	-0.0412738405	-0.0845412749	-0.8537574705
-1.9671632458	0.0090462274	0.0223981112	-0.2726015832	-0.2232204804	-0.0054566206	-0.0042928582
0.7792185448	2.8192930390	0.0343918186	1.6159624942	-0.2458106907	0.4350878169	0.0281152142
2.0513109197	2.2930674636	2.2423824286	0.4648792818	-0.0775416090	-0.0530040220	0.6859978362
1.7690982622	1.8343746640	0.0074416582	0.2228453186	-0.0033087916	0.1681045238	0.1128536987
1.8949971408	0.8546648777	0.8901170554	-0.0270466128	0.0812929689	0.0604385412	0.5996981753

Table A.2. Weight matrix of output layer ( $\{W_o\}^T$ )  $30 \times 2$

-0.0004541622	0.0000273188
-0.5319764023	-0.2854954664
-0.0003441894	0.0020501344
3.4485039345	2.9823836965
-0.0026814267	0.0017790829
0.6728015964	-1.4676073651
-0.0424620819	4.9012227511
0.1071915539	1.0486447359
0.3540589198	2.4794056635
-0.2406135325	-2.2797364150
0.0660927725	0.0131872262
-0.0784408322	-0.6937694464
-0.0000809956	0.0012045041
-0.0257153826	-0.7990459002
-0.3543544747	-0.5570866821
-0.0000338010	-0.0004154328
0.5343639513	0.0001690348
-1.1230772520	-0.9754326032
-1.0148119671	-1.2519356627
4.6673950835	0.0418199512
0.0025770924	0.0421398049
0.4776881387	0.0638182619
0.0001530807	-0.0108757497
-0.0210340207	-0.0802638151
-1.1890409709	-0.8740937595
2.2795114077	-0.0109041739
-2.1486638467	-0.0089769779
-0.8535619818	-2.5508673313
-1.9290284266	-0.1925514522
-0.1484283663	-1.3222400416

Table A.3. Bias vector of hidden layer ( $\{\theta_h\}$ )

-3.0616130999
1.0205489722
-2.8700373923
-5.6829550145
-1.6872974514
6.3853760640
6.7099795510
-2.5490097298
3.0943573210
2.6133967996
-0.9815471852
-1.8274943010
-0.3481111581
3.9815298864
-0.7442932738
0.4406686713
-3.4627331571
-0.9309557294
0.8141177485
7.8982233347
-4.3565714936
4.8343113598
-2.6300894503
1.1432477900
-5.8120399978
-3.8218529984
6.6715161918
7.7630998210
5.1370766056
4.8252687100

Table A.4. Bias vector of output layer ( $\{\theta_o\}$ )

3.4739486442
2.7188758502