

Review Paper:

Literature Review of Traffic Assignment: Static and Dynamic

Krishna Saw^{1*}, B. K. Katti², G. Joshi³

Received: 24. 01. 2015

Accepted: 20. 03. 2015

Abstract:

Rapid urban growth is resulting into increase in travel demand and private vehicle ownership in urban areas. In the present scenario the existing infrastructure has failed to match the demand that leads to traffic congestion, vehicular pollution and accidents. With traffic congestion augmentation on the road, delay of commuters has increased and reliability of road network has decreased. Four stage travel demand modelling is one of the transportation planning tools that used to evaluate the impact of future changes in demographics, land use and transportation facilities on the performance of city's transportation system. However, this planning tool does not cover the dynamic properties of flow precisely and ineffective for traffic management and this planning tolls has several unrealistic assumption such as travel time on link do not vary with the link flows, trip makers have precise knowledge of the travel time on the link. Therefore, it is needed to revisit the available tool and explore new planning tool which is sensitive to present traffic pattern of the city. Evolution and operation of Information Transportation System; Advanced Traveller Information System (ATIS) and Advanced Traveller Management Systems (ATMS) give rise of dynamic based travel demand modelling which covers dynamic nature of flow over time and space. Dynamic travel demand modelling provides better planning and management scope in view of this research focus has been diverted to dynamic traffic assignment (DTA). The main aim of DTA is to manage traffic in a network through real-time measurement, detection, communication, information provision, and control. Here, effort has been made to study the Static Traffic Assignment (STA) and Dynamic Traffic Assignment (DTA) with special focus on limitations of STA.

Keywords:Static traffic assignment, dynamic traffic assignment, intelligent transportation system, dynamic flow, dynamic flow

Corresponding author Email:Kriscivil_10@yahoo.com

1- Ph.D. Candidate, Sardar Vallabhbhai National Institute of Technology, Surat, India

2- Professor, Sardar Vallabhbhai National Institute of Technology, Surat, India

3- Associate Professor, Sardar Vallabhbhai National Institute of Technology, Surat, India

1. Introduction

1.1 General

Rapid economic development and reforms caused the rapid growth of urbanization which is resulting into growth in urban population and expansion of city which triggers a fast increase in urban travel demand and growth in private vehicle ownership [Zou et al. 2013]. However, urban transportation infrastructure and traffic management fail to match the rapid growth rate of travel demand and supply resulting into congestion on the road along with traffic accident and pollution. With congestion intensification, delay of commuters has increased and reliability of network has decreased. In this scenario the importance of travel demand modelling tool is enlarged and efficient toll may only help to manage such developing situation in city in effective and efficient way.

Travel forecasting models are used in transportation planning to evaluate the impact of future changes in demographics, land use or transportation facilities on the performance of city's transportation system. The four stage travel demand model is generally applied which consists trip generation, trip distribution, mode choice and trip assignment. Trip generation methods are used to predict production and attractions for given areas whereas trip distribution models are used to forecast flows between origin and destinations. Subsequently, modal choice gives idea about the absolute trips made by the particular mode and Trip assignment predicts network flow or route choice of travellers for particular O-D.

With the congestion problem on the road network, transportation planning has also focused on improvement of management through application of intelligent transportation system along with infrastructure development. Network flow or route choice of traveller is most useful output of travel demand model to understand and address the congestion and also application of Intelligent Transportation System (ITS); Advanced Traveller Information System (ATIS) and Advanced Traveller Management Systems (ATMS). However, it is time dependent and vary with time but in the case of static planning methods cannot precisely describe time-dependent travel demand and dynamic characteristics of flow [Zou et al. 2013]. Transportation planning can

be either static or dynamic. Within dynamic models, the traffic can change through time, whereas in case of static model the traffic demand is constant through time [Leeuwen, 2011]. Recent development of ATIS and ATMS and their application have increased the attention towards the dynamic traffic assignment.

1.2 Traffic Assignment

Traffic assignment is the process of allocating given set of origin-destination pair to the existing suitable road network based on specific traveller's route choice criteria. The route choice criteria is the travel impedance of the transportation network be minimized for a given origin-destination pair. Present studies shows that travel impedance for the transportation network includes travel time, travel cost, travel distance. However, due to congestion on particular route on particular time period, route choice not only depends on the shortest path but also departure time and reliability of link of network. Therefore, planner should focus optimal path rather than the shortest path. Optimal paths may be defined as the paths having minimum travel time or cost along with higher travel time reliability and safety.

Basically, traffic assignment methods can be broadly classified into two categories as enlisted below.

- a. Static Traffic Assignment Method
- b. Dynamic Assignment Method

Static traffic assignment is mainly focus the aspect of the transportation planning process that determines traffic loading on road network whereas dynamic assignment methods focus on the traffic control and management along with transportation planning and it use to generate time varying traffic flow of the transportation network that illustrate how congestion levels vary with time.

2. Literature Review

Traffic pattern is time dependent and vary with respect to time for example O-D pair and link flow vary with time [Zou et al. 2013] whereas static traffic assignment methods assume that the O-D demand is uniformly distributed over time to estimate the traffic pattern [Jaykishnan et al. 1995]. This method also cannot describe time dependent demand and dynamic flow characteristics so, it is very difficult to solve the congestion issue

and apply traffic management policies [Liu et al. 2005]. Traffic control system and development of traffic queues is important part of the route network which is not considered in the static traffic assignment efficiently where as it is considered in dynamic traffic assignment [Akamatsu, 1999]. Static Assignment models use traffic flow for travel time. A problem in using traffic flow as the variable is that the travel times do not follow a convex function with respect to flow. In reality, average speed decreases with respect to average flow and beyond the maximum flow, the flow decreases when speeds become very low resulting in a travel time (reciprocal of speed) function that turns back and reaches high values. In other words, it is incorrect to assume that there are low travel times at low flows, which is well-known to traffic engineers, as some of the worst travel times occur under low-flow, stop-and-go conditions [Jaykishnan et al. 1995]. However, static assignment is still dominated and most widely preferred tool for the strategic transportation planning due to its simplicity and computation efficiency ([Michiel, 2012] and also very fast and can handle very large networks [Michiel et al. 2013]. But they fail to capture the essential feature of traffic congestion and soon dynamic traffic assignment will replace the static assignment [Jeihani, 2007]

Traffic congestion is widely distributed in the network and it is dynamic in nature [Pei, 2003]. Dynamic Traffic Assignment (DTA) models are the only choice that describes the dynamic characteristics of the traffic and transportation network and captures time varying changes of traffic flow on the basis of time-dependent travel demand [Minngqiao et al. 2013]. The main characteristics of the DTA are that they capture spatio-temporal trajectory of each individual trip starting from origin to destination. The trajectory of each vehicle includes departure time, route choice and position of individual vehicle or trip [Mitsakis, 2011]. DTA capabilities required for evolution and operation of Intelligent Transportation System (ITS) such as Advanced Traveller Information Systems (ATIS) and Advanced Traffic Management Systems (ATMS) to address traffic congestion problems [Peeta, 1995]. DTA uses historical and real time data to estimate and predict the traffic conditions. Dynamic Traffic Assignment can analyze network performance characteristics under emergency

such as emergency evacuation and provide support for decision-making and it gives better result as compared to Static Traffic Assignment [Han, 2010]

3. Static Assignment

Static Traffic Assignment theory has been the basic framework not only for estimating traffic demands on particular network but also widely used for the problems of transportation planning and demand management policies decisions with respect to infrastructure investment [Akamatsu, 1999]. Although focus of research in traffic assignment is shifting towards the dynamic assignment, static assignment is still preferred tool for the strategic transportation planning due to its simplicity and computation efficiency [Michiel, 2012] and also very fast and can handle very large networks [Michiel et al. 2013].

There are various traffic assignment models as enlisted below and has been used for the various transportation planning and management.

- All or Nothing Assignment
- Stochastic Traffic Assignment
- Capacity Restrained Assignment
- Incremental Assignment
- User Equilibrium Assignment
- System Optimum Assignment

3.1 All or Nothing Assignment (AON)

AON is simplest type of traffic assignment is based on the principle that the route followed by traffic is one having the least travel resistance. In this method, out of three; travel cost, travel time, and travel distance, one parameter is considered as travel resistance. However, it is unrealistic because shortest route is not always preferable as commuters also take reliability and safety into consideration for choosing the route. It has one of the major limitations that capacity of link is not taken into consideration. Further, it has certain assumption those are unrealistic such as the travel time on links do not vary with the link flow and all trip makers has precise knowledge of the travel time on the links.

3.2 Stochastic Traffic Assignment Method

Stochastic methods of traffic assignment emphasize the variability in driver's perceptions of costs and com-

Literature Review of Traffic Assignment: Static and Dynamic

posite measure; they seek to minimize (distance, travel time and generalized costs). This assignment assumes the route choice is based on perceived travel times or costs rather than measured link travel time or costs. The travel times perceived by motorist are assumed to be random variable [Sheffi, 1985]. Therefore, in this methods need to consider second-best routes (in terms of engineering or modelled costs or time); this generates additional problems as the number of alternative second-best routes between each origin-destination pair may be extremely large. Stochastic models deal with the probability distribution of flow states and/or the expected flow state rather than the flow evolution trajectory [He et al. 2010].

3.3 Capacity Restraint Method

Capacity restraint method is resultant of improvement of AON assignment which also takes capacity of link into consideration. The capacity restraint procedure explicitly recognizes that as traffic flow of network increases from certain point the speed of traffic decreases. Capacity restraint is used in trip assignment by loading the network and adjusts assumed link speeds after each loading to reflect volume/capacity restraints. These loadings and adjustments are done incrementally until balance is obtained between speed, volume and capacity. The bureau of Public Roads (BPR) method is most suitable method to load the traffic on network. α and β value for each link may change from region to region even it changes from one area type to other, as well as from one link to another [Pulugurtha et al. 2010]. However, it is very difficult to determine the efficient value of α and β for each corridor. It also does not capture the driver behaviour as well as dynamic characteristics (speed, travel time and flow) of traffic flow. It is applicable to highly continuous traffic [He et al. 2013].

$$T_Q = T_o [1 + \alpha \frac{Q}{Q_{max}} + \beta \frac{Q^2}{Q_{max}^2}] \quad (ii)$$

Where,

T_Q = Travel Time at traffic flow Q

T_o = Free flow travel time

Q = Traffic Flow (Veh/hr)

Q_{max} = Practical Capacity

α, β = Parameters

3.4 Incremental Traffic Assignment

Incremental assignment is a process in which fixed proportions of total demand are assigned in steps based on all or nothing assignment. After each step link travel times are recalculated based on link volumes. When there are many increments used, the flows may look like an equilibrium assignment; however, this method does not yield an equilibrium solution. Consequently, there will be inconsistencies between link volumes and travel times that can lead to errors in evaluation measures. Incremental assignment model assumes that each trip maker chooses a path so as to minimize his / her travel time in addition that the travel time on the links vary with the flow on that link. Under such an assumption, the ideal way to assign traffic volume would be to assign a single trip to the road network assuming that the travel time on links during the assignment is constant. One could then update the travel times and repeat the process till all the trips are assigned. However, this procedure is not practical as any network would typically have a very large number of trips. Incremental assignment models therefore try to approximate this ideal process by dividing the total number of trips into few smaller parts and assign each part assuming a constant link travel time.

3.5 User Equilibrium Traffic Assignment

Traffic equilibrium models are commonly in use for the prediction of traffic patterns on transportation networks that are subject to congestion phenomena. User Equilibrium Assignment is based on the Wardrop's first principle as stated as "*It states that the traveller time between a specified origin and destination on all used routes is equal and less than or equal to the travel time that would be experienced by a traveller on any unused route.*"

User equilibrium assignment takes congestion into consideration. However, user equilibrium condition can only be achieved in an artificially small or virtually uncongested network; for a highly congested transportation network, equilibrium can only be closely estimated. The model does not include traffic control as a variable and further it has certain assumption those are unrealistic in nature such as each driver wants to choose the path between their origin and their destination with

the least travel time, drivers have perfect knowledge of link travel times.

3.6 System Optimum Assignment

Wardrop also proposed an alternative way of assigning traffic into a network and this is usually referred to as Wardrop's second principle as stated as "*Under equilibrium conditions traffic should be arranged in congested networks in such a way that the total cost (all trips) is minimized.*"

In contrast with his first principle of Wardrop's, that endeavours to model the behaviour of individual drivers. The second principle is oriented to the organization of traffic to minimize travel costs and therefore to achieve an optimum social equilibrium [Willumsen, 2000]. Although the latter is often held to be more descriptive of traveller's behaviour in traffic networks, System Optimum (SO) assignment is also highly useful: it forms a key component of many first best pricing models, it provides a lower bound on network congestion, provides indications for how networks should be improved, and is applicable in network problems where the routed assets can be centrally controlled.

However, SO traffic assignment assumes deterministic network conditions including a fixed capacity on every link. In reality, incidents, weather, and other phenomena result in considerable variability in roadway capacity and system congestion. At the same time, travellers can learn about the system state as they traverse the network, either through the use of technology or through their own observations [Unnikrishnan, 2008]

3.7 Limitations of Statistic Methods

The assignment methods described so far have a number of natural limitations. It is worth mentioning these limitations here in order to give a feeling of what can be expected from assignment to networks. Only the most important limitations are outlined below,

- In the Static Traffic Assignment the network is not modelled in detail. There are "end effects" due to the aggregation of trip ends into zones represented by single centroids; banned turning movements are not specified in the networks; and inter-zonal trips are ignored in the assignment.
- The assumption of perfect information about

costs in all parts of the networks. Although this is common to all models it is essentially unrealistic. Drivers only know partial information about traffic conditions on the same route last time they used it and on problems in order parts of the network as filtered by their own experience, disposition to explore new routes and the reliance on traffic information services.

- Day-to-day variations in demand. These prevent true equilibrium ever being reached in practice. In that sense, Wardrop's equilibrium represents only "average" behaviour. While its solution has enough desirable properties of stability and interpretation to warrant its use in practice, it is still only an approximation to the traffic conditions on any one day.

4. Dynamic Traffic Assignment

4.1 General

Traffic congestion is widely distributed around the network and its intensity varies with respect to time and space due to variation in travel demand, resulting into time varying link or path flows. Generally, Static Traffic Assignment (STA) is used to analyze congestion but it is unable to capture the dynamic characteristics of traffic [Pei et al. 2003]. Dynamic characteristics of traffic include varies travel time, varies traffic flow, and capacity of link or path. Now days, Dynamic Traffic Assignment (DTA) is used to generate forecasts of traffic that shows congestion levels very with time and useful for traffic control and management. It has attracted significant attention in the last decade in the areas of transportation system such as advanced traffic management system (ATMS) and advanced traveller information systems (ATIS), telecommunication and computer science as well as emergency planning [Ann et al. 2013]. A dynamic traffic system can provide real-time traffic simulation to guide or assist pre-trip and en route travel decisions.

The properties of dynamic traffic assignment (DTA) have important contribution on the theoretical advances and computational issues on transportation planning and operations. These properties depend strongly on the two components of DTA: the travel choice principle and the traffic-flow component. The travel choice principle models travellers' tendency to travel, and if so, how they select their routes, departure times, modes,

Literature Review of Traffic Assignment: Static and Dynamic

or destinations [Szeto [et al. 2005]. Traffic model represents temporal link traffic flow and link travel time variations. Dynamic Traffic Assignment (DAT) frameworks are necessary to predict the traffic patterns and congestion formation in the networks so that traffic route guidance/information scheme can be implemented [Jayakrishanan et al. 1995].

4.2 Dynamic Assignment Approach

With the behavioural assumption of individual routing decisions, the DTA problem can be classified into two categories as below Dynamic System-Optimal assignment (DSO), in which the total travel cost in the network is minimized and Dynamic User Equilibrium assignment (DUE) is a temporal generalization of the Static User Equilibrium (SUE) problem with additional constraints to insure temporally continuous path of flow [Janson., 1991], in which the individual chooses a route that, minimizes his travel cost.

DTA models can be classified into two categories; Analytical Based Models and Simulation Based Model [Peeta and Zilaskopoulous, 2001].

4.2.1 Analytical Based Models

Analytical based models are those models which rely on analytical formulations such as mathematical programming, variational inequalities and optimal control theory [Peeta and Zilaskopoulous, 2001]. Most analytical formulations are extensions of their equivalent and seem have two main disadvantages (1) they cannot adequately capture the realities of street network due to simplification (2) they tend to be inflexible for realistic size networks [Zilaskopoulous et al. 2004].

- **Mathematical programming** - Janson in his study applied discretised time interval techniques for mathematical programming. His mathematical programming model is a direct time sliced generalization of the path-flow formulation of the static user equilibrium models, without any attempt at describing the dynamics of link flows, and with additional side constraints to force the temporal consistency of the flows. The substantial research in mathematical programming based DTA approaches highlights its current limitations for developing deployable models for general networks. A persistent issue is the need to trade-off mathemati-

cal tractability with traffic realism. In addition, general mathematical programming DTA formulations tend to have difficulties related to

- i. The use of link performance and/or link exist functions
- ii. Holding-back of traffic at the minor approach at an intersection in favour of major approach
- iii. Efficient solutions for real time deployment in large scale traffic networks
- iv. A clear understanding of solution properties for realistic problem solutions

- **Variational Inequalities (VI)** – Variational inequality theory provides us with a tool for:

- i. Formulating a variety of equilibrium problems;
- ii. Qualitatively analyzing the problems in term of existence and uniqueness of solutions, stability and sensitivity analysis, and
- iii. Providing us with algorithms with accompanying convergence analysis for computational proposes.

The VI approach is more general than other analytical approaches, providing greater analytical flexibility and convenience in addressing various DTA problems. It has been used to illustrate, with relative ease, the notion of experienced travel times for the so-called “simultaneous” and ideal UE DTA problems. However, VI approaches are more computationally intensive than optimization models, raising issues of computational tractability for the real-time deployment.

- **Optimal Control Theory** – This approach uses control variable to optimize the functional. Functional is defined as function of function. A variable quality as a control variable if, the variable is subject to the optimizer’s choice, and the variable will have an effect on the value of the state variable of interest. [Ran and Shimazaki, 1989 b] present an optimal control theory based user equilibrium dynamic traffic assignment model. The flow existing links are treated as set of control variables rather than functions to avoid the generalization issue.

4.2.2 Simulation Based Models

The term simulation based mainly refers to the solution method rather to the problem formulation [Peeta and Zilaskopoulous, 2001]. Simulation based DTA models

use a traffic simulator to replicate the traffic-flow dynamics, critical for meaningful strategies for operating. The critical constraints that describe traffic flow propagation and spatial-temporal interactions, such as the path-link incidence relationships, flow conservation and vehicular movements are addressed through simulation instead of analytical [Han et al. 2010]. This is because analytical representations of traffic flow that adequately replicate traffic theoretic relationship and yield well behaved mathematical formulations are currently unavailable. Simulation provides the convenient tool for modelling complex dynamic phenomena thus overcoming the difficulties which are associated with the use of analytical mathematical formulations [Mitaskis et al. 2011]. A key issue with simulation based method is that theoretical insights cannot be analytically derived as the complex traffic interactions are modelled using simulations. But due to their better reliability vis-à-vis realistic traffic modelling, simulation based models have gained greater acceptability in the context of real world deployment [Mahasammi et al. 1998].

The key advantages of using simulation-based DTA tools for planning applications include: [Sundaram et al. 2011]

- * Simulation-based DTA systems capture the time-dependent interactions between the demand for the network and the supply of the network.

- * Simulation-based DTA systems are able to predict the locations and impacts of traffic congestion by modelling the dynamic nature of the network and explicitly capturing critical aspects such as congestion build-up, queues, spill-backs and congestion dissipation.

- * Simulation-based DTA systems are able to capture the effects of segment-level operational changes such as ramp meters and traffic lights by incorporating the operation control logic (through its impact on link capacities) into the representation of the supply simulator.

- * Simulation-based DTA systems can model effectively various ITS strategies, in particular ATIS/ATMS strategies and the impact of information. This is possible by incorporating rich traveller behaviour models to represent individual travel behaviour and simulation to model traffic dynamics at the required level of detail.

- * Simulation-based DTA systems can represent travel choices in great detail

5. Conclusion

Static Traffic Assignment models are used for predicting the route choice of commuters but do not cover the dynamics of traffic flow and its time dependent characteristics. In this model, the traffic networks are assessed by predicting long term average steady state supply and demand from day to day and different periods of day and link travel cost function is used which depend only on the traffic flow on this link without taking into account the effects on the traffic flow of other links. Therefore, great difficulties have been faced in the representation and evaluation of real time policies such as congestion management strategies, dynamic route guidance/information and Variable Message Sign (VMS).

Dynamic Traffic Assignment (DTA) models are used for estimating and predicting time dependent traffic condition in road networks. The main characteristics of DTA models are that they are able to capture the spatial-temporal trajectory of each individual vehicles starting up from the origin up to destination. The trajectory of each individual vehicle includes the departure time from the origin, chosen route choice of each individual vehicle and position of each individual vehicle along the its route for each time step. Therefore, dynamic user equilibrium is most suitable method for the dynamic traffic assignment. However, it is complex method due to huge computational requirements. There are two approaches have been developed so far; Mathematical or Analytical Approach and Simulation Based Approach. Mainly three approaches are available for the mathematical formulation of Dynamic User Equilibrium Traffic Assignment. Those are optimization approach, control theory approach and variational inequality approach. Present research indicates that the variational inequality approach is most suitable for the mathematical formulation of dynamic user equilibrium traffic assignment because this approach provides greater analytical flexibility and convenience in addressing various DTA problems. However, mathematical approach is complex approach because it requires great computational effort to cover important aspect of DTA.

Simulation provides the convenient tool for modelling complex dynamic phenomena, thus overcoming the difficulties which are associated with the use of analytical mathematical formulations. Simulation based DTA

Literature Review of Traffic Assignment: Static and Dynamic

models handle the flow of traffic and spatial and temporal interaction with the use of simulation model instead of an analytical mathematical formulation. A main aspect of simulation based DTA models is the use of traffic simulator for simulating traffic conditions especially in networks where traffic exists and where the application of mathematical formulation is difficult for capturing the dynamic aspects of traffic. Principally, it utilizes an iterative procedure between the traffic simulator, the time dependent shortest path computation procedure and the dynamic network loading procedure.

6. References

- Akamatsu, T. (2000) "A dynamic traffic equilibrium assignment paradox", *Transportation Research Part B* 34, PP. 515-531
- Anna, C. Y. L., Nozick, Linda, Davidson R., Brown N. , Jones D. A. and Wolshon B. (2013) "Approximate Solution Procedure for Dynamic Traffic Assignment" *Journal of Transportation Engineering, ASCE*, Vol. 139, No. 8
- Codina, E. and Barcel, J. (1995) "Dynamic traffic assignment: considerations on some deterministic modelling approaches", *Annals of Operations Research* 60, pp.1-58
- Han, S., Fang, S., Wang, X., Chen, X. and Cai, Y. (2010) "A Simulation-based Dynamic Traffic Assignment Model for Emergency Management on the Hangzhou Bay Bridge", *Integrated Transportation Systems – ASCE*, pp. 883-95
- He, X., Guo, X. and Liu, H. X. (2010) "A link-based day-to-day traffic assignment model", *Transportation Research Part B* 44, pp.597–608
- Jayakrishnan, R., Tsai, W. K. and Chen, A. (1995) "A dynamic traffic assignment model with traffic-flow relationships", *Transpn. Res.-C*, Vol. 3, No. 1, pp. pp.51-72
- Jeihani, M. (2007) "A review of dynamic traffic assignment computer packages", *Journal of the Transportation Research Forum*, Vol. 46, No. 2, pp. 35-46
- Lafortune, S. and Sengupta, R. (1993) "Dynamic system-optimal traffic assignment using a state space model", *Transpn. Res-B*. Vol. 27B, No. 6, pp. 451-472
- Lam, W. H. K. and Huang, H. J. (1994) "Dynamic user optimal traffic assignment model for many to one travel demand" *Transpn. Res.-B*. Vol. 29B, No. 4, pp. 243-259
- Leeuwen, A. S. (2011) "Static traffic assignment with queuing", Unpublished Master Thesis Applied Mathematics, University of Twente
- Liu, R., Van Vleet D. and Watling D. P., (2006) "Microsimulation models incorporating both demand and supply dynamics", *Transportation Research A*, 40(2), pp. 125-150.
- Mitsakis, E., Maria Salanova, J. and Giannopoulos G. (2011) "Combined dynamic traffic assignment and urban control models", *Procedia a Social AND Behaviour Sciences* 20, ELSEVIER, pp. 427-436.
- Peeta S. and Mahmassani, H. S. (1995a) "Multiple user classes real-time traffic assignment for online operations: A rolling horizon solution framework", *Transpn. Res.-C*, Vol. 3. No. 2. pp. 83-98
- Peeta S. and Mahmassani H. S. (1995b) "System optimal and user equilibrium time-dependent traffic assignment in congested networks" *Annals of Operations Research* 60, pp. 81 – 113
- Pei Y. and Lang, Y. (2003) "Research on dynamic traffic assignment applied in traffic congestion analyze and management" *IEEE*, PP. 1032-35
- Sheffi, Y. (1985) "Urban transport networks: equilibrium analysis with mathematical programming methods", Prentice-Hall, INC., Englewood Cliffs, New Jersey 07632
- Shin, M. S., Oh, R. S. and Choi, K. (1999) "A path-based analytical dynamic traffic assignment model for real time evaluation of simulation based dynamic traffic

-Sundaram, S., Koutsopoulos, H. N., Ben-Akiva, M., Antoniou C. and Balakrishna, R. (2011) “Simulation-based dynamic traffic assignment for short-term planning applications” *Simulation Modelling Practice and Theory* 19, pp.450–462

-Szeto, W.Y. and Lo, H. K. (2005) “Properties of dynamic traffic assignment with physical queues”, *Journal of the Eastern Asia Society for Transportation Studies*, Vol. 6, pp. 2108 – 2123

-Szeto, W.Y. and Lo, H. K. (2006) “Dynamic traffic assignment: properties and extensions”, *Transport Metrica*, Vol. 2, No. 1, pp.31-52

-Willumsen, L. G. (2000) “Chapter 10: Travel networks” “*Handbook of transport modelling*”, Edited by D.A Hensher and K. J. Button, Elsevier Science Ltd.

-Ziliaskopoulos, A. K., Walle, S. T., Li, Y. and Byram, M. (2004) “Large-scale dynamic traffic assignment: implementation issues and computational analysis”, *Journal of Transportation Engineering, ASCE* Vol. 130, No. 5

-Zou, M., Chen, X., Yu, H., Tong, Y., Huang, Z. and Li, M. (2013) “dynamic transportation planning and operations: concept, framework and applications in China” *Elsevier Procedia - Social and Behavioral Sciences* 96, pp. 2332 – 2343