

Capacity Drop Estimation Based on Stochastic Approach Applied to Tehran-Karaj Freeway

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Abstract:

Existence of capacity drop phenomenon, as the difference between pre-queue and queue discharge flow rates, has been one of the controversial concepts of traffic engineering. Several researches have focused on capacity drop existence and also its estimation issues. This paper aims to estimate capacity drop based not only on a comparison between breakdown and queue discharge flow rates, but also on the estimation of the capacity distribution function before and after breakdown. In the empirical case, speed and flow rate data are collected in a section of Iran's most crowded freeway for four months, based on which the threshold speed as the boundary between congested and non-congested flow is determined, and breakdown flow rates and their subsequent queue discharge flows are detected. Paired t-test between pre-queue and queue discharge flow rates is conducted to find the mean difference. Also, the distribution function of capacity under non-congested and congested flow is estimated using maximum likelihood and product limit methods. Based on the 11,600-record data set, it was observed that end results of both methods are consistent, revealing roughly five percent drop in capacity for the section under investigation.

Keywords: Capacity drop, stochastic approach, probability distribution function

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1. Introduction

Capacity of a freeway as an important characteristic of the facility is found to have a random nature due to variable prevailing (roadway, traffic and control) conditions, environmental conditions, and changing drivers' characteristics. Consequently, capacity is more comprehensively understood while investigated in light of stochastic definition. In the stochastic definition, capacity of a single road is treated as a random variable as it is perpetually changing. Random variables are generated from populations and each population is better recognized when its corresponding probability distribution function is estimated. Estimation of capacity distribution function as a powerful quantitative measure helps reaching a more thorough understanding of capacity which may result in better design and management of facilities.

Several researchers ([Brilon, Geistefeldt and Regler, 2005], [Banks, 2006], [Hall and Agyemang-Duah, 1991], [Chung, Rudjanakanoknad and Cassidy, 2007], [Oh and Yeo, 2012] and [Persaud, Yagar and Brownlee, 1998]) have reported the presence of two different capacities in freeway facilities; one before the breakdown under non-congested traffic condition and the other after the breakdown under congested traffic condition. These two capacities are called "pre-queue capacity" and "queue discharge capacity", respectively. Pre-queue capacity is usually reported to have a higher value than the queue discharge capacity. This difference is called "capacity drop" or "two capacity phenomenon". Several reasons have been hypothesized to cause or intensify this phenomenon:

1- Driving behavior before and after the formation of the queue: Drivers tend to accept smaller headways before formation of queue in comparison with the time that queue is discharged. It happens because the drivers have given up the idea of passing their front vehicles after the formation of the queue. As a result, number vehicles in queue discharge flow decreases [Brilon, Geistefeldt and Regler, 2005].

2- Number of lanes: As the number of lanes increases, amount of capacity drop decreases. This is because in freeways with numerous lanes, shoulder lanes rarely reach the capacity and consequently difference between the overall capacities before and after the breakdown

decreases [Oh and Yeo, 2012].

3- Location that capacity is measured: Detecting capacity upstream of the bottleneck instead of the bottleneck itself might result in over estimation of the capacity drop. This means that it is possible that the section cannot operate at its real capacity in congested condition due to presence of a queue downstream and the value that is regarded as the queue discharge flow (later compared with the pre-queue flow) is smaller than the real capacity of the measurement section within the flow. Thus, it is usually recommended to measure capacity at the bottleneck itself. On the other hand, measuring the capacity in freeway sections located upstream of two-lane off-ramps tends to alleviate the capacity drop.

Highway capacity manual (2000) proposes an average of five percent capacity drop [Highway Capacity Manual, 2000], while different researchers have gained other values. For instance, Banks [Banks, 2006] has reported three percent; Hall and Agyemang-Duah [Hall and Agyemang-Duah, 1991] reported 5.8 percent; and some other researchers (e.g. [Hall and Hall, 1991] or [Maze, Schrock and Kamyab, 2000]) did not observe any capacity drop. In short, capacity drop represents the importance of preventing transition of flow from non-congested to congested condition (e.g. by ramp metering [Banks, 2006] and [Zhong-zhong, 2006]).

Existence of capacity drop is not only a debatable topic in traffic engineering, but different researchers have reported different drop values. This difference emerges from both the sections under investigation and the methodologies used to quantify the drop. As a result, the current study aims to estimate capacity drop using two different methods for a section of Iran's busiest freeway with 11,600 records of data¹ gathered during four months of observation.

This paper is presented in five sections: to give a better insight into different perspectives of capacity drop phenomenon, a review of the literature is made in the next section. The third section is devoted to the methodology used in the current study, followed by the study section characteristics. Next, implementation results of the methodology on the study section are presented to probe the presence of capacity drop. Finally, findings are summarized and some recommendations are given for future research.

2. Capacity Drop Literature Survey

Banks (1990) selected a freeway bottleneck located at San Diego and collected traffic data during 23 morning peak periods. He measured traffic flow characteristics with loop detectors immediately upstream of the bottleneck and used video cameras to identify condition of the flow. Considering the flow conditions, of the 23 days investigated, nine days were recognized appropriate for flow analysis purpose. Banks reported that the queue formed in one lane and grew into other lanes and found that distribution of the flow was more uniform after the formation of the queue. By drawing frequency polygons of the mean 30-second counts and comparing the mean counts before queue formation and after that, he found that the average flow across all lanes as well as left lane decreased when the queue formed. Using a test for significance of variances of difference between two means with different variances, he came to the result that on average a 3 percent drop in the capacity occurs when the queue forms and proposed that this concept could be used for the ramp metering purpose [Banks, 2006].

Hall and Agyemang-Duah (1990) estimated capacity drop by gathering data of commuter traffic for a station located on Queen Elizabeth Way in Toronto, Ontario. By comparing unstable and stable branches of flow-occupancy diagram at two sections of the freeway, they depicted the wrong premise in detecting a place to measure the capacity drop and proposed that capacity drop should only be measured at the bottleneck itself and not upstream of the bottleneck. After omitting flow rates obtained in rainy days and holidays, and exceptionally low peak periods, they estimated capacity drop based on the flow rate data of the 20 remaining days. By performing two test statistics, they found 5.8 percent difference between the pre-queue flow and the queue discharge flow. They also found that queue discharge flows are near-normal distributed for queue discharge flow [Hall and Agyemang-Duah, 1991].

Persaud et al. (1998) examined three bottleneck sites, one located in Gardiner expressway and two located in highway 401 Toronto bypass. They collected speed and flow rate data in 20 second intervals and aggregated them into five minute intervals to calculate capacity drop. With regard to drop in the speed, breakdown

and queue discharge times and their corresponding flow rates were determined visually. By comparing breakdown flow rates with queue discharge flow rates they reached capacity drop ranging from 10 to 26 percent. In addition, by aggregating flow rates into one minute intervals and categorizing them into different groups and dividing frequency of breakdown flow rates to non-breakdown flow rates in each group, authors proposed a preliminary model indicating probability of breakdown versus flow rate. The same model was made using 3, 5, 10, and 15 minute intervals and the researchers found that for the same flow rate, probability of breakdown raises as the aggregation interval increases [Persaud, Yagar and Brownlee, 1998].

Brilon et al. (2005) conducted a thorough research on stochastic concept of capacity in German freeways. The researchers gathered speed and flow rate data in five minute intervals and took advantage of the analogy between life time data analysis and capacity analysis to estimate capacity distribution function. They considered non-congested and breakdown flow rates respectively as censored and uncensored observations and used Product Limit Method and Maximum Likelihood Estimation Method to estimate non-parametric and parametric distribution functions. They found Weibull distribution as the distribution type that provided the best fit to their observations. Moreover, regarding congested and recovery flow rates as censored and uncensored observations, the authors estimated distribution functions of queue discharge flow and compared them with those of the free flow based on the median value to estimate the capacity drop. They found different capacity drop values in each of the three lane sections with an average was 1,180 veh/h [Brilon, Geistefeldt and Regler, 2005].

El-Metwally and Rakha (2009) investigated capacity drop phenomenon through simulation model using INTEGRATION software. The researchers connected one origin and one destination zone to each other through two links with a node in between. Then, they planned two scenarios. In the first scenario, vehicles were allowed to flow between the zones freely with demands of 1800 and 2300 veh/h. Using ten loop detector stations placed across the second link, the researchers measured the capacity in the free flow around 1800 and 2300 veh/h respectively. In the second scenario, an active stop sign

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was placed on the node and vehicles were forced to decrease their speed to reach full stop and then accelerate back again. Once again, flow rate was measured with loop detector stations. This time the maximum queue discharge flow for both demand volumes was much less, around 780 veh/h. The results reveal more than 50 percent capacity drop which is high in comparison with literature [El-Metwally and Rakha, 2009].

Oh and Yeo (2012) used the data gathered through PeMS system for 16 on-ramp merging bottleneck locations with different number of lanes in California and introduced a systematic macroscopic methodology to analyze capacity drop. In bottlenecks under investigation, speeds at downstream and upstream sections were measured and the condition of flow was determined afterwards. Taking the maximum five minute flow during free flow as the capacity and the five minute flow that traffic was most stable after bottleneck activation as the discharge flow, the researchers estimated capacity drop in each of the sections. By comparing the capacity drops reached in sections with different number of lanes, they found that as the number of lanes increased from two to five, capacity drop decreased from 16.33% to 8.85%. In addition, with analysis of different lanes it was found that capacity drop in the shoulder lanes tend to decrease in four and five lane highways. Also, it was observed that existence of two lane off-ramp sections alleviate the capacity drop in three lane highways [Oh and Yeo, 2012].

Different perspectives of capacity drop as one of the most debatable concepts of freeway capacity has been investigated from different points of view. However, not so many research have been made to estimate the capacity drop with regard to comparison of non-parametric and parametric distribution functions of capacity

before and after the breakdown. Thus, the focus of this study is to 1) estimate the difference between breakdown and recovery flow rates using paired t-test 2) estimate both non-parametric and parametric distribution functions of capacity with analogy to life time data analysis and compare them to the results of the paired t-test 3) estimate the overall capacity distribution function based on Markov Chain approach.

3. Methodology

In this paper, capacity is treated in its broad term as a random variable with a stochastic distribution function: “the rate of flow (expressed in pcphpl and specified for a particular time interval) along a uniform freeway segment corresponding to the expected probability of breakdown deemed acceptable under prevailing traffic and roadway conditions in specified direction”² [Lorenz and Elefteriadou, 2000].

Breakdown as an important ingredient in this definition is the transition from non-congested to congested flow, while transition from congested to non-congested flow is defined as recovery. A quantitative measure to detect breakdowns and recoveries in traffic flow, is “threshold speed (v_t)” which constitutes the area in speed-flow diagram where upper and lower branches meet (if speed-flow diagram is discontinuous, it can be estimated from the vacant area between the two branches). The four possible cases arising from a comparison of speeds of two consecutive intervals with threshold speed are presented in Table 1 along with example time intervals in Figure 1.

The two approaches used in this paper to estimate ca-

Table 1. Different flow conditions definition and example

case	(speed relative to threshold speed (v_t) at two consecutive intervals)		Flow condition at interval i	Time interval example in Fig 1
1	$v(i) > v_t$	$v(i+1) > v_t$	non-congested	to 6:15 5:45
2	$v(i) > v_t$	$v(i+1) \leq v_t$	breakdown	6:15
3	$v(i) < v_t$	$v(i+1) < v_t$	congested	to 14:30 6:30
4	$v(i) < v_t$	$v(i+1) \geq v_t$	queue discharge	14:30

capacity drop are: 1- paired t-test between breakdown and queue discharge flow rates to find the mean difference, and 2- comparison of distribution functions of capacity before and after breakdown (i.e. under non-congested and congested flow rates) estimated by life time science.

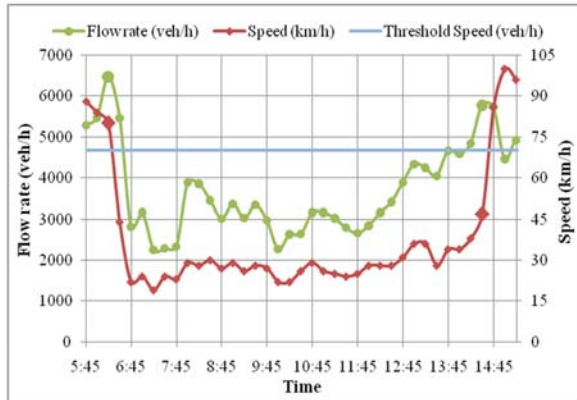


Figure 1. Time series plot of speed and flow rate³

3-1 Non-Parametric Estimation of Capacity Distribution Function

Product Limit Method (Kaplan-Meier Estimator), a non-parametric method, estimates a discontinuous cumulative distribution function for survival of individuals, with a basic form of [Kaplan and Meier, 1958]:

$$\hat{S}(t) = \prod_{i:t_i < t} \left(\frac{n_i - d_i}{n_i} \right) \quad (1)$$

where:

- $\hat{S}(t)$ = estimated survival function of time t ,
- n_i = number of individuals with a lifetime $T \geq t_i$, and
- d_i = number of deaths at time t_i .

Based on an analogy drawn between parameters of life time data analysis and roadway capacity analysis (by Brilon, Geistefeldt and Regler, 2005), non-congested flow rates are considered as censored observations (i.e. duration of observation has finished but freeway has not reached capacity to produce the target phenomenon of transition between different states) and breakdown flow rates are regarded as uncensored (i.e. freeway has reached capacity and thus can be used as useful data revealing transition between different states), while congested and queue discharge flow rates are disregarded (i.e. they give no information about capacity in free flow). Opposite steps are followed to estimate distribu-

tion function of capacity after breakdown, where congested flow rates are considered as censored and queue discharge flow rates as uncensored observations (non-congested flow rates are disregarded).

This analogy and use of survival complement function, result in capacity distribution function as:

$$F_c(q) = 1 - \prod_{i:q_i < q} \left(\frac{k_i - d_i}{k_i} \right)$$

where:

- $F_c(q)$ = capacity distribution function,
- q = traffic volume [veh/h],
- q_i = traffic volume in interval i [veh/h],
- k_i = number of intervals with a traffic volume $q_i \leq q$, and
- d_i = number of breakdowns at volume q_i .

A complete distribution function is possible only if maximum observed volume is followed by breakdown/recovery. Else, it would be impossible to reach a complete distribution function as the function will terminate in a value less than one. Consequently, a complete capacity distribution function is rarely reached; and even if reached, it might not be so reliable in higher volumes, unless a huge size of data is gathered.

3-1 Parametric estimation of capacity distribution function

Estimation of parametric distribution function has the advantage of reaching an equation for capacity distribution function which allows applying capacity in more practical ways. Maximum Likelihood Estimation (MLE) is an applicable method for estimating parameters of models with dichotomous response variable (i.e. here censored and uncensored values). MLE criterion for estimation is the most likely probability density function (from among all) that replicates observations [Myung, 2002]. Different distribution types are compared based on their Likelihood value and the one with the maximum likelihood value is selected. Transformed to capacity analysis, Likelihood Function (L) could be written as [Washington, Karlaftis and Mannering, 2003]:

$$L = \prod_{i=1}^n f_c(q_i)^{\delta_i} \cdot [1 - F_c(q_i)]^{1-\delta_i} \quad (3)$$

where:

- $f_c(q_i)$ = density function of capacity at interval i ,
- $F_c(q_i)$ = cumulative distribution function of capacity at interval i .

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interval i ,

n = number of intervals, and

$\delta_i = 1$, if the interval i is uncensored; and 0, if the interval i is censored.

For more convenience, Log-Likelihood Function is maximized instead (due to their monotonicity) and different density (or cumulative) functions are evaluated and compared based on its values. Transformed into capacity analysis, Log-Likelihood function is written as:

$$\ln(L) = \sum_{i=1}^n \{\delta_i \cdot \ln[f_c(q_i)] + (1 - \delta_i) \cdot \ln[1 - F_c(q_i)]\} \quad (4)$$

3-2 Estimation of the overall capacity distribution function

Wu (2004) proposed equation 5 based on Markov Chain (a mathematical random process for transitions from one state to another in a way that next state is predicted only from information of current state) to estimate overall capacity of freeway considering both non-congested (free) and congested conditions [Wu, 2004]:

$$\begin{pmatrix} P_{free}(q) \\ P_{cong}(q) \end{pmatrix} = \begin{pmatrix} 1 - P_{br}(q) & P_{re}(q) \\ P_{br}(q) & 1 - P_{re}(q) \end{pmatrix} \begin{pmatrix} P_{free}(q) \\ P_{cong}(q) \end{pmatrix} \quad (5)$$

Thus, the probability of having a congested flow equals the sum of probability of a free flow traffic multiplied by transition probability from free to congested flow (P_{br}), and the probability of a congested traffic multiplied by probability of remaining in congested condition (i.e. not being followed by recovery: P_{re}). Hence, by replacing $P_{re}(q)$ with $1 - P_{cong}(q)$ and solving the above equation, Wu derived overall capacity distribution function ($F_{C,overall}(q)$) as:

$$F_{C,overall}(q) = P_{cong}(q) = \frac{P_{br}(q)}{P_{br}(q) + P_{re}(q)} = \frac{F_{C,br}(q)}{F_{C,br}(q) + F_{C,re}(q)} \quad (6)$$

4. Tehran-Karaj Freeway Section

To implement empirically the theoretical foundations of capacity drop estimation based on a stochastic approach, a section on Tehran-Karaj suburban freeway (the oldest and busiest freeway of Iran with an ADT of more than 90,000 passenger cars) with regular breakdowns during the afternoon peak hour was selected (Figure 2). To estimate reliable capacity distribution functions based on Product Limit Method and determine the threshold speed from the speed-flow scatterplot, it is necessary to

select an appropriate section to insure sufficient number of breakdowns. Most users are commuters driving to Tehran in the morning and returning home (Karaj) in the afternoon. In both directions, the freeway has three lanes with a width of 3.65 meters each and a shoulder width of 2.5 meters; and posted speed limit of 120Km/h. The study section is 25 kilometers from Tehran and located 445 meters downstream of an off-ramp section with an on-ramp 1260 meters upstream.

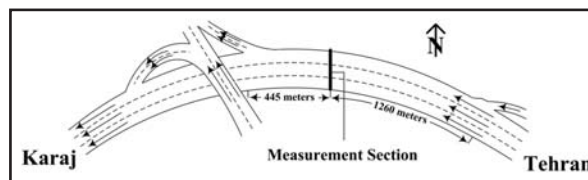


Figure 2. Measurement section in Tehran-Karaj freeway

The study section is located on a horizontal curve of 2,900 meters radius with a 0.6 percent downgrade slope, which may cause breakdowns. As the section of interest is only 445 meters upstream of the off-ramp, it may not be considered as a classic basic freeway section. On the other hand, breakdowns rarely occur on basic freeway sections without any interruptions on the flow. In fact, the best place to analyze capacity of basic freeway sections is immediately at the on-ramp where the additional on-ramp volume causes a breakdown. As no such data was available, current section is selected. Moreover, no specific spillback from the available downstream detectors was observed during the study period.

Data used in this paper is collected by advanced video cameras of Iran Road Maintenance & Transportation Organization equipped with built-in image processing technology allowing them to calculate space mean speed directly. Speed and flow rate data of passenger cars are collected and aggregated in 15-minute intervals for four months.

5. Results and Discussion

Threshold speed as an important parameter of capacity drop estimation is estimated by dividing upper and lower branches of the speed-flow diagram. As indicated in the speed-flow scatter plot of Figure 3, the boundary between free and congested situations suggests a threshold speed of 70 km/h, based on which,

231 breakdown flow rates are detected. Breakdowns occurring due to incidents, accidents and work zones are neglected as such events reduce the capacity of the freeway (as outlying points).

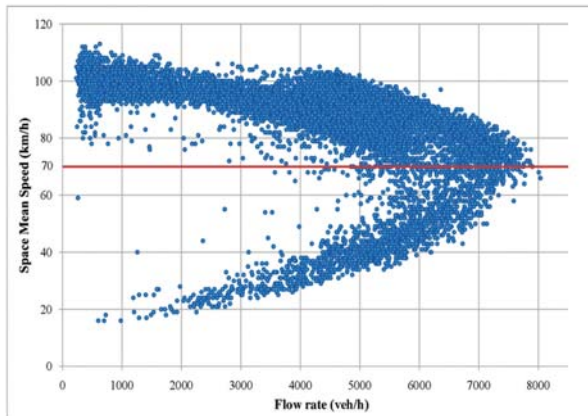


Figure 3. Speed-Flow diagram [Shojaat, 2012]

For a three-lane freeway, blockage of one and two lanes, respectively, may decrease the capacity up to 63 and 77 percent [Qin and Smith, 2001]. Hence, since no specific event (such as accident or incident) is reported, breakdowns occurring in flow rates less than 3600 veh/h (i.e. 1200 veh/h/lane) are disregarded as they are probably due to unreported events [Geistfeldt and Brilon, 2009]. Moreover, only breakdowns followed by recovery are used in capacity estimation, because as a result of discontinuity in data, no records about their queue discharge flow rates were available. After the data cleaning process, 204 breakdowns were identified as reliable and suitable for the purposes of this research, based on which congested, non-congested and queue discharge flow rates are detected (Figures 4 and 5).

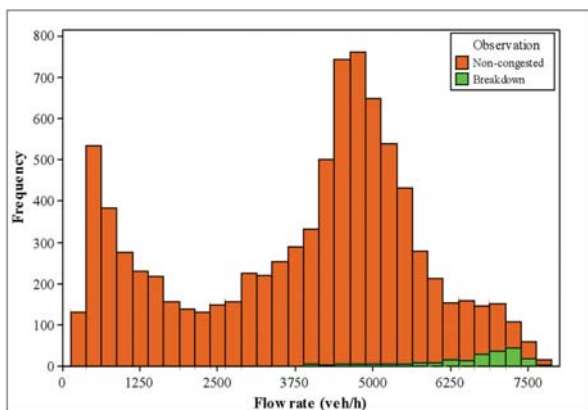


Figure 4. Frequency of the flow rates in the free flow

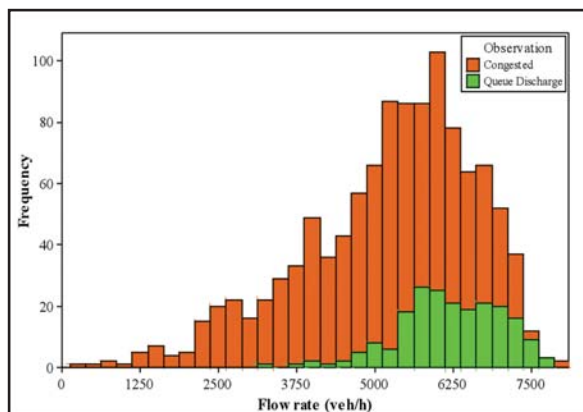


Figure 5. Frequency of the flow rates in the congested flow

5.1 Estimating Capacity Drop Based on Paired T-Test

Paired t-test between breakdown and queue discharge flow rates reveals a mean difference of 333 veh/h, resulting in capacity drop of 5.1 percent (Table 2). Based on the 204-record dataset, breakdown flow of 6530 veh/h falls to 6197 veh/h at queue discharge flow, whose difference is seen to be statistically significant.

Table 2. Results of paired t-test

Ob- serva- tion Pairs	Break- down Flow	Queue Dis- charge Flow	Mean Differ- ence	Std. Devia- tion	T- Val- ue	95% Confidence Inter- val
204	6529.5	6196.8	332.7	877.2	5.42	,211.6 (453.8)

5.2 Estimating Capacity Drop Based on Capacity Distribution Function

The other method involves comparison of capacity distribution functions in non-congested (free) and congested flow. Non-parametric distribution function is estimated using Product Limit Method without any assumption regarding type of capacity distribution function, whereas parametric distribution function is estimated with Maximum Likelihood, with such an assumption. Different functions like Normal, Gumbel, Weibull and Logistic are assumed and calibrated to select the best fit based particularly on their Log-Likelihood values. Table 3 shows the estimated parameters for each of the distribution functions before (breakdown distribution function) and after (queue discharge distribution function) the breakdown. Gumbel distribution is

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the function with best fit in non-congested flow, while Logistic, Weibull, and Normal distributions follow in rank, respectively. However, in the congested situation, Weibull distribution provides the best fit, followed by Gumbel, Logistic and Normal distributions. Figures 6 and 7 demonstrate capacity distribution functions estimated with Product Limit Method and Maximum Likelihood Estimation Method in both conditions.

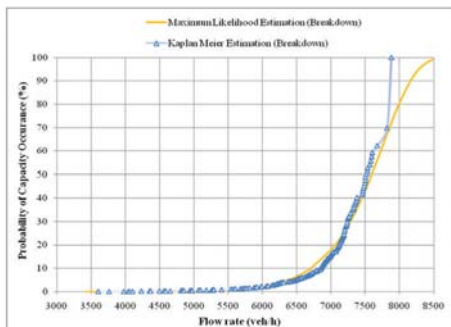


Figure 6. Distribution function of breakdown flow

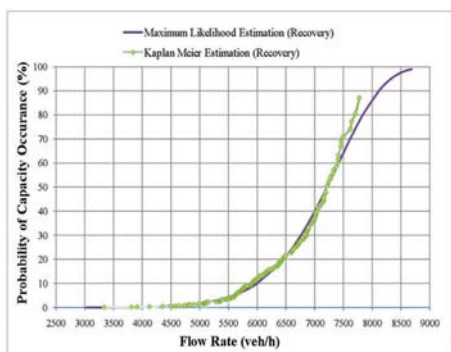


Figure 7. Distribution function of queue discharge flow

As depicted, capacity distribution function based on Product Limit Method reaches the value 1, indicating that the maximum volume observed in this section is followed by breakdown. Considering median value of these distribution functions as capacity, capacity under free flow would be 7590 veh/h, whereas capacity under queue discharge flow would be 7205 veh/h, resulting in a drop of approximately five percent. It is interesting to note that this result is also consistent with the five percent drop suggested by Highway Capacity Manual (2000). applying Gumbel Distribution for capacity before and after breakdown and markov chain approach.

Extreme Value Distribution Type One (Gumbel Distribution) has two forms. One is based on Smallest Extreme and the other on Largest, called minimum and maximum cases respectively.

In this paper, only minimum case is considered as in equation 7 [Milella, 2012]:

$$F(q) = 1 - e^{-e^{-\frac{(q-\mu)}{\beta}}} \quad (7)$$

where:

μ = location parameter, and

β = scale parameter.

Thus, the overall capacity distribution function is estimated as equation 8:

$$(8)$$

$$F_{C,overall}(q) = \frac{(1 - e^{-e^{-\frac{(q-7763)}{470.5}}})}{(1 - e^{-e^{-\frac{(q-7763)}{470.5}}}) + e^{-e^{-\frac{(q-7432.4)}{620.9}}}}$$

Table 3. Calibration results of queue discharge and breakdown distribution function

Distribution type	distribution function	Location Parameter	Scale Parameter	Median	Log-likelihood
Weibull	breakdown	*13.4	7869.2	7656.5	-2005.8
	queue discharge	*10.2	7479.0	7214.7	-1919.8
Gumbel	breakdown	7763.0	470.5	7590.6	-1994.2
	queue discharge	7432.4	620.9	7204.9	-1921.9
Logistic	breakdown	7665.1	444.5	7665.1	-2000.3
	queue discharge	7209.8	542.9	7209.8	-1922.1
Normal	breakdown	**1039.4	***7927.1	7927.1	-2028.3
	queue discharge	**1031.8	***7268.3	7268.3	-1923.4

* shape parameter in the case of Weibull Distribution

** Std.Deviation in the case of Normal Distribution

*** mean in the case of Normal Distribution

Figure 8 demonstrates the estimated capacity distribution functions:

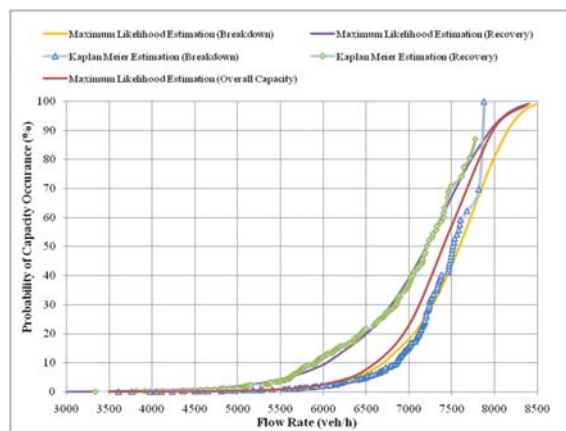


Figure 8. Distribution function of breakdown flow versus queue discharge flow

6. Conclusions

In this paper, existence of capacity drop as a debatable and controversial phenomenon of traffic engineering which has received very little attention was investigated theoretically based on a stochastic approach and implemented empirically for an Iranian freeway.

Empirical study was conducted based on detailed and accurate traffic flow data in the busiest and oldest freeway in Iran collected for four months through video camera recording. Based on the 11,600-record data set, threshold speed was estimated from the speed-flow diagrams, based on which, flow condition was determined and capacity drop was estimated numerically applying two methods. In the first, capacity drop was estimated about five percent (385 veh/h drop from 7590) using paired t-test for breakdown and queue discharge flow rates. In the second method, both parametric and non-parametric capacity distribution functions were estimated based on the analogy between parameters of life time data analysis and roadway capacity. It was found that capacity in free flow (before congestion) follows a Gumbel distribution; while during recovery (after congestion) it follows a Weibull distribution (both being Generalized Extreme Value distributions). The difference between these two for the median (50 percent value) was used as another measure for capacity drop: again five percent drop was observed. Results from both methods are not only compatible with each other but also compatible with the average value proposed

by highway capacity manual (2000). Based on the estimated distribution functions, the overall capacity of the study section was estimated applying Markov Chain approach, whose median was estimates approximately 7400 vphpl.

Deceleration reaction time (including limited acceleration power and human behavior) is shown to be usually shorter than the acceleration reaction time for heavy vehicles [Di Cristoforo, Hood and Sweatman, 2004], thus in future research capacity drop can also be hypothesized to be a result of unequal acceleration and deceleration reaction times: since average delay during acceleration is greater than deceleration, it creates a greater headway in front of the leading vehicle which causes smaller queue discharge flow.

A limitation of the current study is that the minimum interval duration available (i.e. 15 minutes) is rather long. As occurrence of breakdown in traffic flow is sudden, stronger causality between probability of breakdown occurrence and traffic volume can be estimated more precisely using shorter intervals in future studies.

7. Acknowledgement

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9. Endnotes

1- 15-minute observation intervals

2- To define capacity under congested situation, the word “breakdown” could be substituted by the word “recovery” in this definition.

3- Time series is based on real data gathered in one of the most crowded days of the freeway.