Solving a Multi-Depot Location-Routing Problem with Heterogeneous Vehicles and Fuzzy Travel Times by a Meta-Heuristic Algorithm

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Abstract

A capacitated location-routing problem (CLRP) is one of the new areas of research in distribution management. It consists of two problems; locating of facilities and routing of the vehicle with a specific capacity. The purpose of the CLRP is to open a set of stores, allocate customers to established deposits, and then design vehicle tours in order to minimize the total cost. In this paper, a new mathematical programming model for multi-depot location-routing problems is considered. This model considers heterogeneous vehicles and fuzzy travel times, which are innovative and practical limitations compared to the previous studies (e.g., simultaneous pickup and delivery). This makes the model close to real-world situations. After modeling, the fuzzy model is changed to a deterministic model by credibility theory. Since this problem belongs to a class of NP-hard ones because of its computational complexity, it is impossible to find the optimal solution in reasonable time. Therefore, a particle swarm optimization algorithm is proposed and designed to solve the presented model. To show the efficiency of the proposed PSO, its results are compared with the optimal solutions obtained by an exact method embedded in the optimization software. Furthermore, the proposed PSO is able to solve medium- and large-sized problem efficiently.

Keywords: Multi-depot location-routing problem, Simultaneous pickup, and delivery, Heterogeneous vehicles, Fuzzy travel time, Meta-heuristic algorithm.

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1. Introduction

The distribution network design is one of the most important issues in logistics and supply chain management (SCM) because of its great potential to reduce costs and improve the quality of services. The aim of the distribution network design problem is to find the best way to move goods from suppliers to customers by selecting the network structure in such a way that the associated costs are minimized and satisfied customer demands are met.

The design of this network makes two hard combinatorial optimization problems to determine the locations of warehouses and the routes of vehicles to provide customers services from these warehouses. These problems have separately been considered in the literature; however, a continuous progress has integrated as a location-routing problem (LRP).

In today's distributed systems, customers want to meet their needs on time and with the minimum cost considering new assumptions to the applicable issue (e.g., LRP) that make the problem close to real-world situations.

In LRPs, researchers have considered a classical vehicle routing problem (VRP). This means that each vehicle starts from a warehouse, passes through a number of customers, delivers products to every customer and returns to the same warehouse. However, in practice, customers want to pick up and delivery demand and want to do them both at a time. This type of problem in the literature is known as vehicle routing problem with simultaneous pickup and delivery (VRPSPD). According to the application of a VRP with simultaneous pickup and delivery in the areas of a distribution system for perishable products (e.g., dairy and fresh foods) and newspapers, grocery stores (e.g., pallets and beverage industry).

On the other hand, traveling time in urban roads is one of the main parameters in distribution planning problems. Assigning a constant and accurate value for this parameter does not seem to fit the traffic conditions. Using the theory of fuzzy sets in these cases is a pervasive one.

In this article, to bring the problem closer to real situations; in the proposed LRP model the following points are considered:

- Pickup and delivery at the same time.
- Fuzzy travel times.
- Heterogeneous vehicles.
- Restrictions on travel time and service to the customers.

These features are not considered at the same time in any other research.

The proposed model in this article can be used to design distribution networks for food, newspapers, and magazines, as well as for municipal services, support service providers, home care centers, etc.

Since LRP is NP-hard problem [Nagy and Salhi, 2007]; only small-sized LRPs can be solved with commercial solvers. In this article, a particle swarm optimization (PSO) algorithm is introduced for solving real and large-sized problems, as well as a new solution representation.

2. Literature Review

So far, different versions of LRP have been presented in the literature. This study refers to a number of research-related articles.

For example, according to research conducted by us, so far simultaneous picked up and delivered in the area of location routing have been used in a limited number. [Karaoglan et al, 2011] introduced an

LRPSPD, which is a general LRP so that the total cost is minimized. They proposed a branch-and-cut algorithm for effective and exact solutions of the problem and used local search based on a simulated annealing algorithm to gain upper bounds. [Karaoglan et al, 2012] presented a model to minimize the total cost by locating a warehouse and designing the vehicle routes, which are simultaneously in the LRPSPD model for the problem. They provided two mixed-integer linear programming models. In their study, they proposed an innovative two-stage method based on simulated annealing, Tp_Sa, to solve it.

[Ghatreh- Samani and Hosseini-Motlagh, 2017] presented a mixed-integer linear programming (MILP) model for a two-echelon location-routing problem with simultaneous pickup and delivery, in which data uncertainty is taken into account and customers' demand is assumed to be a fuzzy parameter. In the investigated problem, one echelon of facilities, which is called the middle depot echelon, is positioned between central distribution centers and customers echelons. Also, a hybrid heuristic method based on Simulated Annealing (SA) algorithm and Genetic Algorithm (GA) is devised for solving the presented. The results achieved from solving the problem imply that the proposed hybrid algorithm outperforms other algorithms within reasonably computational time. [Alinezhad et al, 2018] presented a Vehicle Routing Problem with Time Windows and Simultaneous Delivery and Pickup (VRPTWSDP) and it is formulated to a mixed-binary integer programming model. In the proposed algorithm, called Improved Particle Swarm Optimization (IPSO), used some removal and insertion techniques and also combined PSO with SA in order to improve the searching ability of PSO and maintained the diversity of solutions as well as extensive computational tests on a class of popular benchmark instances, clearly show the high effectiveness of the proposed algorithm.

For the first time, [Wang, 2013] formulated a unified programming nonlinear multi-objective model for the LRPSPD to minimize the total cost objectives and customer waiting time. In this study, customer's service is in both urban and rural areas, and for two area, two types of vehicles are performed. He also proposed to solve a tabu search algorithm. [Wang and Li, 2017] studied a low carbon for a heterogeneous fleet, simultaneous pickup-delivery, and time windows, and proposed a two-phase hybrid heuristic algorithm to solve the problem. Firstly, they introduced the concept of temporal-spatial distance and used a genetic algorithm to cluster the customer points to construct the initial path. Then, the used variable neighborhood search (VNS) algorithm for local search. [Kartal et al, 2017] introduced a single-allocation *p*-hub median LRP with simultaneous pickup and delivery based on observations from real life hub networks. The aim of their problem is to minimize the cost of transferring the flow between hubs and routing the flow in the network. They proposed several mixed integer programming formulations and two heuristic approaches based on multi-start simulated annealing (SA) and ant colony optimization (ACO) to solve these problems.

Also, the incorporated uncertainty in the form of fuzzy parameters and heterogeneous vehicles, which increase the efficiency in the standard LRPs, known as a new functional field. For example, [Zahedian-Tejenaki and Tavakkoli-Moghaddam, 2015] presented a bi-objective model for routing rail-truck intermodal terminals with the cost and risk as objective functions by considering three sets of the effective factors that lead to hazardous materials transportation accidents in favor of sustainability. Additionally, a fuzziness concept is considered in the presented model. [Tavakkoli-Moghaddam and Raziei, 2016] presented a new bi-objective location-routing-inventory (LRI) problem that considers a multi-period and multi-product problem with heterogeneous fleets in a two-echelon distribution network. The uncertainty in the customers' demand is defined by a fuzzy approach. Two conflicting fuzzy objective functions are included. Numerical results demonstrate the validity of the presented model and show the ability of the model to confront real cases. [Ceselli et al, 2013] provided a model to optimize logistics operations in systems for emergency health care. In particular, the problem of the effective distribution of vaccines or drugs through the use of synchronized distribution centers and vehicles are studied. An exact algorithm based on field production with three different types of columns as well as branch-and-bound methods is designed. [Tavakkoli-Moghaddam et al, 2016] presented a novel bi-objective multi-product capacitated VRP with uncertainty in demand of retailers and volume of products (UCVRP) and heterogeneous vehicle fleets. The objectives are to minimize the cost of the used vehicles, fuel consumption for full loaded vehicles, shortage of products and minimize the shortage of products for all retailers. In order to get closer to a real-world situation, the uncertainty in the demand of retailers is applied using fuzzy numbers and the volume of products is applied using robust parameters, because the possible value of this parameter is not distinct and belongs to a bounded uncertainty set. In order to show the conflict between two objectives in an excellent way, a Pareto-optimal solution with the ε-constraint method is obtained.

[Ghannadpour and Zarrabi, 2017] considered the special application of vehicle scheduling that it is modeled by using the VRP with time windows (VRPTW) to optimal assignment of locomotives to assembled trains. The concepts of fuzzy sets and fuzzy control systems are considered to model the uncertainty in travel times. Besides, a GA with various heuristics is proposed to tackle the proposed model. [Issabakhsh et al, 2018] presented a case study considering the realistic assumptions, such as travel time uncertainty. A VRP model is presented to serve PD patients at home with special logistic services. Thereafter, based on the criticality of timeliness in providing healthcare service, a conservative method, called robust optimization, is applied to handle time uncertainty. The corresponding results show that the proposed method at the maximum uncertainty level has less than 30% variations in results and in comparison with the deterministic model increases the costs only by 1.2%. [Ghannadpour, 2018] presented a new model and solution for the energy minimizing a VRPTW and customers' priority. The detailed mathematical formulation of the proposed model is provided and interpreted as multi-objective optimization, in which the energy consumed and the total number of vehicles are minimized and the total satisfaction rate of customers is maximized. Furthermore, a multi-objective evolutionary algorithm is proposed and its performance on several completely random instances is compared with the Non-dominated Sorting Genetic Algorithm II (NSGA II) and CPLEX Solver.

In [Mehrjerdi and Nadizadeh, 2013] an LRP is considered in which the vehicles and the depots have a predefined capacity to serve the customers that have fuzzy demands. To model this problem, a fuzzy chance-constrained programming model based on the theory of fuzzy reliability design is used. To solve this problem, a greedy clustering method (GCM) includes random simulation is presented. [Ghaffari-Nasab et al, 2013] considered a location routing problem LRP with fuzzy a demand (LRPFD). [Mehrjerdi and Nadizadeh, 2013] designed a fuzzy chance-constrained programming based on the theory of the fuzzy reliability. However, to solve the problem, a combination of SA based on heuristic algorithm combined with stochastic simulations developed and offered. [Fazel Zarandi et al, 2011] introduced a fuzzy version of the capacitated location-routing problem, namely CLRP, considering the warehouse opening fee, limited capacity vehicles and a defined set of customers with specific demands. A number of potential nodes to locate warehouses, the fuzzy time between nodes as well as a time frame to meet each customer's demands were considered. They suggested a multi-depot CLRP that the travel time between two nodes is a fuzzy variable and the credibility theory is used to create a de-fuzzy model. An SA algorithm was proposed to solve this problem. [Fazel Zarandi et al, 2013] suggested an LRP with time windows (LRPTW) under the uncertainty, in which customer demand and travel time was fuzzy variables. A fuzzy chance-constrained programming (CCP) model designed by the credibility theory. To solve this problem, an SA embedded simulation is presented. In these papers, the possibility of simultaneous pickup and delivery, and the use of heterogeneous vehicles have not been considered.

[Golozari et al, 2013] developed a mathematical model for an LRP according to the conditions and restrictions in the real world. The maximum travel time constraint is added, and fuzzy numbers to determine customers demand, travel time and drop time were considered. The proposed problem as a fuzzy linear programming (FLP) was modeled using a fuzzy ranking function. They offered a hybrid algorithm include a simulated annealing algorithm and a mutation operator to solve the Case.

In this study, for the first time, an LRP is modeled considering this assumption at the same time, then a particle swarm optimization (PSO) method is developed to solve the model.

3. Formulation of the Proposed Fuzzy Planning Model

The proposed mathematical model is the expansion of [Karaoglan et al, 2011], in which assumptions of the fuzzy parameter of the travel time between nodes, the use of heterogeneous vehicles and travel time constraints for vehicles were added to the model. Then, the new model is achieved.

3.1 Assumptions

- Each customer is served by exactly one vehicle.
- Each route must begin and ends at the same depot.
- The total vehicle load at any point of the route does not exceed the vehicle capacity.
- Each vehicle is used only one route.
- The total pickup and total delivery load of the customers assigned to an opened depot must not exceed the capacity of the depot.
- The total travel time, which a vehicle spent on each route, is less than or equal to the maximum traveling time.
- The path has a maximum time associated with that path.
- The timing of dispatch of all vehicles from the warehouse is equal and at zero.

- All the customers can return the goods to the warehouse as well as receiving ordered goods. Also, the amount of returned goods can be less or more than the amount received.
- Customers do not exchange any goods with each other; all returned goods are returned to the warehouse.
- There is no preference for sending goods.
- Uncertainty exists in the problem.
-

3.2 Model components

Sets:

I : Set of total depots

J : Set of total customers

K : Set of total vehicles

Parameters:

N : Number of customers

d_j : Delivery demand of customer *j*

p_j : Pickup demand of customer *j*

Q_k : Physical capacity of vehicle *k*

O_i : Maximum physical capacity of depot *i*

t_{jk} : Servicing time at customer *j* by vehicle *k*.

$\tilde{t}_{ijk}$: Fuzzy travel time between nodes (customer, depot) *I* to *j* by vehicle *k*.

T_k : Maximum service time and time along the route, by vehicle *K*.

f_i : The Fixed cost of deploying a new warehouse in candidate's place *I*.

h_k : Shipping cost per unit of distance in the route *K*.

g_k : Fixed vehicle cost of *K*

c_{ij} : Travel distance between node *i* to *j* ($i, j \in I \cup J$)

Decision variables

$$x_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ travels directly from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ij} = \begin{cases} 1 & \text{if node demand j is assigned to depot i} \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if a depot is in node } i \\ 0 & \text{otherwise} \end{cases}$$

U_{ijk} : Demand to be delivered to customers routed after node *i* and transported in an arc (*i*, *j*) if vehicle *k* travels directly from node *i* to node *j* ($\mathrm{i}, \mathrm{j} \in \mathrm{I} \cup \mathrm{J}$), otherwise 0.

V_{ijk} : Demand to be picked-up from customers routed up to node *i* (including node *i*) and transported in an arc (*i*, *j*) if a vehicle *k* travels directly from node *i* to node *j* ($i, j \in I \cup J$), otherwise 0.

3.3 Model Formulation

$$\text{Min } M = \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} g_k x_{ijk} + \sum_{k\in K}\sum_{i\in(I\cup J)}\sum_{j\in(I\cup J)} h_k c_{ij} x_{ijk} \tag{1}$$

s.t.

$$\sum_{k\in K}\sum_{i\in I\cup J} x_{ijk} = 1, \qquad j \in J \tag{2}$$

$$\sum_{j\in I\cup J} x_{ijk} - \sum_{j\in I\cup J} x_{jik} = 0, \qquad i \in (I \cup J), \mathrm{k} \in \mathrm{K} \tag{3}$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \le 1, \qquad \mathrm{k} \in \mathrm{K} \tag{4}$$

$$\sum_{k\in K}\sum_{i\in I\cup J} U_{ijk} - \sum_{k\in K}\sum_{i\in I\cup J} U_{jik} = d_j, \qquad \mathrm{j} \in \mathrm{J} \tag{5}$$

$$\sum_{k\in K}\sum_{i\in I\cup J} V_{jik} - \sum_{k\in K}\sum_{i\in I\cup J} V_{ijk} = p_j, \qquad \mathrm{j} \in \mathrm{J} \tag{6}$$

$$U_{ijk} + V_{ijk} \le Q_k x_{ijk}, \qquad i, j \in (I \cup J), i \ne j, \mathrm{k} \in \mathrm{K} \tag{7}$$

$$\sum_{k\in K}\sum_{j\in J} U_{ijk} = \sum_{j\in J} d_j z_{ij}, \qquad \mathrm{i} \in \mathrm{I} \tag{8}$$

$$\sum_{k\in K}\sum_{j\in J} U_{jik} = 0, \qquad \mathrm{i} \in \mathrm{I} \tag{9}$$

$$\sum_{k\in K}\sum_{j\in J} V_{jik} = \sum_{j\in J} p_j z_{ij}, \qquad \mathrm{i} \in \mathrm{I} \tag{10}$$

$$\sum_{k\in K}\sum_{j\in J} V_{ijk} = 0, \qquad \mathrm{i} \in \mathrm{I} \tag{11}$$

$$U_{jik} \le (Q_k - d_j) x_{jik}, \qquad i \in (I \cup J), j \in J, \mathrm{k} \in \mathrm{K} \tag{12}$$

$$V_{ijk} \le (Q_k - p_j) x_{ijk}, \qquad i \in (I \cup J), j \in J, \mathrm{k} \in \mathrm{K} \tag{13}$$

$$U_{ijk} \geq d_j x_{ijk}, \qquad i \in (I \cup J), j \in J, k \in K \tag{14}$$

$$V_{jik} \geq p_j x_{jik}, \qquad i \in (I \cup J), j \in J, k \in K \tag{15}$$

$$\sum_{i \in I} z_{ij} = 1, \qquad j \in J \tag{16}$$

$$-z_{ij} + \sum_{u \in I \cup J} (x_{iuk} + x_{ujk}) \leq 1, \qquad i \in I, j \in J, k \in K \tag{17}$$

$$\sum_{j \in J} \sum_{i \in (I \cup J)} t_{jk} x_{ijk} + \sum_{i \in (I \cup J)} \sum_{j \in (I \cup J)} \tilde{t}_{ijk} x_{ijk} \leq T_k, \qquad k \in K \tag{18}$$

$$\sum_{j \in J} d_j z_{ij} \leq O_i y_i, \qquad i \in I \tag{19}$$

$$\sum_{j \in J} p_j z_{ij} \leq O_i y_i, \qquad i \in I \tag{20}$$

$$x_{jik} \leq z_{ij}, \qquad i \in I, j \in J, k \in K \tag{21}$$

$$x_{ijk} \leq z_{ij}, \qquad i \in I, j \in J, k \in K \tag{22}$$

$$x_{ujk} + z_{ij} + \sum_{m \in I, m \neq i} z_{mj} \leq 2, \qquad i \in I, \quad u, j \in J . u \neq j, k \in K \tag{23}$$

$$x_{ijk} \in \{0,1\}, \qquad i, j \in (I \cup J), k \in K \tag{24}$$

$$y_i \in \{0,1\}, \qquad i \in I \tag{25}$$

$$z_{ij} \in \{0,1\}, \qquad i \in I, j \in J \tag{26}$$

$$U_{ijk}, V_{ijk} \geq 0, \qquad i, j \in (I \cup J), k \in K \tag{27}$$

The objective function (1) expresses respectively: The fixed cost of deployment of the warehouse in the place I, fixed vehicle cost K. Shipping cost per unit of distance.

Based on Constraint (2), each customer is assigned exactly to the one path and one warehouse. Constraint (3) states that the entry and exit of the arcs are equal to each node (it means that the vehicles enter into each node out of it). Constraint (4) is specified there can be one route between the two customers at most. (Each vehicle is assigned exactly to one path.) Constraint (5) flow protection for demand and constraints (6) provide protection of flow for picking up demand. Two Constraints (5) and (6) eliminate a sub-tour and ensures that the delivery and pickup of the demand for each customer is satisfied. Constraint (7) guarantees that the total load on each arc should not exceed the capacity of the vehicles (i.e., the total amount of loads that have already been taken and the total amount of the loads to be delivered from now on should not exceed the capacity of the vehicle.) According to Constraint (8), the total dispatched delivery load from each warehouse is equal to the total delivery demand to the customers assigned to that storage. Constraint (9) assumes that the total value of the delivered load returned to the warehouse must be zero. According to Constraint (10), the total pickup load entered into each warehouse is equal to the total demand for picking up from customers assigned to the relevant warehouse. In Constraint (11), the total pickups from each warehouse should be equal to zero. Constraints (12) to (15) are boundary limits for additional variables. Constraint (16) specifies that each customer should be assigned to only one warehouse. Constraint (17) expresses the connection restriction, only when a customer can be allocated to a warehouse whose client's path belongs to that warehouse. Failure to exceed the maximum allowable time for the route and service of each vehicle is given in the Constraint (18) (i.e., the constraint of a fuzzy variable). Constraint (19) shows that if warehouse *i* to be deployed, the total delivery load in each warehouse should not exceed the capacity of the warehouse and Constraint (20) if the warehouse i is located, the total load picked up in each warehouse should not exceed the capacity of the warehouse. Constraints (21) to (23) prohibition of banned routes (the prohibition of routes that do not start and end from one warehouse). Constraint (24) - (27) are described for the purpose of introducing the nature of the decision variables.

3.4 Fuzzy Parameter

As noted above, Constraint (18) has a fuzzy parameter of travel time between nodes that causes the fuzzy limitation. In order to solve the model limitation should be in the de-fuzzy state, so to do that fuzzy credit scale theorem has been used.

Generally, credibility-based chance constrained programming is an efficient computational approach to fuzzy mathematical programming, which relies on strong mathematical concepts. There are mainly three important types of measurements in fuzzy sets: possibility, necessity, and credibility [Pishvaee and Torabi, 2010].

Suppose $\tilde{\zeta}$ be a fuzzy variable with membership function $\mu(r)$ for each set A belonging to R. The possibility measure pos of a fuzzy event $\{\tilde{\zeta} \in A\}$ is defined by $Pos\{\tilde{\zeta} \in A\} = \sup_{r \in A} \mu(r)$ and the necessity measure nec of $\{\tilde{\zeta} \in A\}$ is defined by $Nec\{\tilde{\zeta} \in A\} = 1 - \sup_{r \in A^c} \mu(r)$ where the Ac set against A. The credibility measure Cr is an average of the possibility measure and necessity measure, which is:

$$Cr\{\tilde{\zeta} \in A\} = \frac{1}{2}(Pos\{\tilde{\zeta} \in A\} + Nec\{\tilde{\zeta} \in A\}). \qquad (28)$$

The constraint of the fuzzy variable must be converted to the fuzzy chance limit and the constraint must be maintained at an assurance level in terms of validity. As a result, if the level of confidence be $(0 \prec \alpha \prec 1)$ $,\alpha$, then:

$$Cr\left\{\sum_{i\in(I\cup J)}\sum_{j\in(I\cup J)}\tilde{t}_{ijk}x_{ijk}\le T_k-\sum_{j\in J}\sum_{i\in(I\cup J)}t_{jk}x_{ijk},\quad k\in K\right\}\ge\alpha\quad\Leftrightarrow \tag{29}$$

Because the level of assurance is very low for practical issues, it should be considered as larger than 0.5. A review of the fuzzy location routing articles that have used credibility theory is evidenced by this [Mehrjerdi and Nadizadeh, 2013; Ghaffari-Nasab et al, 2013; Fazel Zarandi et al, 2013].

Due to the fact that trapezoidal fuzzy numbers are closer to reality, the fuzzy variable $\tilde{\zeta}$ is completely determined by four definite numbers (a, b, c, d) that are $a\le b\le c\le d$.

The basis of the article from the theorem of [Zhu and Zhang, 2009] is that the fuzzy limitation expressed by the fuzzy chance turns into linear functions:

Proposition: If $\tilde{\zeta}$ is a trapezoidal fuzzy number (a, b, c, d). Two confidence levels α and β $(0.5\prec\alpha,\beta\le 1)$ are given. Then $Cr\{\tilde{\zeta}\ge r\}\ge\alpha\Leftrightarrow r\le W_\alpha$ and $Cr\{\tilde{\zeta}\le r\}\ge\beta\Leftrightarrow r\ge W_\beta$ while $W_\alpha=(2\alpha-1)a+2(1-\alpha)b$ and $W_\beta=(2\beta-1)d+2(1-\beta)c\cdot$

Consequently, given the above case, under the terms: 1) Use trapezoidal numbers 2) $0.5\prec\alpha\le 1$:

1. $Cr\{\tilde{\zeta}\ge r\}\ge\alpha\Leftrightarrow r\le(2\alpha-1)a+(2-2\alpha)b$ (30)
2. $Cr\{\tilde{\zeta}\le r\}\ge\alpha\Leftrightarrow r\ge(2-2\alpha)c+(2\alpha-1)d$ (31)

Suppose $\tilde{t}_{ijk}=(t_{ijk(1)},t_{ijk(2)},t_{ijk(3)},t_{ijk(4)})$, and according to the above statement:

$$\sum_{j\in J}\sum_{i\in(I\cup J)}t_{jk}x_{ijk}+\sum_{i\in(I\cup J)}\sum_{j\in(I\cup J)}\tilde{t}_{ijk}x_{ijk}\le T_k\quad k\in K\ \rightarrow \tag{32}$$

$$\sum_{i\in(I\cup J)}\sum_{j\in(I\cup J)}\tilde{t}_{ijk}x_{ijk}\le T_k-\sum_{j\in J}\sum_{i\in(I\cup J)}t_{jk}x_{ijk} \tag{33}$$

$$\tilde{\zeta}=\sum_{i\in(I\cup J)}\sum_{j\in(I\cup J)}\tilde{t}_{ijk}x_{ijk} \tag{34}$$

$$r=T_k-\sum_{j\in J}\sum_{i\in(I\cup J)}t_{jk}x_{ijk} \tag{35}$$

Given that the restriction is of a less than or equal to, the relation (31) is used:

$$Cr\{\tilde{\zeta}\le r\}\ge\alpha\Leftrightarrow r\ge(2-2\alpha)c+(2\alpha-1)d\quad\Rightarrow \tag{36}$$

$$Cr\left\{\sum_{i\in(I\cup J)}\sum_{j\in(I\cup J)}\tilde{t}_{ijk}x_{ijk}\le T_k-\sum_{j\in J}\sum_{i\in(I\cup J)}t_{jk}x_{ijk}\right\}\ge\alpha\Leftrightarrow \tag{37}$$

$$T_k-\sum_{j\in J}\sum_{i\in(I\cup J)}t_{jk}x_{ijk}\ge \sum_{i\in(I\cup J)}\sum_{j\in(I\cup J)}x_{ijk}[(2-2\alpha)t_{ijk(3)}+(2\alpha-1)t_{ijk(4)}] \tag{38}$$

Therefore, the relationship (18) becomes the definite constrained (38), and the model goes out of fuzzy mode.

4. Proposed Solution Algorithm

As previously mentioned, the location routing problem is from NP-hard problem. For medium and large dimensions of LRP, no solution is reachable at a reasonable time. So the heuristic and meta-heuristic algorithms for near-optimal solutions in a reasonable time are used for this problem.

The particle swarm optimization algorithm (PSO), is a successful algorithm in the field of continuous and discrete optimization and applied in many different fields. In this article, because of the advantages of the algorithm that are rapid convergence, simplicity in its encoding and decoding, fast and easy to compute, it is developed and enhanced for the proposed optimization problem.

Since the model proposed in this study has not been evaluated and solved in any other scientific sources and in studies there is no comparable answer exists in comparison with results of this model, the proposed solution algorithm to the model presented in this study is new. Also to solve small sizes of this problem, gams software that gives global optimal solutions is used.

Since another solution representation scheme of other papers does not cover the proposed problem, because of the heterogeneity of the vehicle; in this study, a new solution representation scheme is presented that reduce the number of variables. And with fewer variables, more passives to be answered which are described below. The advantage of solution representation scheme introduced in this research: A) the length of the chromosome is reduced. B) applies to most constraints (if there is no application in capacity constraints, the penalties are taken into account). C) Covers entire space.

The mentioned problem include *I* storage, *J* customer and *K* vehicle. In showing the answer, there is a string of L length. This consists of two parts and is shown in Figure 1.

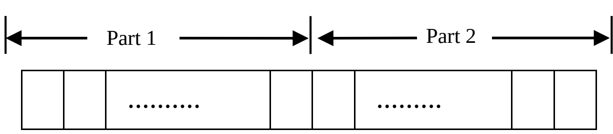

Figure 1. Structure of the solution representation

First part: This section which has the length of $j + k$-1 represents the paths and specifies the customers of each direction. Since the coding is continuous, first $j + k$-1 random numbers are made on the interval [0, 1] then each number is enumerated by a label of 1, ..., *N*.

Second part: the length of this section is *K* and used to allocate vehicles to warehouses. For assigning each of the vehicles to a warehouse a mapping of integer part are used. First, real numbers between 0 and 1 are produced then by mapping of the integer part, turns to integers.

In the PSO algorithm proposed in this study, a randomized primary answer to search and the more selection points at the start of the algorithm is considered each *N* particles as a possible answer to the problem has position and speed and the best position of each particle is considered equal to the current position of the particle. The best total cost of the particles is considered to be infinite at the beginning of the algorithm. The important point when generating the initial population is producing the answer for the problem, the produced answer in the form of representation scheme that was pre-coded and has added to the population.

In the main loop position algorithm and the speed of each particle, under three components updated in each iteration. The speed of the particle Based on the current speed of the particle, Significance distance level from the current position to its best viewing position by particle itself and the coefficient of the current position of the particle in best position found by the total particles is updated. The new position of every bit is equal to its previous position of the particle plus its update speed. To compare all particles (answers) an evaluation function is defined so review the position of each particle is proportional to the demand question and finally particles to be compared with each other, and lead particles to the optimal solution.

In the proposed algorithm evaluation function is a function defined with minimum cost. So the best answer of the particles, particles that have the lowest cost function. In each iteration, the cost function algorithm will be updated and will be calculated the best cost per bit. It is less than the total cost the best public position and the best cost updated by the position of the particle. Calculation of the particle's new position until the number of known duplicates is not shown is done. In this algorithm at each stage, five mutations occur on the particles. It means that on each solution five boundaries are chosen. If a better solution is found among them, it will be replaced with the current solution, and five mutations occur on the global best once every 20 iterations. This action is a local research and improves the performance of the algorithm.

Due to the impact of movements function on the performance of the algorithm and quality of the obtained solutions, it has been tried to examine various types of motion functions. These motion functions generally search for the neighborhood of each answer by applying variations in the solution string.

First, a random number is defined between 1, 2 and 3, if the number is 1 only the first part of the answer field will change, if it is 2, the second part is changed and if it is 3, both solution fields are changed.

To change the first part we use the permutation method as it is one of the three operator swaps, reversion and Insertion will be randomly selected. But for changing the second part the mutations will be used: In this case, one of the produced "A" is chosen for the second part and sum with a random number.

Also, at the beginning, it is assumed that all the vehicles will meet the customers' demands, due to what is shown in representation scheme; even if the vehicle is overloaded. At each stage, the pickup and delivery cost will take into account and the

total load will be calculated. If a vehicle is loaded more than its capacity, a fine will be added to the cost function.

5. Computational Experiments and Analysis Results

The parameters used in the proposed algorithm, particle swarm optimization, are set. The performance of the proposed algorithm compared with the results that have been obtained GAMS 24.0.1 software solutions and meta-heuristic algorithm listed in medium and large problems are calculated.

The performance of an algorithm depends heavily on its parameters. Meaning the parameter setting is to choose the best values for parameters so that the performance of the algorithm to be optimized. In this study, to adjust the parameters of the proposed algorithm and find the best level of each parameter of the Taguchi method is used.

In the algorithm, the best combination of n parameters is calculated in 3 levels. Due to optimal levels for each parameter for each run the total of 3*n* combination of parameters exists due to the given number of issues, the number of tests increases to determine the best combination requires a lot of time and unreasonable, so the Taguchi method using MINITAB 16 is used for this purpose.

The Taguchi method seeks to minimize the effects of errors and determine the optimal level. The aim of the method is to find the best combination of factor inputs to S / N ratio is minimized.

$$S/_{N} = -10\log(\frac{1}{n}\sum_{i=1}^{n} {y_i}^2) \quad (39)$$

Due to the importance of adjusting the parameters in the output quality to determine the best parameters Particle swarm optimization algorithms have the best combination of five parameters: The number of particles, inertia factor, the significance level for the best position of the particle, the significance level for the best position of total particles and the number of repetitions to be calculated and the best combination of parameters in Table 1 determined. Since standard issue for the problem does not exist, the data is randomly generated.

5.1 Performance Evaluation of the Proposed Algorithm

To solve the small size of the proposed optimization software GAMS 24.0.1 with solver CPLEX is used and the results of the application compared with proposed algorithm. To check the quality of the answer

and quickness of solving 10 problems were made for small size with up to 10 clients And 5 candidate sites for the storage and 6 vehicles. In Table 2, features of 10 examples produced that the number of customers and the number of site candidate for the construction of warehouse and vehicle are different and by vehicle - warehouse - the customer is displayed.

Table 1. Optimal parameters of the PSO algorithm

Parameter	c_2	c_1	W	***nPop***	***MaxIt***
Amount	1.5	0.2	0.8	50	1000

Table 2. Calculations results of sample small-sized problems

Problem *J-I-K*	**PSO Objective function**	**PSO Time (second)**	**GAMS Objective function**	**GAMS Time (second)**	**Gap (%)**
6-3-4	44575.90	4.1	44575.90	9.5	0.0
7-3-4	37826.15	7.3	37826.15	78.1	0.0
8-3-4	47088.34	6.1	46285.82	98.9	1.7
8-4-5	78773.08	5.0	78334.64	118.8	0.6
9-3-4	39652.08	4.8	39130.09	121.2	1.3
9-4-5	79144.42	10.2	78310.12	160.1	1.1

9-5-6	81069.01	5.7	79971.78	170	1.4
10-3-4	52511.62	8.0	51631.48	164.3	1.7
10-4-5	95050.75	4.3	94012.16	180.4	1.1
10-5-6	80357.49	11.3	79543.00	275.6	1.0
Average	63723.37	6.68	62962.11	137.69	1.27

As can be seen in two examples of three candidate place for the storage, four vehicles and six and seven customers GAMS and algorithms amounts are equal. By increasing in a number of nodes because of the complexity of the model the difference percentage is increasing but as you see the difference is not significant, also in small examples the PSO algorithm compared to GAMS found the solutions in a lower time.

The performance of the PSO algorithm for medium and large-sized problems with 20 problems with a maximum of 70 clients and seven candidate sites for warehouses are examined with 10 vehicles that are randomly generated.

The algorithm is executed 10 times for each problem and the minimum and maximum values run 10 times. And the average time and improvement of solutions (i.e., the difference between the results for the first possible answer and the possible answer after a certain number of repetitions) in Table 3 for medium- and large-sized problems are provided. According to the calculations, it is concluded that in medium and large sizes, PSO algorithm is efficient for solving the proposed model.

Table 3. Calculations results of sample medium and large-sized problems

Problem *J-I-K*	**PSO**				
	Min	**Max**	**Ave.**	**Time**	**Improvement**
15-4-6	85706	95767	93432	37	34429
15-5-7	101342	141503	114591	70	53443
18-4-6	104290	110412	106839	112	8607
18-5-7	84794	93319	90585	69	67460
18-6-8	90582	124769	96170	95	51143
20-4-6	84811	108312	93020	106	30790
20-5-7	83010	87358	89133	120	73057

20-6-8	88769	108659	93960	112	102761
25-4-6	90039	93151	91892	197	17823
25-5-7	88301	98114	90811	181	48388
25-6-8	93405	130044	112870	251	59508
35-5-8	107383	122358	126240	342	40592
35-6-9	96108	106201	110950	372	81365
35-7-10	133149	157781	145216	307	66235
50-5-8	119471	166883	143799	399	9670
50-6-9	132000	161543	145945	415	6275
50-7-10	171554	217447	187301	456	32801
70-5-8	152876	182850	166339	435	2135
70-6-9	151202	189761	174965	464	10383
70-7-10	183583	221499	199825	518	24971
Average	112119	135887	123694	380	41092

6. Suggestions for Future Research

To improve the solutions obtained in this paper and future investigations, the following items are suggested:

- In this study, the time of the variable between two nodes is regarded as fuzzy parameters. Taking into account other problem parameters (e.g., demand and loading time) as fuzzy numbers and their modeling, can lead to a more realistic model.
- The proposed model is a single-commodity model. This model can be extended to multi-commodity mode.
- Considering hard and soft time windows for the LRP.
- The proposed model in this paper is a single objective model. Studying of the problem in a multi-objective mode by adding other criteria (e.g., minimizing the used vehicles, increasing the customer satisfaction, the possibility of covering, the risk of availability, the risk of transport or combination of them.) is another direction for future research.
- Development of heuristic and meta-heuristic alternative methods to solve the problem and compare it with the proposed method

7. References

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