Modeling Different Decision Strategies in a Time Tabled Multimodal Route Planning by Integrating the Quantifier-Guided OWA Operators, Fuzzy AHP Weighting Method and TOPSIS

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Abstract

The purpose of Multi-modal Multi-criteria Personalized Route Planning (MMPRP) is to provide an optimal route between an origin-destination pair by considering weights of effective criteria in a way this route can be a combination of public and private modes of transportation. In this paper, the fuzzy analytical hierarchy process (fuzzy AHP) and the quantifier-guided ordered weighted averaging (Q-OWA) operators were integrated to calculate the weights of the criteria. Accordingly, a user determines the relative weights with fuzzy AHP method at first. Then, by considering his/her slightly decision strategy, the final weights (the ordered weights) were calculated and *K*-shortest route determined using *K*-shortest route algorithm. In the next step, the proposed model presented the best route to user using TOPSIS method. In this study, subway, BRT, bus, taxi, and walking transportation modes were considered for traveling. Also, time, fare, and minimum changes in mode of transportation were considered as effective criteria. This model is implemented in a web-based geographical information system for an area in the center of Tehran and results proved that on average 85.00% of the users with different decision strategies selected the route proposed by the model as the best route.

Keywords: Multimodal multi-criteria route planning, decision strategies, Fuzzy AHP, OWA operators, TOPSIS

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1. Introduction

Transportation networks constitute one class of major civil infrastructure system that is a critical backbone of modern societies [Zamanifar et al. 2014]. Nowadays, increasing numbers of vehicles to transport goods and passengers have caused congestion in transportation networks, especially in big cities [Eydi et al., 2017]. For traveling within a metropolis, people willingly trend to cross from routes that satisfy their criteria. Consequently, demands for services that address route planning efficiently are growing [Kirchler, 2013]. In the simplest form, the personalized route planning is to find the shortest route from an origin to a destination. For solving this problem, well known algorithms like Dijkstra or A* can be used. In more realistic forms, the personalized route planning is finding the shortest route from an origin to a destination by minimizing a set of personalized criteria known as 'Multi-Criteria Personalized Route Planning' or with different transportation modes known as 'Multi-Modal Personalized Route Planning'. Journey planning on a public transportation system is a hard problem due to its inherent time-dependent and multi-criteria nature [Bast et al., 2016].

For problem definition, suppose a single mode transportation network could be represented as a directed graph $G = (V, E)$, in a way V and E represents nodes and edges sets, respectively. In this paper, five transportation modes were used including subway, BRT, bus, taxi, and walking. Hence, the transportation network consists of 5 sub-graphs. For constructing the network, these 5 sub-graphs were merged together to create connectivity among these sub-graphs. In this regard, subway, bus, and BRT lines were connected to street network in stations by walking links. Consider (i,j) represents a directed edge between two nodes i and j: $\{i, j \in V\}$ and W_{ij} represents its weights set. A route between two nodes s and t $\{s, t \in V\}$ represents by $R(s, t)$, in which $R(s, t) =\{s=i_1, (i_1, i_2) \dots i_{j-1}, (i_{j-1}, i_j), i_j=t\}$ is a sequence of alternative nodes and edges [Yu and Lu, 2012]. Because in this paper we are working on a multi-modal multi-criteria transportation network, each edge has a set of weights $\{W_1, W_2, \dots, W_p\}$ related to different criteria $\{C_1, C_2 \dots C_P\}$. Moreover, each node also has a delay time that has to be added to the next edge impedance. The main research questions are:

- how users' inherent ambiguity in criteria weighting could be modeled,
- how users' different decision strategies in route selecting could be modeled, and
- how the best multimodal route could be determined among a set of semi-optimal routes?

In former studies mentioned above, high tradeoff decision strategies, i.e. the weighted linear aggregation rules, were used to calculate the impedance of the links and the other strategies were neglected. For solving this problem, this paper tried considering different decision strategies that the user may intend to use them for his/her route planning. A decision strategy defines whether a user insists on satisfying all of his/her preferences regarding the selection of one route from a set of routes or he/she will be happy if most or many of criteria are satisfied. In this paper, for modeling different decision strategies, the quantifier-guided OWA operators were used. To model a family of parameterized decision strategies, Yager [Yager, 1998] introduced OWA operators. The quantifier-guided OWA obtained by integrating fuzzy linguistic quantifiers with OWA operators. A fuzzy linguistic quantifier Q can cover a range from "at least one" to "all" when the user would like to "at least one criterion must be satisfied" and "all criteria must be satisfied", respectively. A fuzzy AHP-Q-OWA method proposed in this paper has four steps for assigning weights to the effective criteria; a) pairwise comparison of the effective criteria; b) calculating the relative weights of criteria; c) determining a fuzzy linguistic quantifier Q; and d) calculating the final weights of criteria. After determining the final weights, the first *K*-shortest routes would be determined between the origin and destination nodes and then, the model presents the best multi-modal route to user using TOPSIS method. TOPSIS is a commonly-used method in multi-criteria decision making problems [Beheshtinia and Ahangareian, 2018]. A relative advantage of TOPSIS is the ability to identify the best alternative quickly. Considering different decision strategies, this model provides different routes. In this paper, because a trip in the determined case study is no longer than three hours, Tehran's transport network data between 5:30 AM to 8:30 AM of a day were used.

The rest of this paper consists of 5 sections. Section 2 is consists of a literature review of the route planning problem. In Section 3 problem definition and the proposed algorithm are illustrated. Experimental results as well as the model verification are discussed in Section 4, and finally, conclusions and future works are presented in Section 5.

2. Literature Review

Decision analysis models can be classified in three categories:

a) Single objective decision making (SODM): The main purpose of SODM is to find the best solution with minimizing or maximizing of a single objective function values.

b) Decision support systems (DSS): A decision support system (DSS) is a computer program that provides information in a given domain of application by means of analytical decision models and access to databases, in order to support the decision maker in making decision effectively in ill structured (non-programmable) tasks [Malczewski, 1999].

c) Multi-criteria decision making (MCDM) is a topic that deals with the decision-making process in the presence of different and contradictory criteria [Colson and De Bruyn, 1989]. Despite the widespread use of MCDM, there are some common concepts in all MCDM issues that are shown in Figure 1 of these common features. Each issue can have multiple objectives or criteria. Criteria may conflict. Also, different targets and metrics may also have different measurement scales. Solving these issues can mean designing the best answer or choosing the best answer among available solutions.

As showed in Figure 1, in general MCDM classified in two categories:

a) Multi attribute decision making (MADM): Multiple Attribute Decision Making (MADM) involves "making preference decisions (such as evaluation, prioritization, selection) over the available alternatives characterized by multiple, usually conflicting, attributes" [Hwang and Yoon, 1981].

b) Multi objective decision making (MODM): MODM approach provides a mathematical framework for designing a set of decision alternatives [Kahraman, 2008]. The purpose of MODM is to select the ‘‘best’’ design from a large set of alternatives which satisfy the given

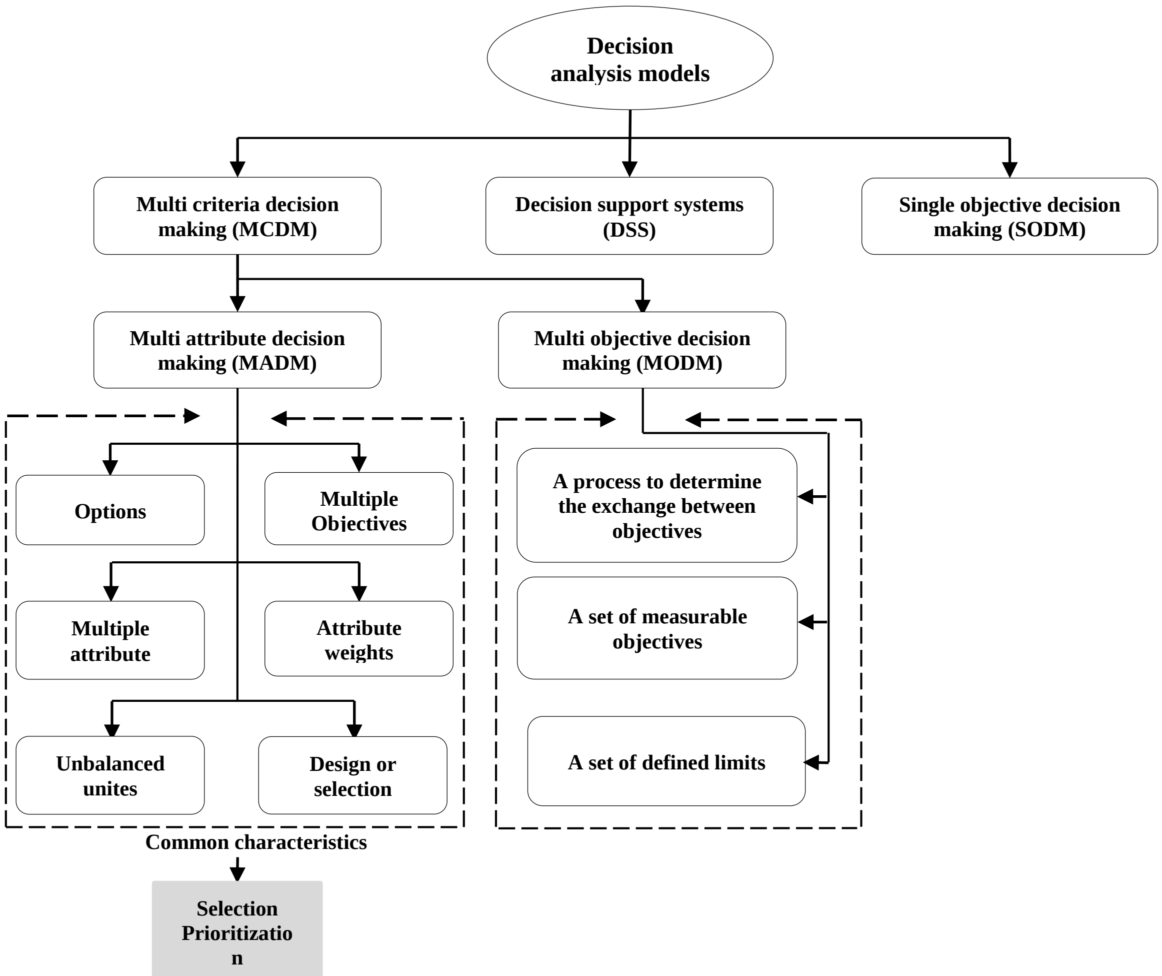


Figure 1. Common characteristics of multi-criteria decision making models [Triantaphyllou, 2000]

requirements and objectives [Pirdashti et al. 2008].

In recent years, many online services have been provided for multi modal route planning such as Google transit. These services are useful in encouraging people to use public transport system instead of their private cars that cause reducing the congestion and CO_2 emission, as well as bettering the traffic flow. The purpose of MMPRP is to provide an optimal route between an origin-destination pair by considering weights of effective criteria. This route may be a combination of different transportation modes including public, i.e., bus and subway, and private, i.e., walking. In the past decades, many researches in personalized route planning have been done. In this paper, these researches divided into three categories including researches in multi-modal route planning, multi-criteria route planning, and multi-modal multi-criteria route planning:

Most of multi-modal researches were related to find multi-modal shortest route in a static transportation network [Nguyen et al., 1988; Delavar et al., 2004].Yu and Lu [Yu and Lu, 2012] used a genetic algorithm (GA) to solve multi-modal route planning problem. They used the variable length chromosomes with several parts to represent routes, where each part describes a kind of transportation mode. Their results showed a various mode combination, and some results adapt experience well. Abbaspour and Samadzadegan [Abbaspour and Samadzadegan, 2011] used an adapted evolutionary algorithm with variable lengths chromosomes to find time-dependent shortest multi-modal route in the complex and large transportation networks. Borole et al. [Borole et al., 2013] tried using real-time transportation network data for solving the multi-modal shortest route problem. They used GPS enabled vehicles for positioning. The result showed that their proposed system provides acceptable route plans in terms of possibility, response time and accuracy.

All the above mentioned researches considered only one criterion.

- In multi-criteria researches, Pahlevani and co authors [Pahlavani et al., 2006] attempted to improve the rate of search in urban multi criteria optimized route guidance by considering unspecified site satisfaction on a real transportation network with multiple dependent criteria. Niaraki and Kim [Niaraki and Kim, 2009] introduced a generic ontology-based architecture using AHP weighting method to design a personalized route planning system. Nadi and Delavar [Nadi and Delavar, 2011] proposed a personalized web based route planning approach using an analytic hierarchy process (AHP) and an ordered weighted averaging (OWA) operators for modeling different decision strategies. In a real word transportation network for tourism scenario, their proposed system showed high performance. Liu et al. [Liu et al., 2012] proposed an oriented spanning tree based Simulated Annealing (SA) for finding the shortest route by considering different criteria for route finding, especially when objectives are nonlinear. By using oriented spanning tree for representing a route, the both of local and global search capabilities of the designed SA are greatly improved. The results showed their proposed model is superior to Nondominated Sorting Genetic Algorithm II (NSGA-II) and Archived Multi Objective Simulated Annealing (AMOSA) algorithms. Pahlavani and Delavar [Pahlavani and Delavar, 2014] proposed a novel approach on the basis of integrating fuzzy algorithms and Artificial Neural Networks (ANNs) for modeling a driver's preferences in multi-criteria route selection. Osaba et al. [Osaba et al., 2016] applied an evolutionary discrete firefly algorithm (EDFA) to the well-known vehicle routing problem with time windows. The proposed technique presents some novelties, such as the use of the Hamming distance to measure the distance between two different fireflies and novel route optimization operators that have been developed for the EDFA. The major weakness of these studies was that they were not carried out on a complex transportation network with different modes of transportation.

These researches only considered one transportation mode for traveling between their considered transportation networks.

- In multi-modal multi-criteria researches, Qu and Chen [Qu and Chen, 2008] proposed a hybrid multi-criteria decision making method to find the multi-modal multi-criteria shortest route. They used fuzzy AHP weighting method to find the suitable initial weights to improve the efficiency of the artificial neural network (ANN). By using a label correcting method, Liu et al. [Liu et al., 2014] designed an algorithm for solving the multi-modal multi-criteria shortest route problem with both transfer delaying and arriving time window. Bouhana et al. [Bouhana et al., 2013] proposed a novel approach that integrated a case-based reasoning with Choquet integral to propose the best multi-modal route by considering the user's preferences. One of the main capabilities of their model was to predict the user's preferences. For this purpose, their model compared the user's preferences with the other preferences for a given context to help the user to adopt the best action when facing a new situation. Ghaderi and Pahlavani [Ghaderi and Pahlavani, 2015] integrated fuzzy AHP weighting method and a simulated annealing (SA) algorithm for finding the optimal multi-modal route in a real static transportation network. The proposed model was implemented in a real public transportation system in MATLAB programming language. The results showed a high efficiency and speed of the proposed algorithm that support their analyses. Dib et al. [Dib et al., 2017] proposed a new formulation that adequately allows representing a public transit network, as well as, yielding the correct results when applying routing algorithms. Also, they introduce several strategies to accelerate the algorithm search process to deal with time consuming aspects of the problem. The results indicated high efficiency in terms of time consuming. Pahlavani and Ghaderi [Pahlavani and Ghaderi, 2017] used a non-dominated sorting genetic algorithm (NSGA-II) to solve the multi-modal multi-objective routing problem. Their algorithm proposed a set of non-dominated routes that had no absolute superiority to each other. Finally, the optimal route was determined using TOPSIS method from this set. The proposed algorithm proposed a better route in 89% and 87% of the routing cases than those of the genetic and the simulated annealing algorithms respectively. Haqqani et al. [Haqqani et al., 2017] proposed an adapted multi-criteria evolutionary algorithm, which incorporates passengers' preferences into the journey planner to solve the multi-modal multi-criteria journey planning.

These researches considered both multi-criteria and multi-modal transportation networks. The proposed web based system provides a multi-modal multi-criteria route planning system which has superior advantages over similar systems which reveals the need for its industrialization:

- Modeling users' ambiguity in the criteria weighting by fuzzy AHP weighting method and their different decision strategies by Q-OWA operators.
- Using TOPSIS method to determine the best routes among the semi-optimal routes which K-shortest path algorithm proposed.

3. Proposed Method

In our proposed hybrid MCDM method, firstly, the effective criteria and their values for each alternative must be determined. Then, in fuzzy AHP Q-OWA weighting method, relative weighs are calculated by the pairwise comparison matrix of fuzzy AHP method. Accordingly, for integrating these weights and calculating the ordered weights, the quantifier-guided OWA is used. The main characteristic of this hybrid multi-criteria decision making (MCDM) is supporting different decision strategies in calculating the impedances. Finally, after deterring the *K*-shortest routes, the best alternative is determined using TOPSIS method (Figure 2).

3.1 Criteria Modeling

A critical problem in multi-modal multi-criteria route planning is criteria modeling. In this study, we considered three criteria: (a) Time: in this study, time modeling for BRT, subway, and bus modes has been done according to the timetable. In this regard, Table 1 presents a part of the subway timetable.

For taxi and walking transportation modes, the time needed to pass each edge/intersection of the transportation network is an average assigned time during 5:30 AM to 8:30 AM.

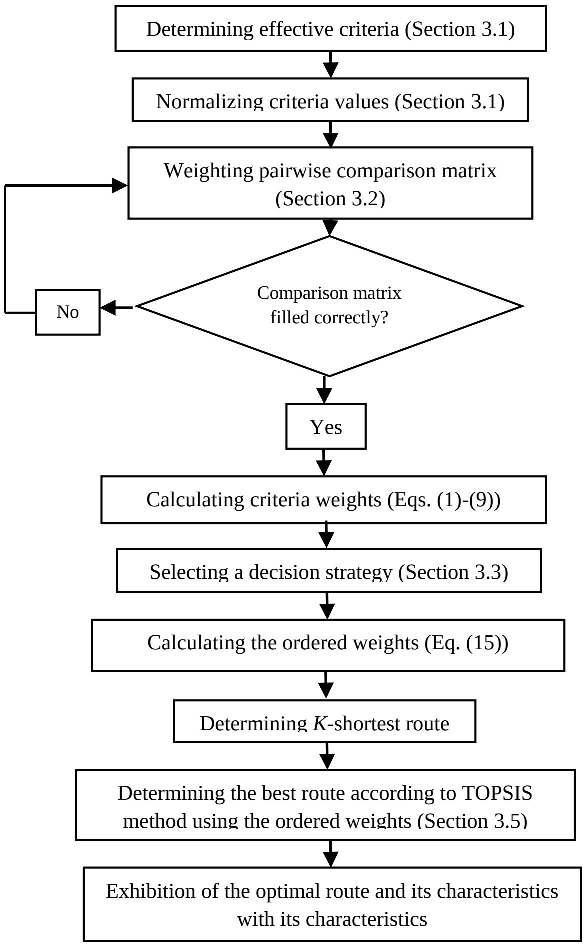


Figure 2. The proposed hybrid MCDM in route planning's flow chart

This time interval divided into smaller intervals with 15 minutes. Therefore, passing-time value retrieves its related time-window according to start time of the travel from the origin. For instance, if a travel is started in 7:00 AM, for the first 15 minutes, taxi and walking modes retrieves from 7:00 AM to 7:15 AM and after that the passing-time retrieves from 7:15 AM to 7:30 AM, and so on. (b) Fare: it is clear that walking mode has no fare but the other modes have their specific fare. For bus and BRT mode, each line has its specific fare but in the subway and the taxi (personalized car) modes the fare depends on the route length. (c) Minimum changes of transportation modes: changes of transportation modes may cause inconvenience to passengers. Therefore, it is better that the proposed route has the minimum changes of transportation modes.

To compare alternatives, various criteria of different alternatives must be normalized [Nadi and Delavar, 2011]. As the determined criteria in this study are benefit criteria, the maximum score method is used as follows [Malczewski, 1999]:

$$x_{i_j}^{N} = \frac{x_{i_j} - x_{\min_j}}{x_{\max_j} x_{\min_j}}, \tag{1}$$

where x_{i_j} is the i^{th} value of criterion j and $x_{i_j}^{N}$ is its normalized value, and $x_{\max_j}$ and $x_{\min_j}$ are the maximum and minimum value for j criterion, respectively in a way $x_{\min_j}$ is

Table 1. A part of the subway timetable

Stations / No.	Shohada Sq.	Darvaze shemiran	Darvaze dolat	Ferdowsi	Valieasr	Enghlab Sq.	Tohid	Azadi
1	5:40	5:43	5:45	5:47	5:50	5:52	5:55	5:57
2	5:50	5:53	5:55	5:57	6:00	6:02	6:05	6:07
3	5:58	6:01	6:03	6:05	6:08	6:10	6:13	6:15
4	6:06	6:09	6:11	6:13	6:13	6:18	6:21	6:23

calculated using Dijkstra algorithm. To calculate $x_{\max_j}$, 500 random routes between the origin and destination points would be generated and the maximum value for criterion *j* is considered as its value.

3.2 Fuzzy Analytical Hierarchy Process (Fuzzy AHP)

The Analytic Hierarchy Process (AHP), is the most useful method where many criteria are investigated in complex problems, involving human judgments and qualitative parameters [Naemi et al., 2014]. Despite the general popularity, the AHP method due to its inability in integrating inherent ambiguity and lack of clarity in mapping perceptions of decision makers with exact numbers is criticized [Deng, 1999]. To solve this problem, Buckley [Buckley, 1985] introduced a fuzzy AHP method. In this weighting method, fuzzy numbers are used for pairwise comparison. In this study, in fuzzy AHP Q-OWA weighting method, triangular fuzzy numbers are used for pairwise comparison. Preference of *i* criterion against *j* criterion and also preference of *j* criterion against *i* criterion, respectively, indicated as follows [Zare et al., 2016]:

$$\tilde{a}_{ij} = (a_{ij}, b_{ij}, c_{ij}), \tag{2}$$

$$a_{ji} = (1/c_{ij}, 1/b_{ij}, 1/a_{ij}). \tag{3}$$

Each triangular fuzzy number has linear representation such that its member function can be represented as follows [Zare et al., 2016]:

$$\mu(z) = \begin{cases} o & z < a \;\; or \;\; z \ge c \\ \dfrac{z-a}{b-a} & a \le z < b \, . \\ \dfrac{z-c}{b-c} & b \le z < c \end{cases} \tag{4}$$

Finally, the pairwise comparison matrix is formed as follows [Zare et al., 2016]:

$$A = \begin{bmatrix} (a_{11}, b_{11}, c_{11}) & \cdots & (a_{1n}, b_{1n}, c_{1n}) \\ \vdots & \ddots & \vdots \\ (a_{n1}, b_{n1}, c_{n1}) & \cdots & (a_{nn}, b_{nn}, c_{nn}) \end{bmatrix} \tag{5}$$

Fuzzy AHP preference scale to form the comparison matrices is presented in Table 2.

Table 2. Fuzzy AHP preference scale to form the pairwise comparison [Prakash, 2003]

Fuzzy AHP scale of importance for pairwise comparison	Numeric rating	Reciprocal
Absolute	(7,9,11)	(1/11,1/9,1/7)
Very, very strong	(5,7,9)	(1/9,1/7,1/5)
Very strong	(3,5,7)	(1/7,1/5,1/3)
Moderate	(1,3,5)	(1/5,1/3,1)
Weak	(1,2,4)	(1/4,1/2,3/1)
Equal	(1,1,1)	(1,1,1)

For normalization the fuzzy numbers in pairwise comparison, geometric mean method is used. Normalized fuzzy numbers for each criterion are calculated using Equations. (6) to (8) as follows [Sivrikaya et al., 2015]:

$$a_i = \left(\prod_{j=1}^{n} a_{ij} \right)^{1/n} , \tag{6}$$

$$a = \sum_{i=1}^{n} a_i \, . \tag{7}$$

Similarly, b_i, c_i, b, c are calculated and finally, the normalized numerical value of criterion *i*, $\mu_i(z)$, can be calculated as follows [Sivrikaya et al., 2015]:

$$\mu_i(z) = \left(\frac{a_i}{c}, \frac{b_i}{b}, \frac{c_i}{a} \right). \tag{8}$$

After forming the normal matrices of criteria, the next step is altering the fuzzy numbers to the exact ones. For this purpose, the centroid defuzzification method is used [Ross, 2009]:

$$w_i = \frac{\int\left(\mu_i\left(z\right).z\right)dz}{\int\mu_i\left(z\right)dz}, \tag{9}$$

where w_i is the final weight of the i^{th} criterion.

Just like the AHP method, the fuzzy AHP also has a checking method for control consistency in the pairwise comparison matrix which is beyond the scope of this paper (see [Leung and Cao, 2000] for details).

3.3 Quantifier-guided OWA operators

OWA operators are a set of multi-criteria combination operators. To model a family of parameterized decision strategies, Yager [Yager, 1988] introduced the ordered weighted averaging (OWA) operators. This method is able to calculate the user's risk taking and risk aversion, as well as entered them for selecting the final option. An OWA operator is an integrating operator, F, with corresponding weight vector $w \in [0,1]^n$, ($\sum_{i=1}^{n} w_i = 1$), in which for an input set $X = (x_1, x_2 ... x_n)$ the resulted F will be:

$$F_w\left(x_1, x_2, ..x_n\right) = \sum_{i=1}^{n} w_i b_i, \tag{10}$$

where b is a permutation that sorts input vector X in a descending order ($b_n \leq b_{n-1} \ldots \leq b_1$). OWA operators have two main characteristic that indicate their behavior [Yager, 1988], including (a) *ORness* degree, and (b) *Tradeoff*. *ORness* or degree of risk appetite indicates the situation of OWA operator between two logical operators "or" and "and" as follows [Llamazares, 2018]:

$$ORness = \frac{1}{n-1}\sum_{i=1}^{n}\left(n-1\right)w_i, \tag{11}$$

where the *ORness* > 0.5 indicates a risk appetite and optimistic decision maker, whereas the *ORness* = 0.5 indicates a neutral decision maker and the *ORness* < 0.5 indicates a risk aversion and pessimistic decision maker. Another important characteristic of the OWA operator is *Tradeoff* that indicates how much a criterion is influenced by the others. *Tradeoff* value represented in Equation (12) [Lenormand, 2017]:

$$Tradeoff = 1 - \sqrt{\left(\frac{n}{n-1}\right)\sum_{i=1}^{n}\left(w_i - \frac{1}{n}\right)^2}. \tag{12}$$

The higher amount of the *Tradeoff* indicates more influence in criteria and vice versa. In this study, a class of relative quantifiers, called "Regular Increasing Monotone (RIM)" is used. For defining this class of quantifiers, Equation (13) would be used [Hong and Kyunggido, 2016]:

$$Q\left(p\right) = p^{\alpha}, \tag{13}$$

where by changing α, different decision strategies and their operators could be achieved. Different decision strategies and their corresponding α have been presented in Table 3. The order weights vector W can be calculated using RIM quantifiers as showed in Equation (14) [Malczewski, 2006]:

$$W_j = \left(\frac{\sum_{k=1}^{j} w_k}{\sum_{k=1}^{n} w_k}\right)^{\alpha} - \left(\frac{\sum_{k=1}^{j-1} w_k}{\sum_{k=1}^{n} w_k}\right)^{\alpha}, \tag{14}$$

where w is the relative criteria weights vector. Since the relative weights used in this study is calculated by fuzzy AHP weighting method, w will be a normal vector and as a result $\sum_{k=1}^{n} w_k = 1$. Hence, Equation (14) is simplified to Equation (15) as follows [Malczewski, 2006]:

Table 3. Different decision strategies and their corresponding α [Nadi and Delavar, 2011]

Decision strategy	At least one (or)	Few	Some	Half	Many	Most	All (and)
α	0.0001	0.2	0.5	1	2	5	10000

$$W_j = \left(\sum_{k=1}^{j} w_k\right)^{\alpha} - \left(\sum_{k=1}^{j-1} w_k\right)^{\alpha}. \qquad (15)$$

3.4 *K*-Shortest Path Routing Algorithm

The *K*-shortest path algorithm is an extension of the well-known shortest path algorithm. Unlike the shortest path algorithm, this algorithm returns an ordered series of routes, i.e., $\{r_1, r_2, \ldots, r_k\}$, between origin and destination nodes so that the reminder routes have bigger cost value than bigger cost value of the maximum cost of the obtained series (cost of the k^{th} route). In this research, an extended Dijkstra algorithm was used for determining this set (Figure 3). In this research, cost value for a route with *l* edge is calculated as follows:

$$Im = \sum_{i=1}^{l}\sum_{j=1}^{m} x_{i_j}^{N} \times w_j, \qquad (16)$$

where $x_{i_j}^{N}$, w_j, l, and m are the j^{th} normalized criterion value for edge *i*, the value of weight of criterion *j* that is calculated using fuzzy AHP method, the number of edges, and the number of criterions (i.e. 3 in this research), respectively.

3.5 TOPSIS Method

Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) was originally developed by Wang and Yoon [Wang and Yoon, 1981]. This method is used for Prioritization of the different alternatives based on distance of the ideal alternative [Wang and Yoon, 1981]. TOPSIS selects the alternative that is the closest to ideal solution and farthest from the worst alternative. The ideal solution is formed as a composite of the best performance value exhibited by any alternative for each attribute and the negative-ideal solution is the composite of the worst performance values [Mateo, 2012].

1. Determine the shortest path *P*1 from *s* to *t* in a graph *G* by using the Dijkstra's shortest path algorithm.

2. Assume that *k*-1 (where *k* = 2, 3 . . . *K*) shortest paths are already determined and stored in list **A** and candidate paths for next shortest path are stored in list **B**.

3. In order to determine the shortest path P_k, get the shortest path P_{k-1} and let the path be $<s, v^1_{k-1}, v^2_{k-1} \ldots v^l_{k-1}, t>$ and the set of vertices to be analyzed is $DS = \{s, v^1_{k-1}, v^2_{k-1} \ldots v^l_{k-1}\}$.

4. For each vertex *v* in *DS* do

    1. If there exists a path P_j in list **A** that has the path $<s, v^1_{k-1}, v^2_{k-1} \ldots v>$ as the sub path. Then set the weight of the edge from *v* to its immediate neighbor to infinity for P_j.

    2. Set the sub path $< s, v^1_{k-1}, v^2_{k-1} \ldots v >$ in P_{k-1} as the root path, R_k. Set the path to be determined from *v* to *t* is as the spur path, S_k. Remove the vertices in the R_k from the graph. So that they are not repeated in spur path.

    3. Compute the shortest path from *v* to *t* by using the Dijkstra's algorithm.

4. If a path is found and returned by Dijkstra's algorithm, then add both R_k and S_k to form a candidate path, for next shortest path. Add this path into list **B** and continue.

5. Choose the path from list **B** with shortest distance as P_k and move it into list **A**.

6. Go to step 3 and continue until K shortest paths have been determined.

Figure 3. Pseudo-code of Dijkstra algorithm for finding *K*-shortest route in a graph [Nagubadi, 2013]

A relative advantage of TOPSIS is the capability to identify the best alternative quickly. This method has six steps [Beheshtinia and Ahangareian, 2018]:

- *Step 1: Forming the decision matrix*

If our multicriteria decision making problem includes n alternatives and m criteria, the decision matrix created as showed in Table 4. In this research this alternatives are K-shortest paths determined using Yen's algorithm.

Table 4. A decision matrix

	C_1	C_2	...	C_m
A_1	r_{11}	r_{12}	...	r_{1m}
A_2	r_{21}	r_{22}	...	r_{2m}
$\vdots$	$\vdots$	$\vdots$		$\vdots$
A_n	R_{n1}	R_{n2}	...	R_{nm}

where, A_i is the i^{th} alternative (i^{th} shortest route), C_j is the j^{th} criterion and x_{i_j} is the value of the j^{th} criterion for the i^{th} alternative.

- *Step 2: Normalizing the decision matrix*

The normalized decision matrix (S) is calculated as follows [Beheshtinia and Ahangareian, 2018]:

$$s_{i_k} = \frac{x_{i_j}}{\sqrt{\sum_{j=1}^{m} x_{ij}^{2}}}, i = 1,2,...,n; j = 1,2,...,m \tag{17}$$

where s_{i_k} is the normalized value of x_{i_k} and $S = \lfloor s_{i_j} \rfloor$ ($i = 1,2...,n$, $j = 1,2....,m$).

- *Step 3: Calculating the weighted normalized decision matrix*

This matrix is calculated according to Equation (18) [Beheshtinia and Ahangareian, 2018]:

$$V_{n\times m} = V_{n\times m} \times W_{m\times m}, \tag{18}$$

where, W is the weights matrix that can be calculated with different weighting methods (in this study, the weights calculated with the fuzzy AHP Q-OWA are used for calculating V matrix) [Beheshtinia and Ahangareian, 2018]:

$$W = \begin{bmatrix} W_1 & 0 & 0 & 0 \\ 0 & W_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & W_m \end{bmatrix}_{m\times m}. \tag{19}$$

- *Step 4: Determining ideal and negative ideal alternatives*

Ideal alternative (A^+) is an assumptive route that has the best level for all considered criteria and the negative ideal alternative (A^-) is an assumptive route that has the worst criteria values calculated as follows [Beheshtinia and Ahangareian, 2018]:

$$\begin{aligned} A^+ &= \left\{(\max_i V_{ik} \mid k \in K^+), (\min_i V_{ik} \mid j \in J^-) \mid i = 1,2,...,n\right\} \\ &= \left\{V_1^+, V_2^+, ..., V_m^+\right\}, \\ A^- &= \left\{(\min_i V_{ik} \mid k \in K^+), (\max_i V_{ik} \mid j \in J^-) \mid i = 1,2,...,n\right\} \\ &= \left\{V_1^+, V_2^+, ..., V_m^+\right\} \end{aligned} \tag{20}$$

where, J^+ and J^- are relating to the benefit and cost criteria, respectively and V_j^+, V_j^- are the values of the ideal and the negative ideal alternative for the j^{th} criterion.

- *Step 5: Calculating distance of the ideal and the negative ideal alternatives*

These two distances are calculated according to Equation (21) [Beheshtinia and Ahangareian, 2018]:

$$d_i^+ = \sqrt{\sum_{j=1}^{n} (V_{ij} - V_j^+)^2}, \quad , i = 1, 2, .., n, \quad d_i^- = \sqrt{\sum_{j=1}^{n} (V_{ij} - V_j^-)^2}, \tag{21}$$

where d_i^+, d_i^- and n are the distance of the i^{th} alternative from the ideal, distance of the i^{th} alternative from the negative ideal alternative and number of alternatives, respectively.

- *Step 6: Calculating the relative closeness coefficient for each alternative*

The relative closeness coefficient for each alternative is calculated based on Equation (22) [Beheshtinia and Ahangareian, 2018]:

$$CC_i = \frac{d_i^-}{d_i^- + d_i^+}, i = 1, 2, ..., n, \tag{22}$$

i=1,2,...,n,

where, CC_i is the relative closeness coefficient of the i^{th} alternative.

- *Step 7: Prioritization of the alternatives*

In this step, prioritization of each alternative is determined according to its relative closeness coefficient. In which, each alternative that has the bigger relative closeness coefficient gets higher priority.

4. Experimental Results

4.1 The Proposed Web-Based System

The proposed method for the fuzzy AHP Q-OWA is implemented as a web-based GIS tool with PHP and C++ languages. PHP is widely used as a general purpose scripting language and it is especially suited for web development [Mitchell, 2016]. Also, it can be embedded to (X)HTML or XML and run on a web server. The proposed hybrid weighting method, i.e. fuzzy AHP Q-OWA and *K*-Shortest path algorithm, was implemented using C++ language. Then, this program was converted to the .exe file and used in the PHP program. MapServer is an open source development environment for building spatially enabled internet applications. It requires and processes requests coming from the user and returns output results to the user [Singh et al., 2012]. PostgreSQL and PostGIS were used as the database. PostgerSQL is an open source and object-relational database and PostGIS is an open source software that adds geographic objects. The way that the PostgreSQL/PostGIS connects to the MapServer is shown in Figure 4. At first, the proposed route planning system provides a pairwise comparison matrix to the user for comparing between each two criteria with an expression (Figure 5, Table 5).

The user can also save and load these weights for the next times. After calculating these weights, the user must select his/her desired decision strategy. Next, the ordered weights will be calculated. Before going to the route planning page, the user can either determine the degree of difficulty of each mode according to his/her preferences or use the default values of his/her route personalized route planning (Figure 5). Afterwards, by selecting the start and end nodes, the proposed method of this study proposes the best route (Figure 6) and its characteristics to the user (Figure 7).

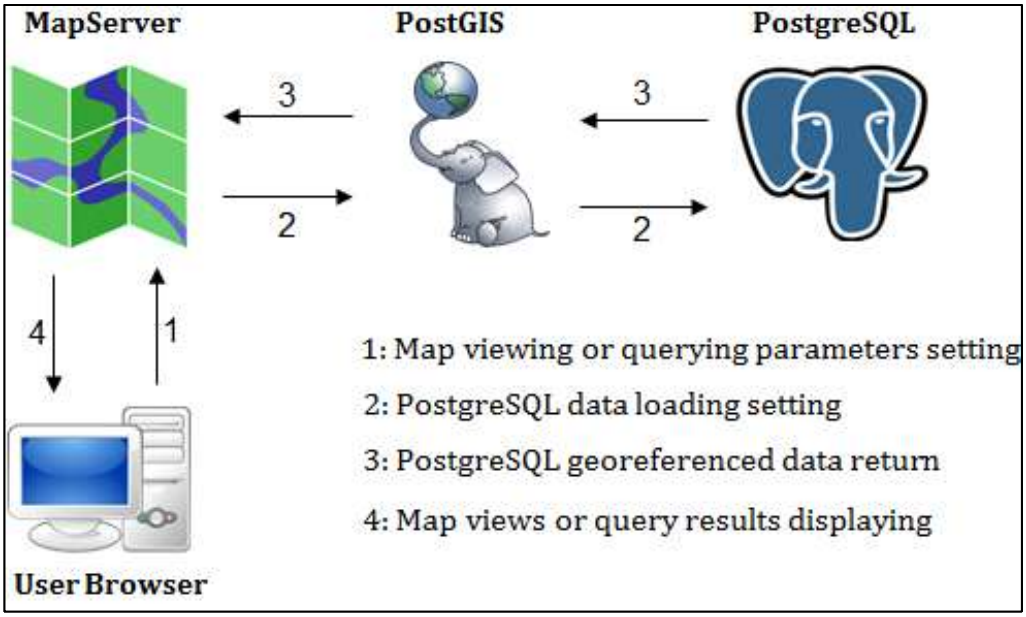


Figure 4. Connection of PostgreSQL/PostGIS to MapServer

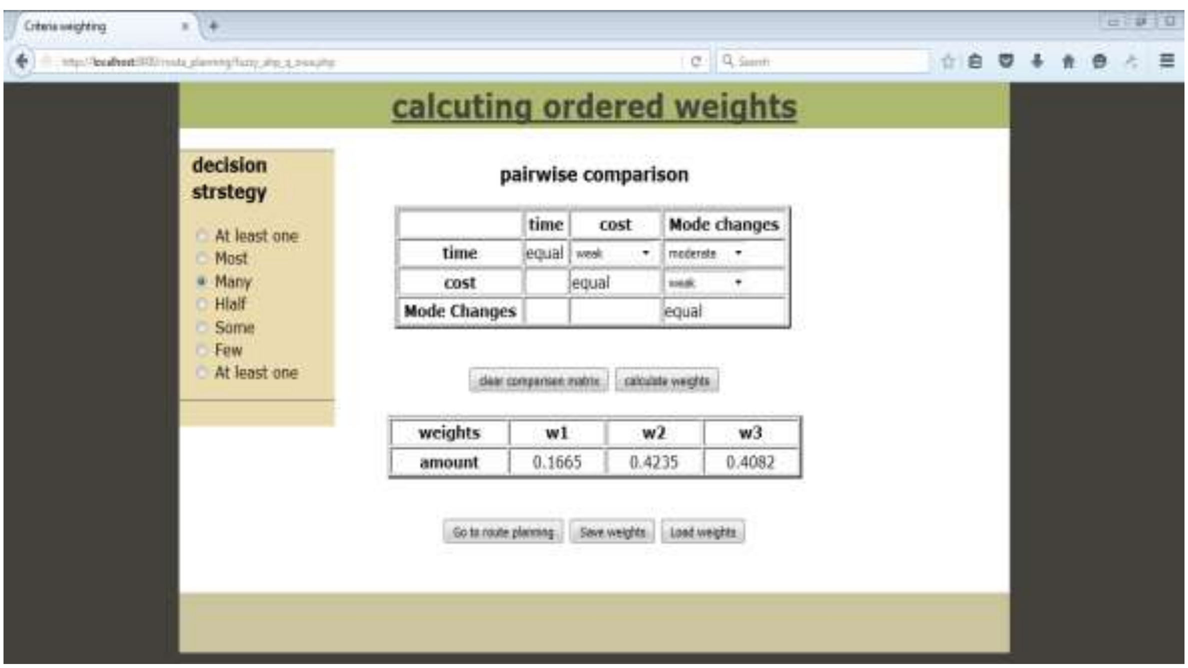


Figure 5. Devoted configuration wizard for the pairwise compressions

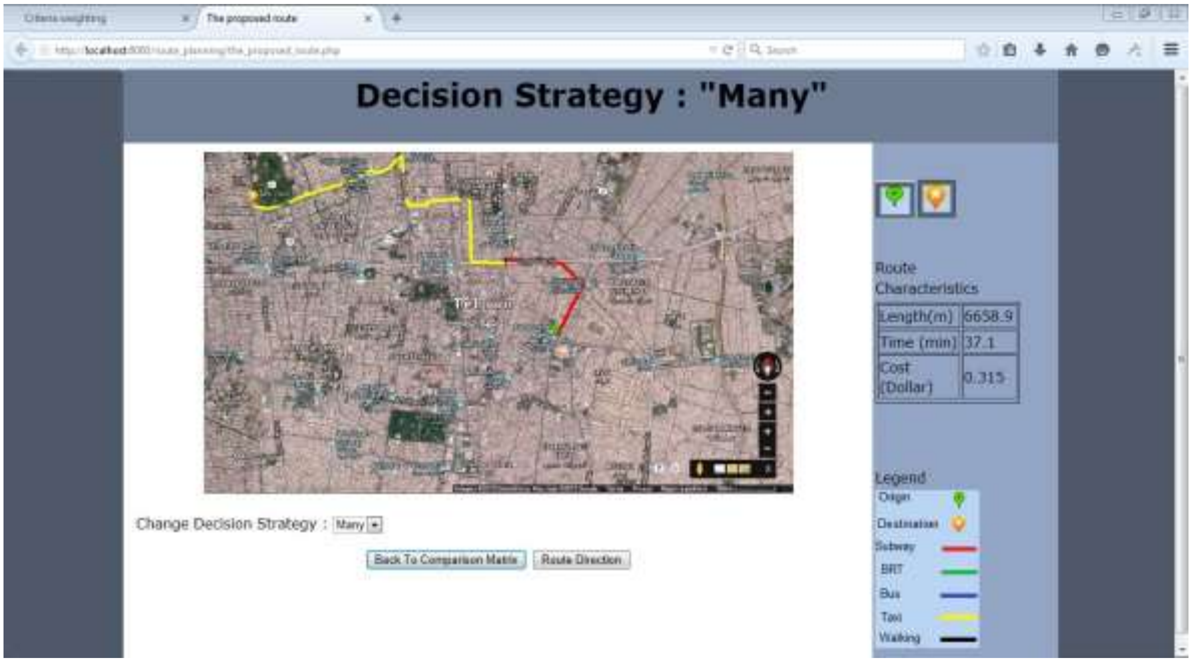
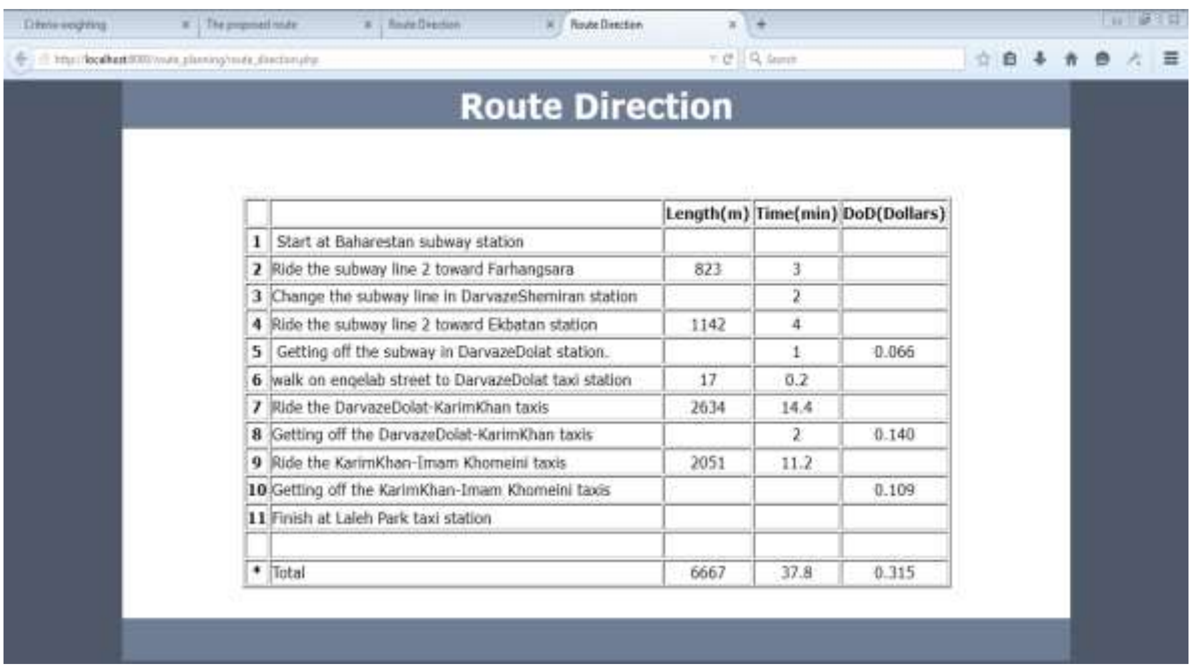


Figure 6. General user interface of the proposed route planning system



Route Direction

		Length(m)	Time(min)	DoD(Dollars)
1	Start at Baharestan subway station			
2	Ride the subway line 2 toward Farhangsara	823	3	
3	Change the subway line in DarvazeShemiran station		2	
4	Ride the subway line 2 toward Ekbatan station	1142	4	
5	Getting off the subway in DarvazeDolat station.		1	0.066
6	walk on enqelab street to DarvazeDolat taxi station	17	0.2	
7	Ride the DarvazeDolat-KarimKhan taxis	2634	14.4	
8	Getting off the DarvazeDolat-KarimKhan taxis		2	0.140
9	Ride the KarimKhan-Imam Khomeini taxis	2051	11.2	
10	Getting off the KarimKhan-Imam Khomeini taxis			0.109
11	Finish at Laleh Park taxi station			
*	Total	6667	37.8	0.315

Figure 7. Route direction

Table 5. The considered criteria and the pairwise preferences between them for an arbitrary user

Criterion	Time	Cost	Mode changes
time	Equal	Weak	Moderate
cost		Equal	weak
Mode changes			Equal

4.2 The Proposed System Verification

To verify the proposed model and illustrate its application in real world multi-modal multi-criteria route planning, the transportation network of a central area of Tehran city was used (Figure 9).

The considered area has 21 km^2 and consisted of 2 BRT lines with 24 BRT stop stations, 28 sweep bus lines with 203 bus stop stations, and 4 sweep subway lines with 17 subway stop station, and totally more than 450 km of roads. The proposed system has been implemented on a computer with these specifications: Core^{TM} i5, RAM 4GB and 64-bit operation system. Maximum running time of the system was not exceeding 8 seconds.

Time and fare criteria for each link of the considered modes were obtained by the departments of United Bus Company of Tehran, Tehran Urban and Suburban Railway Operation Company, and Tehran municipality. The proposed method of this study was implemented for a route from the Baharestan square to the Enghelab square that is one of the most crowed routes in our case study. Initially, the devoted configuration wizard for pairwise compression (Figure 5) was presented to 60 users (12 users for any decision strategy) and they were asked to weight the criteria. Then, the relative importance of each criterion was calculated considering the weights assigned in the pairwise compression matrix. Afterwarads, the users were asked to determine their desirable decision strategies from one of the strategies, including “at least one”, “few”, “half”, “many”, and “most”. Considering these 4 decision strategies, our system proposed 5 alternative routes with a description of their characteristic to the user. Accordingly, the user was asked to select one of these 5 routes. Results showed that 83.33% of the users with “at least one” decision strategy selected the proposed route of the system with the same decision strategy. This value for other decision strategies presented in Table 6.

Table 6. Percent of the users with a specific decision strategy that selected the proposed route of the system with the same decision strategy

Decision strategy	Percent of the users
“at least one “	83.33
“few”	75.00
“half”	91.67
“many”	83.33
“most”	91.67

Table 6 reveals that on average 85.00% of the users with different decision strategies selected the model’s proposed route as the best route. For example, the manner of selecting the best route from 50 alternatives obtained from *K*-shortest path algorithm method according to the weights showed in Figure 6 is as follows:

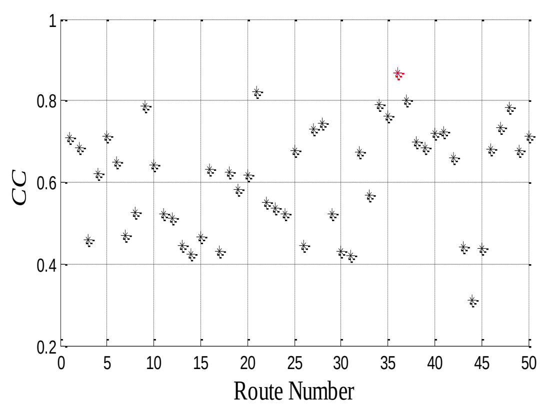


Figure 8. *CC* value for 10000 random routes and determining the best route that has the most *CC* value (*)

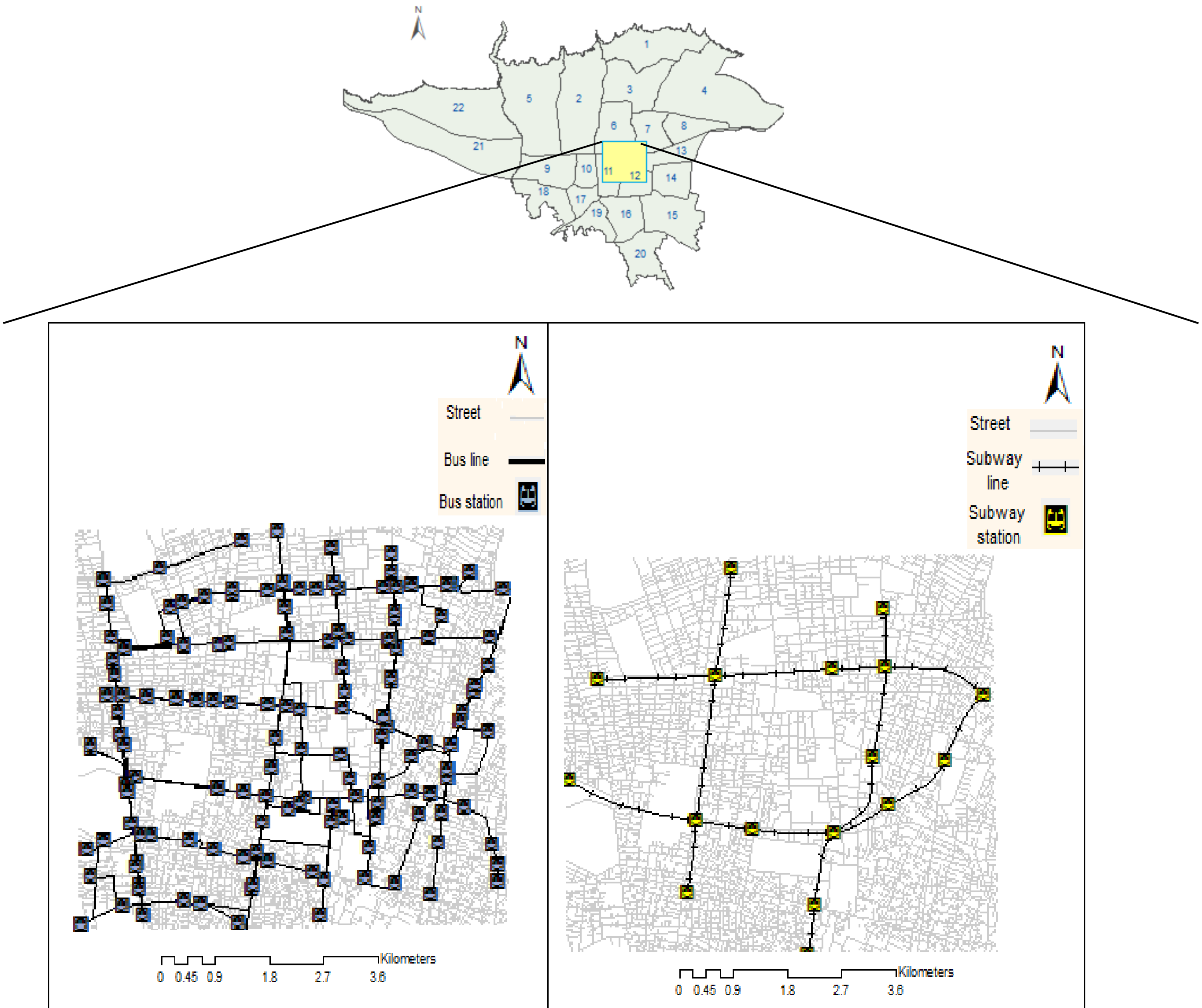
Table 7. Showing 20 first random routes and the best route features and ranking

Route	Time (min)	DoD (rail)	Mode changes	d^-	d^+	CC	Ranking
1	38.52	0.334	4	0.0334	0.0137	0.7092	15
2	40.18	0.329	4	0.0322	0.0149	0.6838	17
3	37.1	0.387	6	0.0253	0.0297	0.4598	41
4	39.08	0.341	5	0.0290	0.0177	0.6211	28
5	40.42	0.319	4	0.0339	0.0137	0.7127	14
6	38.75	0.336	5	0.0303	0.0163	0.6502	24
7	38.53	0.359	7	0.0231	0.0260	0.4711	39
8	44.91	0.311	6	0.0285	0.0258	0.5248	34
9	39.19	0.309	4	0.0371	0.0100	0.7877	5
10	40.53	0.326	5	0.0300	0.0168	0.6407	25
11	43.09	0.338	5	0.0253	0.0231	0.5226	36
12	40.91	0.344	6	0.0237	0.0227	0.5114	38
13	40.38	0.379	5	0.0223	0.0279	0.4440	43
14	40.67	0.368	6	0.0201	0.0273	0.4250	48
15	41.07	0.342	7	0.0221	0.0252	0.4672	40
16	39.67	0.321	7	0.0333	0.0195	0.6304	26
17	37.92	0.377	7	0.0223	0.0296	0.4299	47
18	39.18	0.326	6	0.0296	0.0177	0.6261	27
19	40.04	0.344	5	0.0272	0.0194	0.5835	30
20	39.05	0.359	3	0.0322	0.0198	0.6191	29
⋮	⋮	⋮		⋮	⋮	⋮	⋮
36	37.0	0.315	3	0.0410	0.0062	0.8685	1
⋮	⋮	⋮		⋮	⋮	⋮	⋮

As shown in Figure 8, the best route is route 36 that has maximum *CC* value. Time, fare, and mode changes, *CC* value and final ranking of each route for 20 first random routes and the best route (number 36) are shown in Table 7. In this table, *CC* value for first 20 shortest paths and the best path (number 36 which has maximum *CC* value) among 50-shortest path between the origin and destination points have been determined. Similarly, the best routes for other decision strategies are shown in Figure 10 (a)-(d).

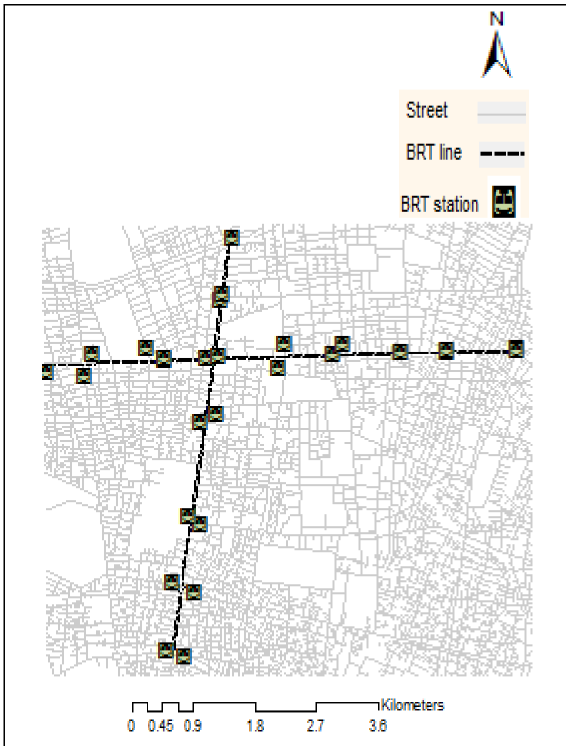
5. Conclusion

This paper proposed a multi-criteria decision making model based on integrating fuzzy-AHP, quantifier-guided OWA operators and TOPSIS method for finding the best multi-modal personalized route in a real transportation network. In this study, subway, BRT, bus, taxi, and walking transportation modes were considered for traveling. Also, time, fare, and minimum changes in mode of transportation were considered as the effective criteria. This model was implemented in a web-based geographical information system for an area in the center of Tehran. For finding this route, these five main steps were considered respectively: (a) determining effective criteria in multi-modal route planning, (b) weighting these criteria with fuzzy-AHP method, (c) calculating the ordered weights using Q-OWA operators, (d) determining a set of optimal routes using K-shortest path algorithm, and (e) using TOPSIS method to determine the best route between these K routes. By considering different OWA operators, different decision strategies were obtained and the model proposed different routes based on different decision strategies.

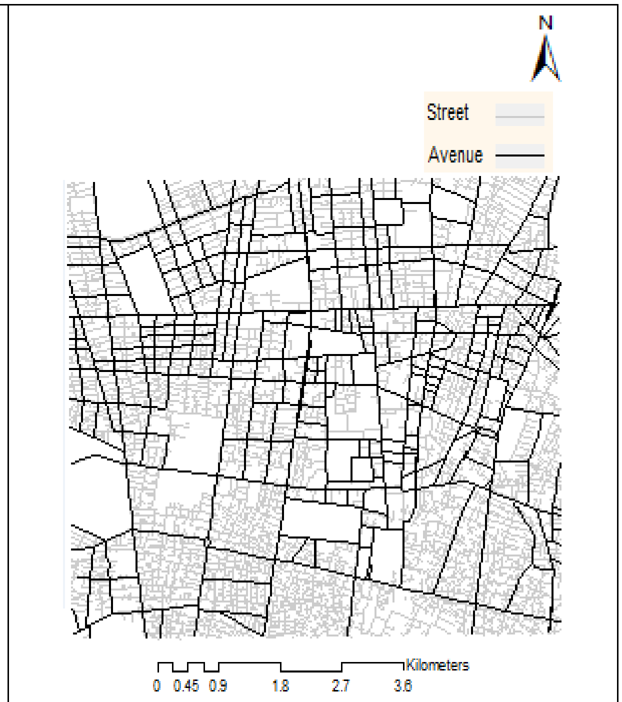


(a) Bus network (b) Subway network

Figure 9. The study area



(c) BRT network



(d) Avenue network

(a) Decision strategy: "At least one"

(b) Decision strategy: "Few"

(c) Decision strategy: "Half"	
(d) Decision strategy: "Most"	

Figure 10. Different proposed routes by considering different decision strategies (the proposed route considering "Many" Decision Strategy was illustrated in Figure 6)

The proposed system used by 60 users (five decision strategies and twelve users for each decision strategy). Results showed that on average 85.00% of the users with different strategies selected the model's proposed route as the best route. In future studies, this model can be implemented in a transportation network dataset of a metropolis with too large and highly complex search space. Also, implementation of the proposed system as a location-based service (LBS) for in-vehicle usage that combines real-time transportation network datasets such as traffic volume into the proposed system could be studied in further researches. As well as, other ranking method such as VIKOR [Beheshtinia and Omidi, 2017] can be used to determine the best route among semi-optimal routes and their resulted can be compared to TOPSIS method.

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