A Fuzzy Two-Stage Capacitated Continuous *P*-Cent median Vehicle Routing Problem: A Self-Adaptive Evolutionary Algorithm

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Abstract

In this paper, a two-stage continuous *p*-center and *p*-median (namely *p*-centmedian) problem is developed. In the first step, a location problem is studied to compare the differences between the p-center and p-median by considering facility disruption. P-center problems are common in emergency situations with aim of minimizing the maximum distance between the facilities and costumers, while p-median problem aim is to minimize the total spent distance. Moreover, an integer linear programming is developed to deal with a time-window multi-depot capacitated vehicle routing problem in order to optimize the flows between facilities. This paper compares the mentioned p-center and p-median effects along with the vehicle routing problem as a two-step integrate problem. Since both steps are NP-hard, to deal with the problem in both stages a possibilistic programming, fuzzy single-objective programming is developed and solved by an efficient algorithm, namely self-adaptive differential evolution algorithm. Considering demand as a fuzzy parameter is an important factor and makes the problem more realistic, this feature is more considerable in emergency situations such as p-center problems. To improve the performance of results, the Taguchi method is used. In order to validate the results of the mentioned algorithms of small-sized test problems are compared with GAMS, also other valid metaheuristics are developed to be compared with the proposed algorithm in large-sized problems. The results show the capability of algorithm to generate near-optimal solutions. Also, the results demonstrate the *p*-median problem is more volatile against variation in the parameters while the *p*-center problem is more expensive.

Keywords: *P*-Median and P-Center problem, capacitated vehicle routing, Taguchi method, Fuzzy set, differential evolution.

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1. Introduction

Multi-facilities location-allocation problem is one of the most common problems in the literature. To determine a reasonable arrangement of new facilities in continuous space or a discrete set when a number of possible locations are finite (e.g., discrete location problem) or infinite (e.g., continuous problem), different approaches are proposed. In the last two decades, researchers have tried to solve continuous location problems with trustable solutions. A hybridized location optimization problem is a popular problem among different kind of location problems.

The main purpose of *p*-median problem is to minimize the total spent distance. (e.g., cities or factories) and first introduced by [Hakimi, 1964] while in *p*-center problem is to minimize the maximum distance as an objective function. *p*-median problems are divided in two main groups; namely discrete and continuous *p*-median problems. In discrete problems, there are some pre-determined locations to be assigned while in continuous problems everywhere of solution space can be considered as a place for facilities.

In other hand, a vehicle routing problem (VRP) is the second step of this paper to minimize the transportation costs between the facilities. VRP is the problem of visiting and serving by number of vehicles [Montoya-Torres et al. 2015]. Different heuristic and meta-heuristic approaches are proposed to solve the *p*-median problem. An algorithm based on the new graph theory developed by [Rabbani and Yousefnejad, 2013]. A general algorithm to solve a wide range of continuous location problems proposed by [Blanco, Puerto and Ben-Ali, 2016]. A genetic algorithm with precise and fast answers. In some cases, continues problems can be changed to the discrete problems [Stanimirovic and Ciric, 2011]. A genetic algorithm with greedy search to solve the *p*-median problem considered by [Neema ,Maniruzzaman and Ohgai, 2011]. A new genetic algorithm using greedy search with better efficiency to deal with continuous problems proposed by [Kazakovtsev et al. 2015]. Meta-heuristic algorithms are able to generate near-optimal solutions, but do not guarantee any optimal solution. They can be merged with other algorithms to improve their efficiency [Mehrjerdi and Nadizadeh, 2013].

In a VRP, vehicles leave a depot in the network and return to the first place after completion of their assigned rout. This problem was firstly proposed in the literature by [Dantzig and Ramser, 1959]. A capacitated VRP (CVRP) is one of the most prevalent branches of the VRP, which considers a specific number of loads for every vehicle. Different approaches are proposed to solve the CVRP, a linear programming to solve the CVRP proposed by [Lightner-Laws et al. 2016], a metaheuristic to solve CVRP proposed by [Szeto, Wu and Ho, 2011] . A Bat algorithm to solve the CVRP developed by [Zhou et al. 2016]. An uncertain CVRP which minimize the cost of vehicles, fuel consumption and products shortage presented by [Tavakkoli-Moghaddam et al. 2016]. A capacitated location-routing problem is formulated as a reverse logistic system by using a greedy clustering method to cluster the costumers and ant colony system to find the best routs for vehicles by [Nadizadeh and Hosseini Nasab, 2019]. A novel model of the capacitated vehicle routing problem with time windows is developed and solved by simulated annealing method [Rabbani et al. 2018]

The literature review demonstrates that there is a gap in *p*-continuous and *p*-median problems, considering the flow of products between the facilities led us to develop a second stage for the locating problem to compare the differences between the routing in *p*-median base located facilities and *p*-center base located facilities. Considering other new aspects of the problem makes this paper more applicable, especially when there is an emergency situation considering time windows. Facility disruption is another feature, which is possible in sites, so when there is a nonstable situation and nominated locations are possible to be out of reach, the decision makers face with a multi-aspect problem. To best of our knowledge this is the first paper that takes into account such features in *p*-median and *p*-center problems. A new particle swarm optimization method comprised of local and global search processes to deal with a capacitated location-routing problem (LRP) with stochastic demands proposed by [Marinakis, 2015]. A hybrid evolutionary algorithm to deal with the LRP considering time constraints developed by [Koç et al. 2016]. A multi-depot inventory-routing problem (IRP) to minimize the transportation and holding costs with a variable neighborhood search (VNS) algorithm along with a simulated annealing (SA) algorithm to avoid trapping in local optimum solutions solved by [Nikkhah Qamsari, Hosseini Motlagh, and Jokar, 2017]. Uncertainty is the feature that makes the model more compatible with the real situation. In this regard a fuzzy mathematical model to deal with the VRP with backhauls used by [Yalcın and Erginel, 2015]. One of the most important parameters from the manager’s point of view is the amount of demands. Since this parameter is an uncertain one, a two-echelon LRP with simultaneous pickup and delivery under fuzzy demands introduced by [Ghatreh Samani and Hosseini-Motlagh, 2017].

Besides the demand, there are other important factors that can makes the model more realistic. Time windows are prevalent factors in a VRP with wide usage in different cases. However, this parameter increases the complexity of the VRP and needs to be solved by efficient approaches. A hybrid particle swarm optimization (PSO) with SA to use the benefits of them for the problem used by [Alinezhad et al. 2018]. This parameter can be considered as an uncertain parameter [Ghannadpour and Zarrabi, 2017; Issabakhsh et al., 2018]. As previously described, a fuzzy uncertainty is used to deal with non-determinative parameters (e.g., costumer demands). Thus, this approach is used in our research that has increased the complexity of the solution method.

To overcome these shortcomings and fulfill these gaps, we develop a mathematical model for a continuous *p*-center and *p*-median (i.e., *p*-centmedian) problem with capacity of open centers and a mathematical model for a multi depot capacitated vehicle routing problem to investigate the trade-off between the total cost of proposed models. Moreover, an efficient algorithm, called self-adaptive differential evolution algorithm is proposed to solve the developed mathematical model in a reasonable amount of time and obtain near-optimal solutions.

In this paper, two types of continuous location problems are proposed. Our intention is to compare the difference between the total costs of both *p*-median and *p*-continuous problem in first step and compare the problems with vehicle routing in next step and demonstrate the results. Since the decision process of this problem is comprised of different aspects, we consider other management decision-making dimension to bring the problem closer to real case including the facility disruption probability, which is possible for various reasons (e.g., firefighting facility and earthquake). The basic questions of this study are as follows:

- Which model is more compatible under facility disruption?
- Which model is more economical?
- Which model is more efficient when there is a time window for delivery?

Because of extensive applications of a *p*-median problem in delivery centers such as distribution companies and suppliers of raw materials and applications of *p*-center problems in emergency activities such as fire Stations and disaster relief location problems, it is necessary to extend this topic and add new feature like scheduling and facility disruption risk. Table 1 depicts the summary of recent researches in this area.

Table 1. Overview of the literature on the *p*-median and *p*-center problems.

Paper	Objective	Methodology	Feature			
			Capacity of vehicles	Capacity of open locations	Time window	Reliability
[Dantrakul et al., 2014]	Minimization of setup cost and transportation cost	Greedy algorithm	✓	–		
[Kazakovtsev et al., 2015]	Minimization of distance	Greedy algorithm	–	–		
[Nematian and Sadati, 2015]	Minimization of the maximum distance	Possibility theory and fuzzy random chance-constrained programming	–	✓		
[Irawan and Salhi, 2015; Nematian and Sadati, 2015]	Minimization of distance	Variable neighborhood search	–	✓		
[Colmenar et al., 2016]	Minimization of the maximum distance	Heuristic method – based on the greedy random search	–	–		
[Maleki and Abbasi, 2015]	Minimization of the maximum distance	Firefly method	–	–		
[Basappa et al., 2015]	Minimization of the maximum distance	Cole's parametric search	–	–		
[Callaghan et al., 2017]	Minimization of distance	Reexamined and efficient neighborhood reductions	✓	✓		
This paper	Minimization of transportation, safety and disruption costs	A self-adaptive evolutionary approach	✓	✓	✓	✓

The main features of this paper are as follows:

- Designing a novel integer linear mathematical model for a continuous *p*-center problem.
- Designing a novel integer linear mathematical model for a continuous *p*-median problem.
- Considering a capacity for the number of open facility for both *p*-center and *p*-median problems.
- Proposing a chance constrained method in order to increase the reliability of locations capacity limitation.
- Formulation of safety factors based on the scale of natural disasters in the nominated places
- Designing an integer linear mathematical model for a multi-depot VRP for the second stage and time windows for demands.
- Developing an efficient new hybrid evolutionary algorithm to efficiently solve the large-sized instances in a reasonable amount of time.
- Proposing a possibilistic programming approach based on the ME measure and fuzzy multi-objective programming approaches.
- Parameters tuning by the Taguchi method for the self-adaptive evolutionary approach.

2. Problem statement and methodology

The mathematical model of a facility location problem is considered as a binary integer-programming problem, based on [Dantrakul, Likasiri and Pongvuthithum, 2014]. In this paper, the vehicles are considered unequal with a specific uniform speed range and variable usage cost, which are dependent to the vehicle type in distance. There is a possibility for unpredictable accidents in facilities based on the expert judgments. The following notations are presented to express the mathematical model. Following, we propose a *p*-median and *p*-center problem as the first step of our research.

I	Set of clients ($I = \{1,\ldots,n\}$)
J, K	Set of potential facility sites ($J = \{1,\ldots,m\}$)
c_{ij}	Transportation cost from client *i* to facility *j*
VC_{ij}^{v}	Mean variable cost of vehicles
t_{ij}^{v}	1 if client *i* is assigned to facility *j* by vehicle *v*; and 0, otherwise
I, J	Node set of a complete graph
x_{ij}	1 if client *i* is assigned to facility *j*; and 0, otherwise
y_j	1 if facility *j* is opened; and 0, otherwise
P	Number of active locations
V	Set of available vehicles
W_v	Capacity of each vehicle *v*
$\tilde{d}_i$	Demand of client *i*
$time_{jk}$	Mean travel time from node *j* to node *k*
U_i	Upper bound of costumer time window
L_i	lower bound of costumer time window
P^j	Probability of disruption at facility cite
Fc_j	Estimated cost of rebuilding facility
$speed_{jk}$	Mean speed of vehicle from node *j* to node *k*
cap_i	Required capacity of costumer order
$Tcap_j$	Total capacity of facility j
SLL_j	Safety level of location j
MSC_j	Cost factor related to the safety of the selected location
$ROND$	Range of safety level
$MRSL$	Minimum required safety level

For achieving a reliabilie system a chance constraint method is taken into account as the facility exceedance probability which depicts the situation of allocation over the capacity of facility. Constraint (1) represents the exceedance probability.

$$P\{\sum_{i\epsilon I} x_{ij} cap_i \geq Tcap_j\} \leq \gamma_j \tag{1}$$

This equation can be reformulated based on cumulative distribution function as follows:

$$F_{cap_j}(x_{ij}) \leq \gamma_j \tag{2}$$

Since cumulative distribution functions are monotonic one to one, the reverse of Constraint (2) is proposed:

$$x_{ij} \leq F_{cap_j}(\gamma_j)^{-1} \tag{3}$$

To deal with Constraint (3), capacity of facilities is considered to be uniform random variable. Upper and lower bounds of the capacity is determined based on facilities design, where θ_{ij} is a fraction to set the minimum capacity of facilities. Finally, Equation (4) demonstrates the inverse cumulative distribution function of the capacity.

$$F_{cap_i}(\gamma_i)^{-1} = \theta_{ij} DTcap_j + \gamma_i DTcap_j(1-\theta_{ij}) \tag{4}$$

It is noticeable that $DTcap_j$ determines the deterministic value of the facility capacity. As a result, the facility capacity reliability constraint is proposed by:

$$x_{ij} \leq DTcap_j[\theta_{ij}(1-\gamma_i)+\gamma_i] \tag{5}$$

The *p*-median problem is formulated as follows:

$$\text{Min} \sum_{i\epsilon I}\sum_{j\epsilon J} c_{ij}x_{ij} + \sum_{j\epsilon J} P^j Fc_j + \sum_{j\epsilon J} MSC_j \frac{SLL_i}{ROND}\mathrm{y_j} \tag{6}$$

s.t.

$$\sum_{j\epsilon J} \mathrm{y_j} \leq P \tag{7}$$

$$\sum_{j\epsilon J} x_{ij} = 1 \qquad \forall i \tag{8}$$

$$x_{ij} \leq DTcap_j[\theta_{ij}(1-\gamma_i)+\gamma_i] \qquad \forall i \tag{9}$$

$$\sum_{j\epsilon J} SLL_j \mathrm{y_j} \geq MRSL \tag{10}$$

$$x_{ij}.\mathrm{y_j} \,\epsilon\, \{0,1\} \tag{11}$$

For the *p*-center problem, let Z be the maximum distance of facilities and costumers then the proposed model is formulated:

$$\text{Min } z + \sum_{j\epsilon J} P^j Fc_j + \sum_{j\epsilon J} MSC_j \frac{SLL_j}{ROND}\mathrm{y_j} \tag{12}$$

s.t.

$$\sum_{j \epsilon J} y_j \leq P \qquad (13)$$

$$\sum_{j \epsilon J} x_{ij} = 1 \qquad \forall i \qquad (14)$$

$$x_{ij} \leq DTcap_j[\theta_{ij}(1-\gamma_i)+\gamma_i] \qquad \forall i \qquad (15)$$

$$\sum_{j \epsilon J} SLL_j y_j \geq MRSL \qquad (16)$$

$$x_{ij}.y_j \,\epsilon\, \{0,1\} \qquad (17)$$

For the second step, an uncertain constrained VRP is proposed below:

$$\text{Min } Z = \sum_{(i;j)\in A} \sum_{v\in K} VC_{ij}^{v} t_{ij}^{v} \qquad (18)$$

s.t.

$$\sum_{i}\sum_{j} \tilde{d}_i Xij^{v} \leq W_v \qquad \forall v \epsilon V \qquad (19)$$

$$\sum_{i;j} Xij^{v} = \sum_{j;i} Xji^{v} \qquad \forall v \epsilon V \qquad (20)$$

$$\sum_{j}\sum_{i} Xij^{v} = 1 \qquad \forall v \epsilon V \qquad (21)$$

$$\sum_{j} X1j^{v} = 1 \qquad \forall v \epsilon V \qquad (22)$$

$$L_i \leq time_{jk} \leq U_i \qquad \forall i \epsilon I.\, j.\, k \epsilon J \qquad (23)$$

$$time_{jk} speed_{jk} \leq c_{jk} x_{ij} y_j \qquad \forall i \epsilon I.\, j.\, k \epsilon J \qquad (24)$$

$$o_i - o_j + (M)Xij^{v} \leq M-1 \qquad (25)$$

$$\forall\, i \in I\,\{1\}\ and\ \ \forall j \neq i\ \ and\ \ \forall v \epsilon V$$

The objective function (6) is to minimize the total transportation cost, expected cost of facility disruption and safety cost of facility location. Constraint (7) determines the number of active centers and Constraint (8) ensures that each client is assigned to some facility. As mentioned before Constraint (9) considers facility capacity reliability constraint. Constraint (10) oblige the system to consider the minimum required amount of safety to select the location of facilities between the nominated zones based on the scale of the natural disasters that happened in the past constraint. Constraint (11) determines the type of variables. Objective function (12) minimizes the maximum transportation cost, expected cost of facility disruption and safety cost of facility location, Constraints (13) to (17) are as same as Constraint (7) to (10). Objective function (18) minimizes the transportation cost of vehicles. Constraint (19) is about the capacity of each vehicle. Constraint (20) is added in order to establish the balance between centers met by vehicles. Constraint (21) force vehicles to meet all the demands. Constraint (22) makes sure that every vehicle starts from depot. Constraint (23) and Constraint (24) enforce the time window restrictions. Constraint (25) removes the sub-tours. Figure 1 demonstrates an overview of the process.

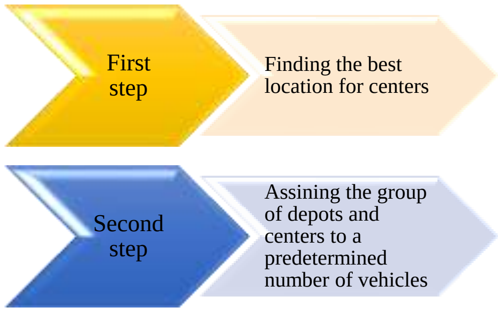

Figure 1. Overview of the process

3. Solution Procedure

In Section 2, we propose a two-step linear programing, while in this part we introduce the procedures used to solve the models. The proposed model is an uncertain single-objective two-step linear programming one. In the first, the uncertain model is converted to its equivalent crisp one, and then in the second step, the crisp model is solved using the developed evolutionary algorithm. Since the core of every meta-heuristic approach is the relation between the feasible solution, objective function and process of comparing the other solutions in a specific algorithm, in the beginning the solution representation is described for each step.

3.1 Solution representation

To produce different solutions in a continuous space different combination of random locations are chosen in specific continuous space, and a random function determines the order of assigned centers to depots which is shown in Table 2. Also, in the second step, another random function determines the assigned locations and depots to a vehicle to find the best objective function is depicted in Table 3.

Table 2. Ordination of the depots assignment

1	2	4	7	**9**	6	3	5	**8**

Table 3. Ordination of the vehicles assignment

1	2	**10**	4	7	9	6	8	5	3	**11**

3.2 Equivalent auxiliary crisp model

The expected value and chance-constraint programming approach popular methods to capture the uncertainty in the parameters of model [Mousazadeh, Torabi and Pishvaee, 2014]. Usually the decision maker's behavior is divided into optimistic and pessimistic, which are the basic measures of fuzzy approaches. However, a flexible method, which can consider both decisions simultaneously developed by [Xu and Zhou, 2013]. According to [Xu and Zhou, 2013], the fuzzy measure Me is defined as follows:

$$Me\{A\} = Nec\{A\} + \lambda(Pos\{A\} - Nec\{A\}) \qquad (13)$$

where $(\theta. P(\theta). Pos)$ is the possibility space and A is a set in $P(\theta)$. Moreover, λ is the optimistic-pessimistic parameter.

the expected value operator based on the Me method is demarcated by [Xu and Zhou, 2013]:

$$E[\xi] = \frac{(1-\varepsilon)}{2}\xi_1 + \frac{1}{2}\xi_2 + \frac{\varepsilon}{2}\xi_3 \qquad (14)$$

where $\xi = (\xi_1. \xi_2. \xi_3)$ is a triangular fuzzy variable.

In order to deal with the uncertain parameters of the model, the chance-constrained programming approach is used which is described below:

$$\begin{aligned} &\text{Min } \tilde{c}x \\ &\text{s.t.} \\ &Me\{\tilde{A}x \geq \tilde{b}\} \geq \alpha \\ &Me\{\tilde{N}x \leq \tilde{d}\} \geq \beta \\ &x \geq 0 \end{aligned} \qquad (14)$$

where $\tilde{c} = (\tilde{c}_1. \tilde{c}_2. \dots. \tilde{c}_n)$. $\widetilde{A} = [\tilde{a}_{ij}]_{m\times n}$. $\widetilde{N} = [\tilde{n}_{ij}]_{m\times n}$. $\tilde{b} = (\tilde{b}_1. \tilde{b}_1. \dots. \tilde{b}_n)^t$ and $\tilde{d} = (\tilde{d}_1. \tilde{d}_1. \dots. \tilde{d}_n)^t$ show the triangular fuzzy numbers include in the objective function and constraints. According to [Xu and Zhou, 2013], the mentioned model can be divided to approximation models, namely the lower approximation model (LAM) and upper approximation model (UAM) presented by:

$$(UAM)\begin{cases} \text{Min } E[\tilde{c}]x \\ s.t \\ \quad Pos\{\tilde{A}x \geq \tilde{b}\} \geq \alpha \\ \quad Pos\{\tilde{N}x \leq \tilde{d}\} \geq \beta \\ \quad x \geq 0 \end{cases} \qquad (16)$$

$$(LAM)\begin{cases} \text{Min } E[\tilde{c}]x \\ s.t \\ \quad Nec\{\tilde{A}x \geq \tilde{b}\} \geq \alpha \\ \quad Nec\{\tilde{N}x \leq \tilde{d}\} \geq \beta \\ \quad x \geq 0 \end{cases} \qquad (17)$$

The above models can be changed into a definite equivalent model as Eq. (18) and (19).

$$(UAM)\begin{cases} \text{Min } \left(\frac{1-\lambda}{2}C_{(1)} + \frac{1}{2}C_{(2)} + \frac{\lambda}{2}C_{(3)}\right)x \\ s.t \\ \quad A_{(2)}x + (1-a)(A_{(3)} - A_{(2)})x \geq b_{(2)} - (1-a)(b_{(2)} - b_{(1)}) \\ \quad N_{(2)}x - (1-\beta)(N_{(2)} - N_{(1)})x \leq d_{(2)} + (1-\beta)(d_{(3)} - d_{(2)}) \\ \quad x \geq 0 \end{cases} \qquad (18)$$

$$(LAM)\begin{cases} \text{Min } \left(\frac{1-\lambda}{2}C_{(1)} + \frac{1}{2}C_{(2)} + \frac{\lambda}{2}C_{(3)}\right)x \\ s.t \\ \quad A_{(2)}x - a(A_{(2)} - A_{(1)})x \geq b_{(2)} + (1-a)(b_{(3)} - b_{(2)}) \\ \quad N_{(2)}x + (1-\beta)(N_{(3)} - N_{(2)})x \leq d_{(2)} - \beta(d_{(2)} - d_{(1)}) \\ \quad x \geq 0 \end{cases} \qquad (19)$$

According to the above explanations, our model is presented for the second step of the model with uncertain demands as follows:

$$\text{Min } Z = \sum_{(i;j)\in A} \sum_{v\in K} c_{ij} Xij^v \qquad (20)$$

s.t.

$$\sum_{\substack{i\\ i\neq k}} \left(\left[d_{i(2)} - (1-\alpha)\left(d_{i(2)} - d_{i(1)}\right)\right]\right) X_{ik} \le W_v \qquad \forall v \epsilon V \qquad (21)$$

$$\sum_{i;j} Xij^v = \sum_{j;i} Xji^v \qquad \forall v \epsilon V \qquad (22)$$

$$\sum_{v} \sum_{i} Xij^v = 1 \qquad \forall v \epsilon V \qquad (23)$$

$$\sum_{j} \mathrm{X1j^v} = 1 \qquad \forall v \epsilon V \qquad (24)$$

$$o_i - o_j + (M) Xij^v \le M - 1 \qquad (25)$$

LAM:

$$\text{Min } Z = \sum_{(i;j)\in A} \sum_{v\in K} c_{ij} Xij^v \qquad (26)$$

s.t.

$$\sum_{\substack{i\\ i\neq k}} \left(\left[d_{i(2)} - (1-\alpha)\left(d_{i(3)} - d_{i(2)}\right)\right]\right) X_{ik} \le W_v \qquad \forall v \epsilon V \qquad (27)$$

$$\sum_{i;j} Xij^v = \sum_{j;i} Xji^v \qquad \forall v \epsilon V \qquad (28)$$

$$\sum_{v} \sum_{i} Xij^v = 1 \qquad \forall v \epsilon V \qquad (29)$$

$$\sum_{j} \mathrm{X1j^v} = 1 \qquad \forall v \epsilon V \qquad (30)$$

$$o_i - o_j + (M) Xij^v \le M - 1 \qquad (31)$$

3.3 Developed evolutionary algorithm

In the literature, it is proved that *p*-median and *p*-center problems are NP-hard, furthermore to deal with such a problem developed evolutionary algorithm is applied .Differential evolution (DE) introduced by [Storn and Price, 1997]. This algorithm is belonged to the class of evolutionary algorithms(EA). Similar to other EAs, DE is dependent to primary population. In each generation, a mutation operator is employed to generate a new solution vector as shown by:

$$MV_i = \omega_{v1} + F(\omega_{v2} - \omega_{v3}) \qquad (32)$$

where MV_i is a mutant vector and $\omega_{v1}.\omega_{v2}.\omega_{v3}$ are different arbitrary vectors nominated from $(i = 1 \ldots. N_{pop})$ and they are not equal to i. F is the amplification coefficient (i.e.و $F \in [0.2]$).

A Binomial crossover is the other function to increase the diversity of solutions. By using the binomial crossover operator, we can obtain a combined vector with some common information achieved by sharing the data of mutated vector and another predetermined (target) vector to create trial vector $\mu_i = \{\mu_{i1} \ldots \mu_{ik} \ldots \mu_{in}\}$ as shown below.

$$\mu_{ik} = \begin{cases} MV_{ik} & if\ rand(k) \le CR\ and\ k = rnbr(i) \\ \omega_{ik} & if\ rand(k) > CR\ and\ k \neq rnbr(i) \end{cases} \qquad (33)$$

where *rand(k)* is the *k*-th element of an *N*-dimensional uniform random number $\in [0.1]$, CR linked to the crossover rate and *rnbr*(*i*) is an arbitrarily chosen index $\in \{1 \ldots N\}$, which guarantees that the new solution vector gets at least one of the dimensional values of mutated vector. Lastly, the new solution with better results will be replaced with old one. Table 4 shows the Pseudo code of the DE algorithm.

Table 4. Pseudo code of the DE algorithm

```
Set the parameters
Generate preliminary population
Calculate objective function of each solution
While termination state is not satisfied continue
  a. Choose parents
  b. Apply mutation
  c. Apply Crossover
  d. Compute the objective functions of new solutions
     If TC_new vector < TC_target vector then
          Replace the old vector with new one
     End if
End while
```

According to [Zhalechian, Tavakkoli-Moghaddam and Rahimi, 2017], a new DE algorithm, named self-adaptive DE (SADE), is proposed to increase performance. In the proposed SADE algorithm, different types of mutations and crossover are utilized. In first phase, different types of mutation and crossover apply on the initial population, in main phase, the SADE algorithm explores in solution area using the self-adopted mutation and crossover operators. Table 5 depicts the different types of mutation (based on [Zhalechian, Tavakkoli-Moghaddam and Rahimi, 2017]).

Table 5. Different mutation strategies

Mutation strategy	Operator
Rand/1	$MV_i = \omega_{v1} + F(\omega_{v2} - \omega_{v3})$
Best/1	$MV_i = \omega_{best} + F(\omega_{v1} - \omega_{v2})$
Current to best/1	$MV_i = \omega_{v1} + F(\omega_{best} - \omega_i) + F(\omega_{v1} - \omega_{v2})$
Best/2	$MV_i = \omega_{best} + F(\omega_{v1} - \omega_{v2}) + F(\omega_{v3} - \omega_{v4})$
Rand/2	$MV_i = \omega_{v1} + F(\omega_{v2} - \omega_{v3}) + F(\omega_{v4} - \omega_{v5})$

3.4 Simulated annealing algorithm

To assess the SADE algorithm in large-sized problems, an improved simulated annealing (SA) algorithm is proposed based on [Mahmudy, 2016], which is used to deal with a time window VRP problem and has similarities with this model. The SA algorithm is created based on the physical process of annealing for a specific metal. This algorithm starts with an initial temperature and searches different neighbor solution in each loop. This algorithm replaces other neighbor solutions with the current solution by a specific probability. This probability decreases as the algorithm continuous. Table 6 shows the pseudo code of the SA algorithm.

After some experiments and based on [Mahmudy, 2016], the main parameters of the SA algorithm are set as follows:

1. Initial temperature is 0.9
2. The cooling rate of temperature is 0.9
3. The final temperature is 0.01
4. Number of iteration for each temperature is 5000

Table 6. Pseudo code of the SA algorithm

Set the initial temperature
Generate the initial solutions
Calculate the objective function of solution
While $T \geq Temp1$
  a. Generate neighbor solutions for each iteration
  b. Calculate the objective function of solutions
    If $Z_{\text{new sol}} \leq Z_{\text{current sol}}$ **then**
    Replace the old solution with new one
    Else if $Z_{\text{new sol}} > Z_{\text{current sol}}$
    Accept the change with this probability
    $\text{Prob} = e^{-\left(\frac{(Z_{\text{new sol}} - Z_{\text{current sol}})k}{Z_{\text{current sol}}}\right)}$
    Else do nothing
    End if
Update temperature
End while

3.5 Taguchi method

One approach of factorial design of experiments is the Taguchi method (TM), which aims to improve the quality of manufacturing processes. A full factorial experiment is an experiment whose design includes two or more factors, each of factors are comprised of discrete uncertain values or 'levels', and all the possible mixtures of these levels should be investigated in every experimental units; i.e., such an experiment will consider all possible combinations for a given set of factors [Azadeh et al., 2017]. In order to decrease the number of experiments, a good combination of parameters to run the SADE Taguchi's signal-to-noise (S/N) method is used. Figure. 2 shows the related results. Table 7 shows the levels of SA factors and the results of the Taguchi method are shown by:

Table 7. Design factors and their levels

	Levels		
Parameters	1	2	3
Maximum iteration	100	350	500
Population size	50	80	100
Crossover rate	0.3	0.6	0.8
F	0.6	0.4	0.2

Figure 2. Taguchi's signal-to-noise (S/N)

4. Computational results

To compare the results of two different problems, different test problems have been generated. To validate the results of the proposed algorithm the presented model is coded in GAMS 24.7 and compared with the results of SADE in MATLAB R2014b in small-sized problems. Also, the genetic algorithm (GA) and simulated annealing (SA) results are illustrated to prove the performance of SADE, 10 test problems are generated in small sizes and compared with GAMS while 10 test problems are generated in large sizes. Notably, all the calculations were performed on a laptop with 2.66 GHz CPU and 6 GB RAM. Information of mathematical model to deal with different created test problems are shown in Table 8. To rich the near optimum solution we have set the parameters of GA and SADE based on Taguchi as mentioned before. In order to assess the *p*-median and *p*-center results in small sizes, their results are compared with an exact method (results of GAMS) and the outcomes are summarized by using gap as formulated here $\left[100 \times \left(\mathrm{G_{Opt}} - \mathrm{G_{Alg}}\right)/\mathrm{G_{Alg}}\right]$. Table 9 shows the final appropriate parameters tuned to run the SADE and GA algorithm. Tables 10 and Tables 11 demonstrate the results of first step, this result are related to p-median and p-center problem respectively which validates the performance of metaheuristics in comparison with GAMS results. Table 12 shows the results of large-sized test problems for *p*-median and *p*-center location problem and the gap is calculated based on $[100 \times (\mathrm{S_{SADE}} - \mathrm{S_{GA}})/\mathrm{S_{GA}}]$. Figure 3 shows the results of specific size p-center location problem between the costumers, coded in MATLAB. Table 13 shows the results of the vehicle routing problem between the costumers and facilities as the second step of problem in different test problem sizes. Table 14 and Table 15 show the same results of two last mentioned tables considering the point that the algorithm SA is replaced with GA and gap shows $[100 \times (\mathrm{S_{SADE}} - \mathrm{S_{SA}})/\mathrm{S_{SA}}]$.

Table 8. Parameters of the model

Parameters	Values
C_{ij}	$\sim U(10.50)$
P	$\sim U(5.50)$
d_i	$\sim U(10.20)$
W_v	$\sim U(50.75)$
M	100000000

Table 9. Parameters of the GA and SADE

Algorithm	Parameter	Value
SADE	*Population size*	100
	Crossover rate	0.3
	F	0.8
	Iteration	500
GA	*Population size*	20
	Crossover rate	0.3
	Mutation rate	0.6
	Iteration	500

Table 10. Computational results of a continuous *p*-median problem

Data set	*I*	*J*	SADE		GA		SA	
			Gap (%)	Time (Sec.)	Gap (%)	Time (Sec.)	Gap (%)	Time (Sec.)
1	4	2	0	5	0	3	0	2
2	6	2	0	5	0	5	0	4.5
3	8	3	0	5	0	5.1	1	5
4	9	4	0	5	0	6	1.5	5.12
5	10	5	1	6	1.01	7.8	2	6.5
6	12	5	1	6	1.097	8.6	2.45	7.1
7	14	5	1.01	8	1.18	12	2.7	10
8	16	5	1.1	15	1.2	17	3.1	13
9	18	5	1.34	18	1.6	22	3.5	16
10	40	20	1.9	23	2.3	31	4	21

Table 6. Computational results of a continuous *p*-center problem

Data set	*I*	*J*	SADE Gap (%)	SADE Time (Sec.)	GA Gap (%)	GA Time (Sec.)	SA Gap (%)	SA Time (Sec.)
1	4	2	0	5	0	2.7	0	2
2	6	2	0	5	0	5	0	4
3	8	3	0	5	0	6.2	0.9	5
4	9	4	0	5	0	7.2	1.2	5
5	10	5	1	6	1.04	8.3	1.9	6
6	12	5	1	6	1.98	9.5	3	6
7	14	5	1.02	8	1.1	12	3.5	7.1
8	16	5	1.11	15	1.3	17	3.8	12.4
9	18	5	1.7	18	1.95	20	4.5	15
10	40	20	1.9	27.4	2.04	29	5	24

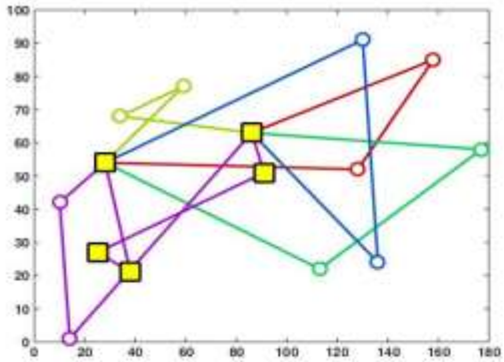

Figure 3. Results of the second step for a *p*-continues in a continuous mode

5. Sensitivity Analysis

In this section cost value of both p-center and p-median problems along with vehicle routing problem in a specific size (*I*=50, *J*=10, *V*=10) are compared in different size of test problems. and the results are illustrated in Figure 4 and Figure5.

Moreover, the difference between the results of p-center and p-median problems caused by changing the capacity of vehicles and run time of the proposed algorithm are compared for both the *p*-center and *p*-median problems in Figure 6. Figure 7 and Figure 8 shows the process time of SADE, GA and SA algorithms, based on the results SA algorithm takes less time rather than other algorithms.

The fluctuations of the total objective function (summation of location and routing problem) are demonstrated in Figure 9, which look into the variations of results caused by changes in the important factor of this formula as the objective functions used in the algorithm to achieve the optimum answer:

Total z = (eta) (p-median z) + (1 - eta) (p-center z)

In those figures, the number of vehicles is assumed to be 5, also the number of *I* and *J* are based on the mentioned data in Table 12. To sum up the results show the superiority of SADE algorithm to reach near optimum answers in large size test problems however, SA algorithm takes less time between the mentioned algorithms.

Table 7. Computational results of the continuous *p*-center and *p*-median problems

Data set	*I*	*J*	*p*-median time SADE	*p*-center time SADE	*p*-median time GA	*p*-center time GA	*p*-center GAP (%)	*p*-median GAP (%)
11	45	20	77	73	92	87	3	4
12	45	30	80	77	114	123	3.5	5.5
13	50	20	88	90	129	134	5	6.1
14	60	25	119	123	156	150	7	8
15	60	45	138	143	189	209	8	7.7
16	100	30	186	181	223	228	5.6	8.1
17	100	40	209	190	245	256	7.1	9
18	150	40	261	258	290	294	6.9	7.8
19	200	50	316	320	350	365	8	6
20	200	100	391	404	443	459	10	12

Table 8. Computational results of the continuous *p*-center and *p*-median routing problems

Data set	*(I, J)*	*V*	*P*-median routing time (SADE,GA)	*p*-center routing time (SADE,GA)	*p*-center GAP (SADE,GA)%	*p*-median GAP (SADE,GA)%
1	(10,5)	3	(7.5,8)	(8.1,12)	(0,0)	(0,0)
2	(20,10)	5	(11,14)	(13.5,19)	(0.9,1)	(1,1)
3	(20,15)	10	(20,25)	(22,27)	(1.5,2)	(1.4,2.1)
4	(30,10)	10	(30.4,37)	(31,41)	2.4	2.33
5	(40,10)	15	(38,42)	(40.1,48)	3.3	3.9
6	(50,20)	20	(50,55)	(52,60.3)	4.7	4.5
7	(100,40)	30	(103,113.1)	(110,120)	5.6	5.7
8	(150,40)	40	(123,128)	(128,136)	10.09	10.11
9	(200,50)	50	(173,180.3)	(180,194)	14.2	16
10	(200,100)	50	(184,197)	(190,205)	18	21

Table 9. Computational results of the continuous *p*-center and *p*-median problems

Data set	*I*	*J*	*p*-median time SADE	*p*-center time SADE	*p*-median time SA	*p*-center time SA	*p*-center GAP (%)	*p*-median GAP (%)
11	45	20	77	73	60	58	5	4.6
12	45	30	80	77	65	66.5	6.1	7
13	50	20	88	90	76	75	7.6	9
14	60	25	119	123	100	114	9	10.1
15	60	45	138	143	113	123	10.1	11
16	100	30	186	181	140	134	11	12.3
17	100	40	209	190	169	154	13	14.2
18	150	40	261	258	198	187	14.5	13
19	200	50	316	320	247	223	17	18.7
20	200	100	391	404	304	293	20	23

Table 10. Computational results of the continuous *p*-center and *p*-median routing problems

Data set	*(I, J)*	*V*	*P*-median routing time (SADE, SA)	*p*-center routing time (SADE, SA)	*p*-center GAP (SADE, SA) %	*p*-median GAP (SADE, SA) %
1	(10,5)	3	(7.5,3)	(8.1,4.1)	(0,0)	(0,0)
2	(20,10)	5	(11,7)	(13.5,5)	(0.9,2)	(1,3)
3	(20,15)	10	(20,17)	(22,11)	(1.5,5)	(1.4,4.1)
4	(30,10)	10	(30.4,23)	(31,22)	5	7
5	(40,10)	15	(38,31)	(40.1,27)	8.1	9
6	(50,20)	20	(50,42)	(52,36)	12	11
7	(100,40)	30	(103,88)	(110,74)	13.2	14
8	(150,40)	40	(123,91)	(128,100.5)	17	20
9	(200,50)	50	(173,137)	(180,123)	19.5	22
10	(200,100)	50	(184,150)	(190,141)	21	25

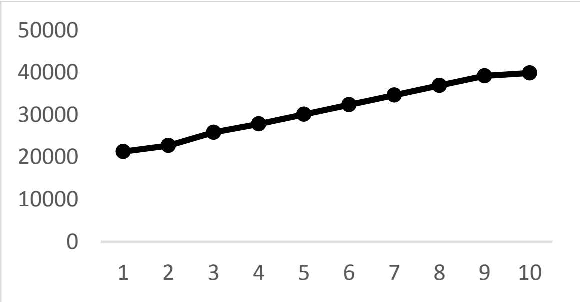

Figure 4. Objective function of the *p*-median location-routing problem

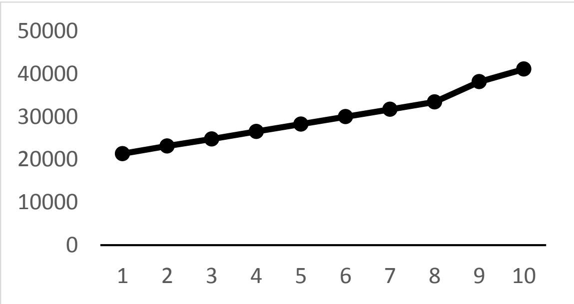

Figure 5. Objective function of a *p*-center location-routing problem

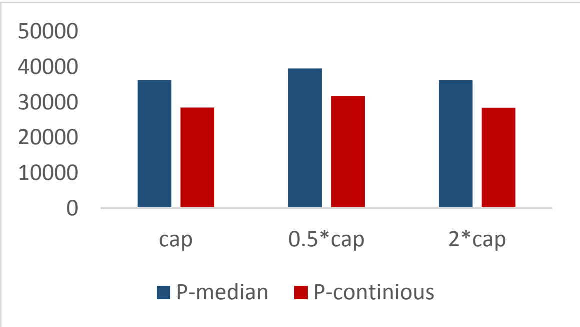

Figure 6. Objective function fluctuations due to changes in capacity of the vehicles

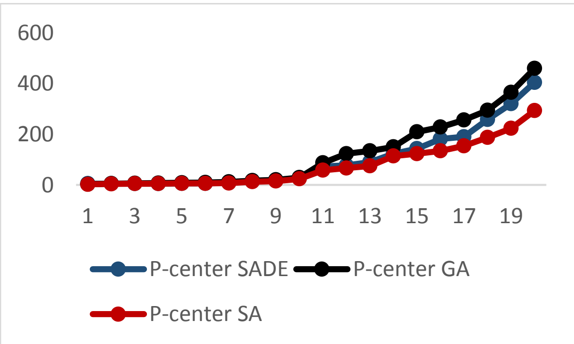

Figure 7. Run time comparisons of *p*-center problem for the proposed algorithms

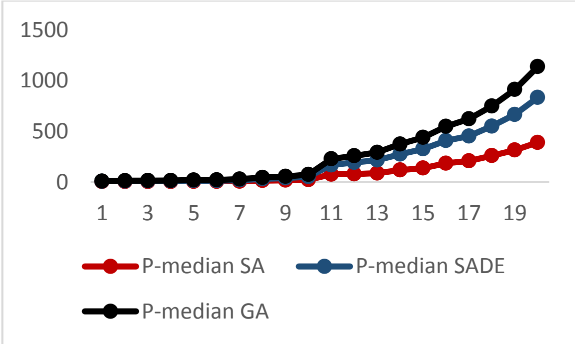

Figure 8. Run time comparisons of the *p*-median problem for the proposed algorithms

It is obvious that the *p*-median problem is more volatile rather than the *p*-center problem.

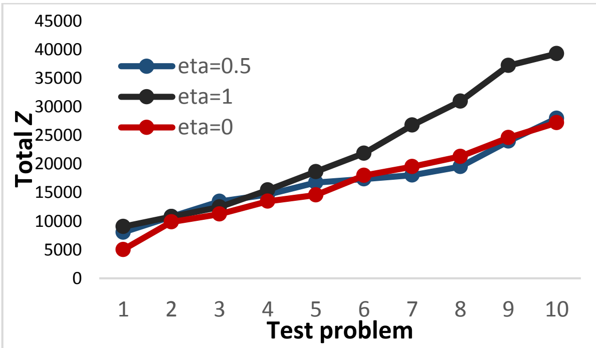

Figure 9. Total objective function comparison with variation of the eta

6. Conclusion

In this paper, a fuzzy two-stage integer programing introduced to compare the differences between the *p*-center and *p*-median location problem in a continuous space by considering facility disruption and in second step the achieved results utilized for time-window multi-depot capacitated vehicle routing problem. To cope with uncertain parameters, an efficient possibilistic programming approach based on Me measure was applied. Since the problem was NP-hard in both stages a self-adaptive evolutionary approach applied to deal with the problem. Furthermore, some numerical experiments and sensitivity analyses were shown to validate the presented model. Moreover, the performance of the proposed self-adaptive differential evolution algorithm was improved by Taguchi method where the results showed that developed self-adaptive differential evolution algorithm out performs rather than genetic algorithm and simulated annealing algorithm. According to the results, the *p*-median problem shows to be more volatile under variation of vehicle capacity while the *p*-center problem costs more. Some extensions in this paper can be about developing the model (for example considering financial constraint and scheduling of vehicles) and designing exact algorithms to solve the model.

7. References

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