Evaluation of Methods for Computing Free-Flow Speed and Its Significance in the HCM 2010; Case Study: A Ramp-Weaving Segment

Meisam Akbarzadeh[1], Ahmad Mohajeri[2]

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Abstract

The core methodology of the Highway Capacity Manual (HCM) 2010 for freeway analyses is based on Free-Flow Speed (FFS). Moreover, weaving segments are major elements of freeway facilities that form where two one-way traffic streams intersect by merging and diverging maneuvers. Hence, this study used three different methods to compute FFS of a ramp-weaving segment, and then employed the proposed HCM 2010 model to analyze the weaving. Comparison of the model results and the field data indicated that the HCM 2010 method for field measurement of FFS outperformed the other two methods. Nonetheless, the considerably poor performance of the HCM 2010 model in predicting the speed of non-weaving vehicles adversely affected the ultimate outcome of the model and caused under-predicted results. Thus, by app.lying slight modifications to the field measurement method of FFS for non-weaving flow, this study proposed a novel method which produces significantly more favorable results compared to other methods. Considering these results alongside the context in which manuals like the HCM are calibrated in, elegantly demonstrates the merit of this study because the HCM traffic models depend on several contextual factors including vehicle technical characteristics and traffic behavior. This implies that most recent editions of manuals may not always be the most accurate ones for the traffic conditions throughout the highways of Iranian cities.

Keywords: Free-flow speed, weaving segment, Highway Capacity Manual, Isfahan.

Corresponding author E-mail: makbarzadeh@cc.iut.ac.ir

1. Assistant Professor, Department of Transportation Engineering, Isfahan University of Technology, Isfahan, Iran

2. MSc. Grad., Civil Engineering, Transportation Research Institute, Isfahan University of Technology, Isfahan, Iran

1. Introduction

Urban freeways perform an essential role in roadway transportation system [Kiattikomol et al. 2008]. On the other hand, traffic flow in these facilities is greatly affected by weaving maneuvers [Wang et al. 2014]. Weaving segments are a common design element of freeway facilities [Skabardonis and Mauch, 2015], which form when one-way traffic streams cross by merging and diverging maneuvers [Roess and Prassas, 2014]. Intense lane-changing activities in these segments are the primary cause of fixed bottlenecks near ramp junctions which ultimately lead to traffic congestion [Wan et al. 2017; Chen and Ahn, 2018]. This serious issue raises the risk of crashes and reduces the discharging flow [AASHTO, 2010]. Thus, weaving segments on urban freeways should be carefully evaluated [Pande and Wolshon, 2016].

The Highway Capacity Manual (HCM) is considered as a comprehensive manual for traffic operational analysis throughout the United States and many other countries. The HCM is comprised of methods to guide analysts [Findley et al. 2016]. Weaving is considered as one of the most important elements in the freeway capacity and the Level of Service (LOS) analysis in the HCM [Liu et al. 2012]. The concept of FFS was introduced in the fourth edition of this manual for analysis of freeway facilities. According to the HCM, FFS is the theoretical speed when both density and flow rate of a segment are close to zero [TRB, 2010]. On the other hand, studies demonstrate an inextricable link between human behavior and traffic system operation; moreover, it is believed that desire, emotion and knowledge are the principal sources of human behavior [NCHRP Report 600, 2012; Scott-Parker, 2017]. Therefore, traffic system components depend upon driver characteristics [Hill, Elefteriadou and Kondyli, 2015]. For example, capacity of freeways is highly sensitive to human behaviour and environmental conditions [Mamdoohi, Saffarzadeh and Shojaat, 2015]. Thus, the behavior of traffic participants depends on economic, cultural and social factors [Ghavidel Abraghan and Afandizadeh Zargari, 2016].

2. Overview of HCM Methodologies

A methodology for design and analysis of weaving segments was first presented in the 1950 HCM [Yi, Lu and Ma, 2011]. This simple and general method is basically a rational app.roach based upon several judgmental principles, aided by the limited available data on weaving that came from multilane highways and few existing freeways [Roess and Prassas, 2014]. The result was a graphical model which predicted both the capacity and the operating speeds of weaving segments [Zhang, 2005].

To use this model in designs, doubling the traffic volume triples the length of the required weaving segment and doubles the number of lanes required for the weaving vehicles [Yi, Lu and Ma, 2011]. In 1953, the U.S. Bureau of Public Roads started a revision of the 1950 procedure and published a new procedure in the 1965 HCM [Zhang, 2005].

In fact, the 1965 HCM provided three different models for the analysis of weaving segments. A model (which was developed by Leisch and Normann) app.lies to all weaving configurations on all types of freeways [Roess and Prassas, 2014]. Two other models are presented in Chapter 8 to analyze the ramp configuration: one to analyze the configuration under free-flow conditions (LOS A to C) which was developed by Hess, and the other to analyze the ramp configuration under heavy traffic conditions

(The Level D Method) which was developed by Moskowits and Newman [TRI and Kittelson, 2008].

Although the Leisch/Normann method followed the conceptual framework of the HCM 1950 app.roach, it shows some differences, such as the introduction of LOS criterion to the methodology. Also, it has used much boarder range of lengths and weaving volumes compared to the HCM 1950. Therefore, in this model, the maximum length of weaving segment was extended from 3600 ft to 8000 ft and a total of weaving volume was increased from 3600 veh/h to 4000 veh/h [Roess and Prassas, 2014]. Another unique aspect of this model was the clear definition of "out of the realm of weaving'' [TRI and Kittelson, 2008].

Weaving became a significant challenge on urban freeways by mid 1960's. Therefore, weaving proved as a good start point for the development of a third-edition of the HCM [Roess and Prassas, 2014].

Concurrent with this procedure, new approaches to weaving segment analysis began to be developed. This led to two concepts; configuration and operation types of weaving segments. These concepts were first presented in the NCHRP Project 3-15 model, which was developed by the Polytechnic Institute of New York (PINY) between 1968 and 1971.

Studies carried out in those 20 years to release the HCM in 1985 were very important and influential. In the way that in 1984, the Highway Capacity and Quality of Service Committee had three options to write the weaving chapter of the 1985 HCM: a) a procedure developed by PINY in 1979 which was a revision to NCHRP 3-15, b) a procedure developed by Leisch in 1984 c) the Reilly procedure in 1984. Eventually, the committee opted to go with the form of the algorithm developed by Reilly with some modifications and recalibrations to reflect the impact of configuration types (included in the PINY and Leisch methodologies), and the issue of types of operation (included in the PINY methodology) by Roess [TRI and Kittelson, 2008; Skabardonis and Kim, 2010].

This led to six equations for the prediction of weaving speed, and six for non-weaving speed. These equations were used for speed estimation of tri-configuration types (A, B and C) and constrained vs. unconstrained operations [TRB, 1985]. The initial calibration tried to adopt regression app.roach but the results were not satisfactory. Because the data base was statistically inadequate to supp.ort such development [Zhang, 2005; TRI and Kittelson, 2008], to achieve better result and an acceptable level of sensitivity, the model was modified by a trial-and-error app.roach [TRI and Kittelson, 2008].

Since the publication of the HCM in 1985, multitude of concerns were expressed by researchers and professionals regarding this model [Skabardonis and Kim, 2010]. Thereby, this model was revised twice.

In the fourth edition of the HCM, a model was presented to analyze the weaving segment which took an important role from the prior studies and their developments. Actually, it's assembly was based on: a) the speed prediction algorithm in Reilly's model; b) configuration and operation concepts in the NCHRP 3-15 model (1975) and its update in the late 70s; c) the revised model of the 1985 HCM (1997); d) both Leisch and Fazio studies in the late 80s [TRB, 2000].

This model has tried to eliminate the shortcomings of the previous methods. These modifications include: a) recalibration of the constants to reflect further changes in other freeway analyses related to chapters of the Manual, and b) determination of the LOS based on the density in the weaving section and removing the assignment of separate levels of services to weaving and non-weaving vehicles [Skabardonis and Kim, 2010]. Another important change in the HCM 2000 presents a multi-page table for determination of the weaving segment capacity [Roess and Ulerio, 2009].

Despite these beneficial changes, the HCM 2000 has limitations. It retains the tri-classification of weaving configurations, and is still based on a relatively small data base which consists of 10 weaving segments, using hourly data [Roess and Prassas, 2014].

The HCM 2010 provided a methodology for the analysis of weaving segment developed by Roess et al. in an extensive study. This study started in 2006 by the NCHRP's sponsorship. The bases of this methodology are the effective geometry characteristics (length, width, and configuration), FFS, and the demand flow rates. [Roess and Prassas, 2014; TRB, 2010]

In comparison with the HCM 2000, this methodology has significantly changed: a) the two-fold and tri-classification (based on operation and configuration) weaving segments elimination, b) the redefinition of the algorithm for determination of weaving segment length, c) the modification of speed algorithm estimation, d) an introduction of a new model for capacity prediction, and e) the provision of a mathematical method to determine the maximum weaving length (L_{MAX}) that is based on the volume ratio (VR) and configuration.

This study has intended to provide a model for the lane-changing rates estimation, so it has used the data that could observe the lane changes made by flows moving in a weaving segment. The data were collected from seven cities in six states in the United States. The types of configurations included; the lengths varied from 540 ft to 2,820 ft, and the widths varied from 3 to 6 lanes [Roess and Prassas, 2014].

3. Data

Field data were collected from a ramp-weaving segment of the depressed part of the urban Shahid Aghababaie Freeway in Isfahan. The upstream basic freeway segment of this segment has three 3.6-m lanes and 1.5-m right-shoulders. The regulatory signs of the freeway indicate a 70-km/h Speed Limit, a No-Trucks, and a No-Motorcycles signs. The weaving segments under study were divided into seven 50-m sections to record the lane-changing rate and the speed of all vehicles (Figure 1). This sectioning has tried to consider the significant length of the basic freeway segments in upstream and downstream areas.

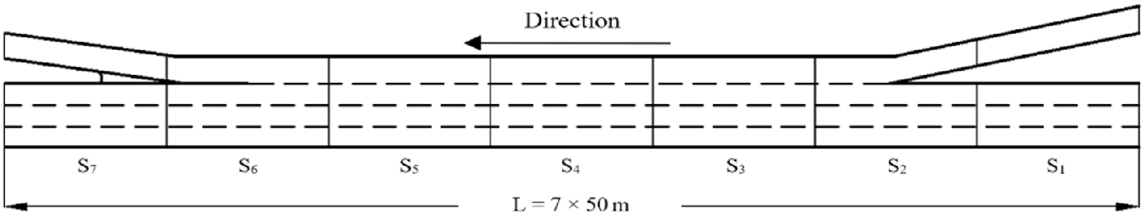


Figure 1. Segment-sectioning of the weaving segment under study

The data were collected by digital video recording and the video camera was mounted on the top of a high building adjacent to the freeway. The procedure included 9 fifteen-minute time steps of recording at different hours, and tried to extract the maximum possible data from recorded videos. These data included the lane-changing rate, the demand flow rate of component flows per vehicle types, the space mean speed, and the density of weaving segment.

Given L_S=197.7 m and L_B=216.7 m, the process of data extraction was performed from the beginning of the second section to the end of sixth section.

Traffic flow in weaving segments is categorized into four distinct categories (freeway-to-freeway flow, freeway-to-ramp flow, ramp-to-freeway flow, and ramp-to-ramp flow) and therefore weaving studies should consider these component flows. In this regard, this study tracked all passing vehicles from the enter point to the exit point of the segment under study, during each time step. Thus, the database of this study contains the flow rates and the lane-changing rates of each component flow, due to the automatic recording of the lane-changing rate of all observed vehicles. Hence, it is not necessary to select a sample for lane-changing rates when the total rate of the society under study is at hand.

To obtain a sample for the density of the weaving segment, in the first place, a sample was drawn from recorded videos (the sample size is 36), and then the number of required samples (N) were computed by the equation of confidence bounds with 97% confidence (N = 130). Due to technical aspects the sample size was extended to 144 (recording the density of segment every one minute in each nine-time step) to facilitate the process of sample selection.

In order to collect a representative sample of the space mean speed of vehicles, this study employed the systematic method (every tenth vehicle in each lane) proposed by the 2010 HCM, as a well-established method of speed studies among transportation researchers. However, it demands more considerable effort. This method led to a sample comprises 1196 space mean speeds (by employing the equation of confidence bounds N equals to 840 with 97% and therefore the larger sample size yields more reliable data). The Table 1 compiled some statistical results of these data.

Table 1. Statistical results of the obtained data

Variables	Weaving Flow	Non-weaving Flow
AVG[1] of speeds (km/h)	65.24	78.14
SD[2] of speeds	10.08	12.96
AVG of LC[3] (lc/h)	1.50	0.19
SD of LC	0.59	0.45
AVG of densities (pc/km/h)	16.31	
SD of densities	5.48	

Note: [1] Average.
[2] Standard Deviation.
[3] Lane-changing rate.

4. Methodology

Global Acceptability of the HCM and the Iranian Code app.lication of the models, presented by this manual for the analysis of weaving segments, led this study to analyze the model of the fifth edition of the HCM, as the most recent edition of this manual.

4.1 FFS Computation

The HCM 2010 presents two methods for computation of FFS, furthermore, there is a rule of thumb for this speed, using which many believe the FFS can be determined.

4.1.1 Field Measurement of FFS

FFS is the mean speed of passenger cars measured during periods of low to moderate flow (up to 1,000 pc/h/ln). For a specific freeway segment, average speeds are almost constant in this range of flow rates. In this method the speed study should measure the speeds of all passenger cars or use a systematic sample (e.g., every tenth car in each lane). A sample of at least 100 passenger-car speeds should be obtained [TRB, 2010].

4.1.2 Estimation of FFS

In this method, FFS estimation is based on the physical characteristics of the segment under study (Equation 1).

$$FFS = 75.4 - f_{LW} - f_{LC} - 3.22TRD^{0.84} \qquad (1)$$

Where
FFS = free-flow speed (mi/h),
f_{LW} = adjustment for lane width (mi/h),
f_{LC} = adjustment for right-side lateral clearance (mi/h), and
TRD = total ramp density (ramps/mi).

4.1.3 The Rule of Thumb

The mean free flow speed and posted speed limit are partially dependent on each other. The Manual on Uniform Traffic Control Devices (MUTCD) [2009] recommends that posted speeds be within 10 km/h (5 mi/h) of the 85th-percentile free flowing speed. Therefore, many studies have used a 'rule of thumb' by adding 10 km/h to the posted limit to estimate free-flow speed without justification. [Fazio, Wiesner and Deardoff, 2014].

4.2 Weaving Analysis

The 2010 HCM methodology is based upon The LOS of weaving segments. In this manual, the weaving analysis procedure begins with determining the operation type in the segment based on the length, after which the values of capacity, lane-changing rates, average speed of vehicles, and ultimately LOS are determined. This procedure is briefly presented in the followings.

The 2010 HCM used the short length (L_S) in its methodology. Even, this length was based for segment classification. In the way that the method computed L_{MAX} in the first step (Equation 2) and if L_S ▯ L_{MAX}, the area would be a weaving segment, otherwise the segment is treated as separate merge and diverge segments.

$$L_{MAX} = [5{,}728(1+VR)^{1.6}] - [1{,}566N_{WL}] \qquad (2)$$

Where N_{WL} is the number of lanes from which a weaving maneuver may be made with one or no lane changes.

The capacity estimation model of the HCM 2010 deliberates two situations; each one of which determines the occurrence of the capacity. The situations are as follows:

1- Breakdown of a weaving segment when the average density of all vehicles in the segment reaches 43 pc/mi/hr.
2- Breakdown of a weaving segment when the total weaving demand flow rate reaches $v_{W(MAX)}$ (2400 pc/h or 3500 pc/h, according to geometrical characteristics of the segment).

In this method:

- When the capacity is controlled by the weaving flow rate, the operation is highly likely to be what is called "constrained" in the 1985 HCM and the 2000 HCM methodologies.
- When it is controlled by density, the operation will likely be what is then called "unconstrained" [Roess and Prassas, 2014].

If demand is less than the estimated capacity ($v/c \leq 1$), the 2010 HCM uses a model to determine the lane-changing rates of the weaving and non-weaving vehicles separately. The sum of these two rates is the total lane-changing rate (L_{CALL}) of all vehicles in the weaving segment.

In the proposed method of the 2010 HCM, the LOS determination is based on density. In the way that after the determination of an average speed of each flow vehicle (Equations 3 to 5) the average speed of all vehicles is computed (Equation 6). Then, the density is computed from the average speed (Equation 7). Ultimately, a table is provided, which shows that the LOS is determined based on the density and the type of facility. It is notable that in this table the LOS F occurs when demand exceeds capacity.

$$S_W = 15+\left[\frac{FFS\text{-}15}{1+W}\right] \tag{3}$$

$$W = 0.226\left(\frac{LC_{ALL}}{L_S}\right)^{0.789} \tag{4}$$

$$S_{NW} = FFS+(0.0072LC_{MIN})\text{-}\left(0.0048\frac{v}{N}\right) \tag{5}$$

$$S = \frac{v}{\left(\frac{v_W}{S_W}\right)+\left(\frac{v_{NW}}{S_{NW}}\right)} \tag{6}$$

$$D = \frac{\left(\frac{v}{N}\right)}{S} \tag{7}$$

Where

S_W = average speed of weaving vehicles within the weaving segment (mi/h),

S_{NW} = average speed of non-weaving vehicles within the weaving segment (mi/h),

S = space mean speed of all vehicles in the weaving segment (mi/h),

W = weaving intensity factor,

LC_{MIN} = minimum rate at which weaving vehicles must change lane to complete all weaving maneuvers successfully (lc/h),

v = total demand (pc/h),

D = average density of all vehicles within the weaving segment (pc/mi/h),

N = number of lanes within the weaving segment,

v_W = weaving flow rate in the weaving segment (pc/h), and

v_{NW} = non-weaving flow rate in the weaving segment (pc/h).

5. Results

Observations revealed that in the segment under study the highest frequency of vehicles belongs to the freeway-to-freeway flow; the freeway-to-ramp, ramp-to-freeway, and ramp-to-ramp flows were placed after that flow respectively.

A lower-than-1000 pc/h/ln flow rate is only observed under the conditions of a time step among a total of nine. The obtained sample from the observations in this time step includes 136 vehicles, 100 of which are associated with passenger cars in the freeway-to-freeway (FF) flow. The cumulative speed distribution curve for this flow is illustrated in Figure 2. A total of 16 lane change was recorded for this 100-item sample. In other words, 86 percent of the passenger cars in the FF flow did not change their lane (optional lane changes). Hence, the FF flow in the ninth time step is highly similar to the through movement of the basic freeway segments. The average speed of the passenger cars was 86.5 km/h in this time step.

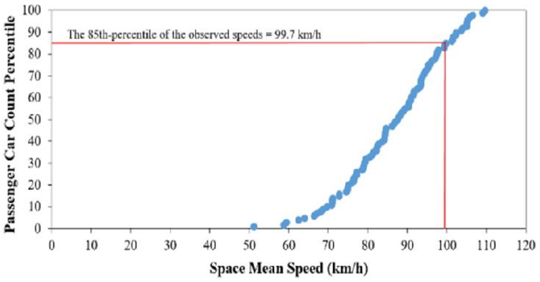


Figure 2. Cumulative speed distribution curve of freeway-to-freeway flow

Since the population under study for computation of FFS is similar to the through movement in the basic freeway segments, and considering the obtained speed is within the specified range by MUCTD (only 1 km/h lower than the upp.er limit), it can be inferred that use of this speed in the study does not cause considerable deviations in the analysis process to produce under-predicted values. Thereby, this study utilized the determined FFS in the weaving segment (based on the FF flow) as a roughly acceptable value in the analysis procedure. It should be noted that the measured and estimated FFS values should match the standard FFS curves for which no interpolations are recommended to be conducted. Hence, the FFS values were rounded to the closest curve according to the known ranges.

Due to the significance of speed in the proposed algorithm, the potential of model in estimating the accurate values of S_{NW}, S_W, S and D was initially evaluated separately, and then the final performance assessment was done based on the LOS criterion (Table 2 and Figure 3).

Comparison of the actual values with the predicted values represents the app.ropriate performance of the HCM 2010 model in prediction of S_W based on the field-measured FFS. On the other hand, the considerably poor statistical performance of this model in prediction of S_{NW} [TRI and Kittelson, 2008; Roess and Prassas, 2014] and the use of weighted average in prediction of S have resulted in the poor performance of the model in prediction of density and consequently of segment LOS based on field-measured FFS compared to the performance of the model based on the estimated FFS. In analysis of the considered segment, the LOS was under-predicted by the 2010 HCM in 4 time steps based on the field-measured FFS, whereas this value was over-predicted in three time steps based on the estimated FFS.

Table 2. Comparison of predicted values with observed values

Basis of Analysis	Parameter							
	S_{NW}		S_W		S		D	
	%Diff	RMSE	%Diff	RMSE	%Diff	RMSE	%Diff	RMSE
FFS$_{\text{MEASURED}}$[1]	-26.2	16.34	0.13	2.11	-18.3	11.76	16.85	4.00
FFS$_{\text{ESTIMATED}}$[2]	-0.09	2.96	13.75	10.70	3.78	4.06	-2.49	1.00
Speed Limit + 10	-45.17	24.29	-8.43	5.50	-33.89	18.94	25.43	6.60

Note: [1] Measured FFS = 86.5 km/h (55 mi/h).
[2] Estimated FFS = 104.7 km/h (65 mi/h).

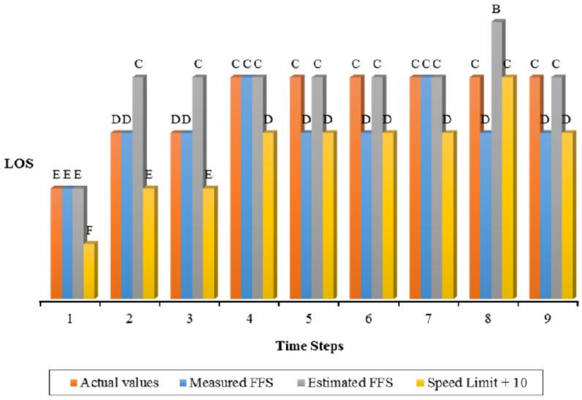


Figure 3. Comparison of predicted LOS with observed LOS

Assumption of limited speed in determining the FFS resulted in significantly poor performance of the model in prediction of all parameters, such that the model was able to predict the LOS merely in one time step.

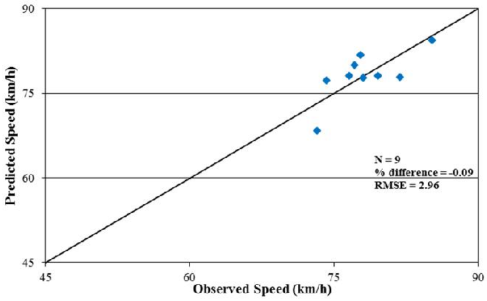
6. Proposed Method

Vehicle speed plays an important role in the proposed model by the HCM for weaving segments. On the other hand, despite the very app.ropriate performance in prediction of S_W, the considerably poor performance of the proposed equation in this model for determining S_{NW} values overshadows the entire analysis procedure, hence causing poor overall performance of the model based on field-measured FFS. Moreover, use of estimated FFS values affects the performance of the model in other aspects.

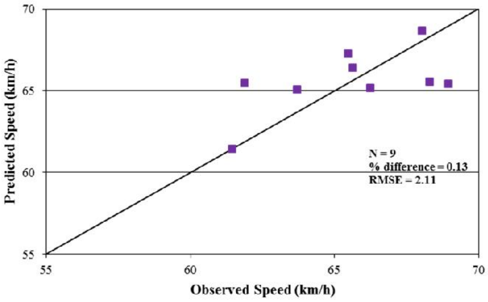
From a design perspective, under- and over-prediction of the model in the cases of using field-measured and estimated FFS values, respectively, results in a non-economical and unqualified design, respectively. Thereby, this study proposed use of operating speed (the 85th-percentile) as the FFS in prediction of S_{NW} (Equation 5) values in order to resolve the highly poor performance of the model in prediction of this value.

In the proposed method, the entire structure of the HCM 2010 model in analysis of the weaving segment as well as the field method for measurement of FFS has been preserved, with the exception in prediction of S_{NW} values (Equation 5), for which the FFS was not equal to the average speed of passenger cars (during periods of low to moderate flow), but to their operating speed under the same conditions. Hence, the FFS values for weaving and non-weaving flows were obtained as 86.5 km/h and 99.7 km/h, respectively. An FFS value of 65 mi/h was assumed for non-weaving vehicles by considering the defined range for standard speed-flow curves.

The obtained values from observation and those predicted by the HCM 2010 based on the proposed method are compared in Figure 4, according to which, the proposed method was able to successfully resolve the weakness of the HCM 2010 model by correctly predicting the LOS value in all the time steps.

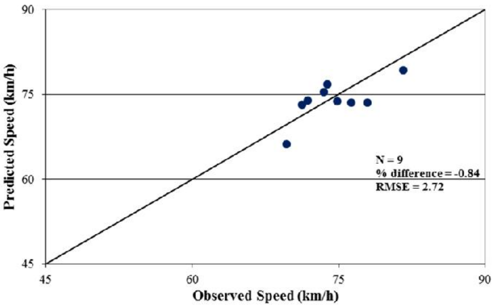


(a) S_{NW}



(b)

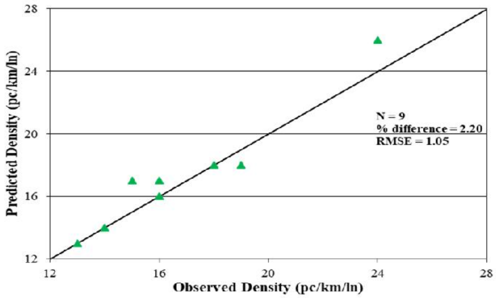
S_W



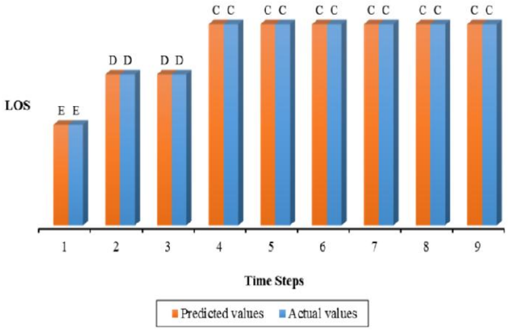
(c) S

Figure 4. Observed versus proposed method-predicted values

(continued)



(d) Density



(e) LOS

Figure 4. (continued) **Observed versus proposed method-predicted values**

7. Conclusion

Comparison of the field-measured values to the results obtained by using the obtained FFS values through the three different methods for analysis of a ramp-weaving segment using the HCM model revealed the accuracy of field method for measurement of FFS as well as its relative superiority compared to the other two methods. However, the major weakness of the HCM 2010 model in prediction of S_{NW} values caused over-prediction of density and consequently deviation in the model output. In estimation of FFS values, the HCM 2010 method caused decreased accuracy of the model and its inefficient overall output due to its over-predicted speed and consequently segment LOS values. Despite researchers' belief, the rule of thumb for computation of FFS produced considerably poor results. Therefore, the field measurement is the most accurate method in computation of FFS, and it is the poor performance of the 2010 HCM model in prediction of S_{NW} which is to be correctly modified.

In order to improve the weakness of the HCM model in prediction of S_{NW}, the current study proposed a method in which the operating speed of passenger cars was used as their average speed in computation of FFS for non-weaving segments. Comparison of the field data to the analysis results obtained through this method indicates its app.ropriate and acceptable performance in covering this weakness.

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