A Study of Pedestrian Movement on Crosswalks Based on Chaos Theory

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Abstract

Walking, as an important transportation mode, plays a large part in urban transportation systems. This mode is of great importance for planners and decision-makers because of its impact on environmental and health aspects of communities. However, this mode is so complex in nature that makes it difficult to study or model. On the other hand, chaos theory studies complex dynamical nonlinear systems that are sensitive to their initial conditions. A small change in initial conditions and/or parameters, may cause a big variation in the results. That is the situation that could happen in many fields of transportation. In the current study, the pedestrian behavior in crosswalks was studied in terms of chaos theory. The well-known social force model was chosen to model pedestrian movement in crosswalks, and based on the model, sensitivity analysis with respect to its parameters was carried out. Pedestrian road crossing behavior based on Helbing social force model was simulated in Matlab codes. Then pedestrian crossing behavior was investigated to detect the chaotic behavior. It was concluded that the speed of a pedestrian when the other pedestrians are closer than 100 cm and when the number of crossing pedestrians is more than 6 is chaotic. Moreover, increasing the number of pedestrians or decreasing the distance between pedestrians increase the occurrence of chaos. Chaotic behavior of speed causes turbulence in pedestrian crossing path, and that makes the path longer. Finally, some solutions for taking the system out of chaos, and consequently making its performance better, were proposed.

Keywords: Chaos Theory, Largest Lyapunov Exponent, pedestrian, road crossing, urban road, Social Force Model.

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1. Introduction

Wide use of vehicles alongside the increase in urbanization and population in recent years have caused a lot of problems. Noise pollution, air pollution and other problems have made the communities to look for possible solutions [Clifton *et al.*2016, Rahul and Verma, 2017, Skayannis, Goudas and Rodakinias, 2017, Tiwari, Jain and Rao, 2016]. As a result, walking received considerable attention as a non-motorized mode [Skayannis, Goudas and Rodakinias, 2017], and decision-makers encouraged people to substitute motorized modes with walking as much as possible.

Development of walking mode needs careful considerations regarding its safety and convenience aspects. A key step in a typical pedestrian study is to determine pedestrian behavior through simulation of walking movement. The more detailed and thorough the analysis are, the safer and more useful the design of pedestrian facilities is [Lu et al. 2016, Zheng et al. 2015].

However, studying and modeling human behavior is difficult because human nature is a very complex phenomenon to model. Helbing and Molnar stated that pedestrian walking behavior is chaotic [Helbing and Molnar, 1995]. Chaos theory is a branch of mathematics in which, the nonlinear dynamic complex systems are studied and analyzed. Chaotic patterns, while at first glance seem to be stochastic, are deterministic with a high sensitivity to its parameters and initial conditions [Barenghi, 2010, Eckmann and Ruelle, 1985, Frazier and Kockelman, 2004, Gokyildirim, Uyaroglu and Pehlivan, 2016].

In the current study, social force model was used to study chaotic pattern of pedestrian urban road crossing behavior. To achieve it, by using a Matlab code, the chaotic region of the model's parameters were detected. Having the domain of the presence of chaos, the paper suggested some solutions for taking the system out of chaos.

The rest of the paper is organized as follows: Some studies regarding pedestrian movement modeling, and the application of chaos theory to transportation are briefly reviewed. Then, the model is introduced and its sensitivity to the parameters is analyzed. finally, the results and conclusions are presented.

2. Literature Review

Chaos theory has been applied to many aspects of traffic engineering and transportation planning. Van Zuylen *et al.* [van Zuylen, Geenhuizen and Nijkamp, 1999] investigated the problem of ill predictability in transportation decision-making. In their study, which is more conceptual rather than quantitative, they have viewed the process of transportation decision-making as a complex and nonlinear phenomenon that acted chaotically.

There are many studies devoted to detecting chaos from time series data [Narh *et al.*2016]. Narh *et al.* claimed that chaos theory was a prime candidate for application to urban traffic control, particularly due to the emergence of new sources of high-resolution temporal and spatial data [Narh *et al.*2016]. Adewumi *et al.* presented an approach to predict urban traffic flow based on chaos theory and using nonlinear time series modeling techniques [Adewumi, Kagamba and Alochukwu, 2016].

Zhang and Jarrett [Zhang and Jarrett, 1998] found out chaos in the dynamic gravity model. Frazier and Kockelman [Frazier and Kockelman, 2004] introduced chaos and its application to transportation problems. They provided a short term traffic prediction model for California by using this theory. Ching Loa and Jang Cho [Lo and Cho, 2005] provided a chaotic model by using the Greenshield's model and flow-velocity-density fundamental relation. Xu and Gao [Xu and Gao, 2009] proposed a two-stage flow prediction model based on gravity model and user equilibrium.

Jieni and Zhongke [Xue and Shi, 2008], and Wanga and Shi [Jin Wanga, 2013] provided a short term traffic prediction model. Rassafi *et al.* [Rassafi, Poorzahedy and Vaziri, 2006] proposed an alternative definition for sustainable transportation using dynamical concepts of the stability and chaos theories. Mahmoudabadi [Mahmoudabadi, 2014] investigated the chaotic behavior in daily traffic volume. Krese and Govekar [Krese and Govekar, 2013] detected chaotic behavior in traffic flow dynamics. Yaling and Zhongke [Yaling and Zhongke, 2015] proposed a modified discrete dynamic coupled map traffic model and detected chaos in traffic flow through the model. Sharifi et al [Sharifi *et al.*2015] employed an existing model of traffic flow on a small expository network to show how chaotic situations occur at certain demand and supply parameter values. Mahmoudabadi and Seyed Hosseini [Mahmoudabadi and Hosseini, 2013] presented a mathematical model based on chaos theory to solve hazmat routing problem. Xu et al [Xu, Ye and Shan, 2015] provided co-development process of road network and vehicle ownership prediction model that has a chaotic behavior in some situations. Cheng et al [Chenga *et al.*2017] identified chaotic characteristics of traffic flow and demonstrated that phase space base on support vector regression prediction model has relatively strong fitting capability.

Pedestrian behavior hasn't been studied from the chaos theory viewpoint yet [Abshar, 2006, Shahabi, 2010]. However, there are studies of pedestrian behavior that resulted in different pedestrian models. For example, Helbing presented a model for simulating pedestrian movement [Helbing, 1991]. He and Molnar[Helbing and Molnar, 1995] developed this model and introduced it as a social force model. It was modified by Helbing *et al.* in 1999 and 2002 [Helbing *et al.*2002, Helbing *et al.*2001]. General form of this model is presented in Equation 1:

$$\overrightarrow{F_\alpha}(t) := \overrightarrow{F_\alpha^0}(\vec{v}_\alpha, v_\alpha^0 \overrightarrow{e_\alpha}) + \sum_\beta \overrightarrow{F_{\alpha\beta}}(\vec{e}_\alpha, \overrightarrow{r_\alpha} - \overrightarrow{r_\beta}) + \sum_B \overrightarrow{F_{\alpha\beta}}\left(\vec{e}_\alpha, \overrightarrow{r_\alpha} - \overrightarrow{r_B^\alpha}\right) + \sum_i \overrightarrow{F_{\alpha\imath}}(\vec{e}_\alpha, \overrightarrow{r_\alpha} - \vec{r}_\imath, t) \quad (1)$$

Kosinski and Grabowski [Kosiński and Grabowski, 2011] provided a modified social force model by using Langevin equations. Shariat-Mohaimani *et al.* [Shariat-Mohaimani *et al.*2014] simulated movement of Tehran pedestrians by VISWALK software and the social force model. They found out that angular intensity effect factor is very sensitive. Traffic and Transportation Deputy of Tehran Municipality [Transportation and Traffic Deputy of Tehran Municipality, 2014] calibrated this social force model for Tehran's different pedestrian facilities. Zeng *et al.* [Zeng *et al.*2014, Zeng *et al.*2017, Zeng, Nakamura and Chen, 2014] studied on pedestrian crosswalk behavior at signalized intersections based on social force model. Lu *et al.* [Lu *et al.*2015] provided a new approach based on modified social forced model for specifying the design of the signalized crosswalk width. Kretz [Kretz, 2015] studied on oscillations in the social force model. Seer et al [Seer *et al.*2014] presented a procedure to estimate model parameter values of social force based model.

To the best of our knowledge, the quantified analysis of chaos in social force model with the parameters calibrated from prior dependent studies has not been previously done.

3. Terms and Methods

Pedestrian micro simulation models provide particular characteristics of pedestrian behavior such as velocity and acceleration. Study of pedestrian behavior from chaos theory viewpoint needs detailed data for each pedestrian, thus this study needs to use micro simulation models. Some of widely used pedestrian micro simulation models are presented in Figure 1.

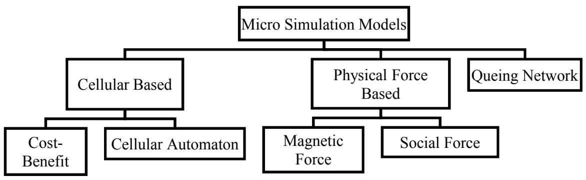

Figure 1. Widely used pedestrian micro simulation models

Social force model is based on Newton's second law [Barenghi, 2010]. Helbing presented this model for the first time and it is developed and modified gradually by Helbing *et al.* [Helbing, 1991]. This model consists of four main parts. Each part is presenting an exerted force to pedestrian that is created by interaction between a pedestrian and its environment. These parts are reflected by the following pedestrian preferences:

- Pedestrians prefer to reach their destinations through the shortest path and with the desired velocity. Difference between the desired velocity and the actual one causes an acceleration in pedestrian walking.
- Pedestrians prefer to keep a certain distance from other pedestrians while walking. The amount of this distance is affecting on the pedestrian's velocity and acceleration. This effect is calculated by elliptical monotonic equipotential decreasing function.
- Pedestrians try to keep a certain distance from walls and barriers. This effect is modeled by monotonic decreasing equipotential function.
- Pedestrians are sometimes attracted by other pedestrians or objects. This effect can be modeled by monotonic increasing equipotential function. Usually this part is omitted to simplify the model [Helbing and Molnar, 1995].

Social force model in a situation with no barriers and attractions is presented in Equation 2 [Rafe and Karimi, 2014, Transportation and Traffic Deputy of Tehran Municipality, 2014]. It means that the third and fourth parts of Equation 1 have been neglected here. In this equation, A and B show the interaction strength and the interaction range and b stands for the half of the semi-minor axis of the elliptical equipotential lines that is virtually assumed in the pedestrian's surrounding, respectively. V_0 is the desired velocity, e_{ax} and e_{ay} are movement direction vectors, v_{ax} and v_{ay} are the velocity of pedestrian α in the directions of x and y, t is the relaxation time, $d_{\alpha\beta}$ is the distance between pedestrians α and β, Δt shows the reaction time, λ denotes the amount of effects of pedestrian acceleration in various angels and v_β is the velocity of the pedestrian β.

$$a_x\vec{\imath} + a_y\vec{\jmath} = \frac{(V_0\overrightarrow{e_{ax}} - \overrightarrow{V_{ax}})\vec{\imath} + (V_0\overrightarrow{e_{ay}} - \overrightarrow{V_{ay}})\vec{\jmath}}{\tau} + \sum_{i=1}^{n} A e^{\frac{-b}{B}} \cdot \frac{|d_{\alpha\beta}| + \||d_{\alpha\beta}| - (|v_{\beta i}| - |v_\alpha|)\Delta t\|}{2b} \cdot \left(\lambda_\alpha + (1-\lambda_\alpha) \cdot \frac{1 + \left[\left(\frac{v_{\alpha x}}{|v_\alpha|} \cdot \frac{-d_{(\alpha\beta)x}}{|d_{\alpha\beta}|}\right) + \left(\frac{v_{\alpha y}}{|v_\alpha|} \cdot \frac{-d_{(\alpha\beta)y}}{|d_{\alpha\beta}|}\right)\right]}{2}\right) \cdot \frac{1}{2}\left[\left(\frac{d_{(\alpha\beta)x}}{|d_{\alpha\beta}|} + \frac{d_{(\alpha\beta)x} - (v_{(\beta x)i} - v_{ax})\Delta t}{\||d_{\alpha\beta}| - (|v_{\beta i}| - |v_\alpha|)\Delta t\|}\right)\vec{\imath} + \left(\frac{d_{(\alpha\beta)y}}{|d_{\alpha\beta}|} + \frac{d_{(\alpha\beta)y} - (v_{(\beta y)i} - v_{ay})\Delta t}{\||d_{\alpha\beta}| - (|v_{\beta i}| - |v_\alpha|)\Delta t\|}\right)\vec{\jmath}\right] \quad (2)$$

The presented social force model has five parameters. These parameters should be calibrated for each pedestrian facility and each situation. Traffic and Transportation Deputy of Tehran Municipality [Transportation and Traffic Deputy of Tehran Municipality, 2014] and Shariat-Mohaimani *et al.* [Shariat-Mohaimani et al.2014] calibrated parameters of this model for different pedestrian facilities of Tehran. The results are provided in Table 1.

Parameters resulted from the studies in the facilities with the presence of vehicles are the same as those resulted from the study of signalized intersections, unsignalized intersections and midblock crossings. Thus, these parameters are used in this study for simulating pedestrian's walking in different situations of urban road crossings in Tehran.

In order to detect the chaotic behavior in pedestrian movement, it has to be dynamically simulated and the dependence of the dynamic model on its parameters should be verified by a set of sensitivity analyses. Social force model is iterative and in each iteration the accelerations of the pedestrian in x and y directions are calculated. The x direction indicates the width of the road while the y direction denotes the length of the crosswalk. Acceleration is calculated by Equation 2 in which the first and second parts of Helbing social force model (Equation 1) are considered. The first part denotes the acceleration that is resulted from the difference between the desired and actual velocities of pedestrians. The second part denotes the acceleration that is resulted from pedestrians' interactions. After that, the velocity is calculated by using 1-second time steps. To calculate the velocity, it is assumed that the velocity is monotonic. Position of the pedestrian in x and y directions are calculated in the same way as the velocity. The next iteration is continued likewise until the pedestrian crosses the road completely. This algorithm is provided in the Figure 2.

There are different ways for detecting chaos in a dynamical system, such as Largest Lyapunov exponent (LLE), Fast Fourier Transform and fractal dimension, among

Table 1. Social force model calibrated parameters for Tehran [Shariat-Mohaimani *et al.*, 2014, Transportation and Traffic Deputy of Tehran Municipality, 2014]

Facility	Δt Sec	B Cm	A Cm/s^2	λ	τ Sec	Reference
(Facilities) without presence of vehicle	1.8	0.23	2.99	0.98	0.88	[Transportation and Traffic Deputy of Tehran Municipality, 2014]
(Facilities) with presence of vehicle	1.2	0.27	1.85	0.62	0.62	[Transportation and Traffic Deputy of Tehran Municipality, 2014]
Signalized intersection	---	0.27	1.85	0.62	0.62	[Shariat-Mohaimani *et al.*, 2014]
Unsignalized intersection	---	0.27	1.85	0.62	0.62	[Shariat-Mohaimani *et al.*, 2014]
Mid-block crossing	---	0.27	1.85	0.62	0.62	[Shariat-Mohaimani *et al.*, 2014]
Sidewalk	---	0.23	2.99	0.98	0.88	[Shariat-Mohaimani *et al.*, 2014]
Walk way	---	0.23	2.99	0.98	0.88	[Shariat-Mohaimani *et al.*, 2014]
Stair	---	0.23	2.99	0.98	0.88	[Shariat-Mohaimani *et al.*, 2014]

which LLE is one of the most popular techniques [Frazier and Kockelman, 2004]. Wolf *et al.* [Wolf *et al.*1985] and Eckmann and Ruelle [Eckmann and Ruelle, 1985] have presented an approach to compute LLE.

In this study, it was assumed that all of the pedestrians used crosswalks, and the road was a divided urban highway with three lanes in each direction. The segment of the road that pedestrians use to cross is a rectangle with 30m length and 6m width. Desired velocity of a pedestrian was assumed 120 cm/s or 4 ft/s that was determined by Highway Capacity Manual (HCM) [Transportation Research Board, 2010] and Manual on Uniform Traffic Control Devices (MUTCD) [Federal Highway Administration (FHWA), 2003]. Maximum velocity was assumed 180 cm/s [Essex County Council, 2006, Iran's Planning and Budget Organization, 1996, Zębala, Ciępka and Reza, 2012].

Having the above-mentioned assumptions, the simulation process of Figure 2 was conducted and related parameters and indicators were calculated. The next section presents the results of this calculations.

4. Results and Analysis

In order to be able to better observe the parameter variations, and hence finding the more accurate pattern of changes, the simulation was first carried out for just two pedestrians. Then, after finding the chaotic parameter and its associated range, the simulation was extended to more numbers of pedestrians in the next steps. After the simulation, the results of speed and acceleration of each pedestrian during his or her movement across the crosswalk were obtained. For the sake of brevity only some of the excerpted results have been reported herein.

Figure 3 depicts the fluctuation of speed in the x direction for the pedestrian with 30 cm distance with the other pedestrian at the start of the crossing (initial distance). In this diagram LLE is positive and equals to 3.7080 that shows the chaotic behavior of pedestrian's speed.

Also Figure 4 indicates the fluctuation of speed (in the x direction) for the pedestrian with 68 cm initial distance with the other one. Same as Figure 3, LLE in this diagram is positive indicating the chaotic behavior of pedestrian's speed but it is less than LLE in Figure 3.

Figure 5 shows the crossing paths of two pedestrians with 20 cm initial distance from each other. According to this figure both pedestrians don't cross directly from the road and their paths present a large amount of turbulence.

The analysis of changes in different parameters of the social force model showed that if pedestrian's distance from the other one decreased from 100 cm, his or her movement speed became chaotic. In this situation presence of chaos was observed in the speed and its x and y components. Changes in the initial position of the pedestrian and its initial speed didn't affect the chaotic behavior of speed.

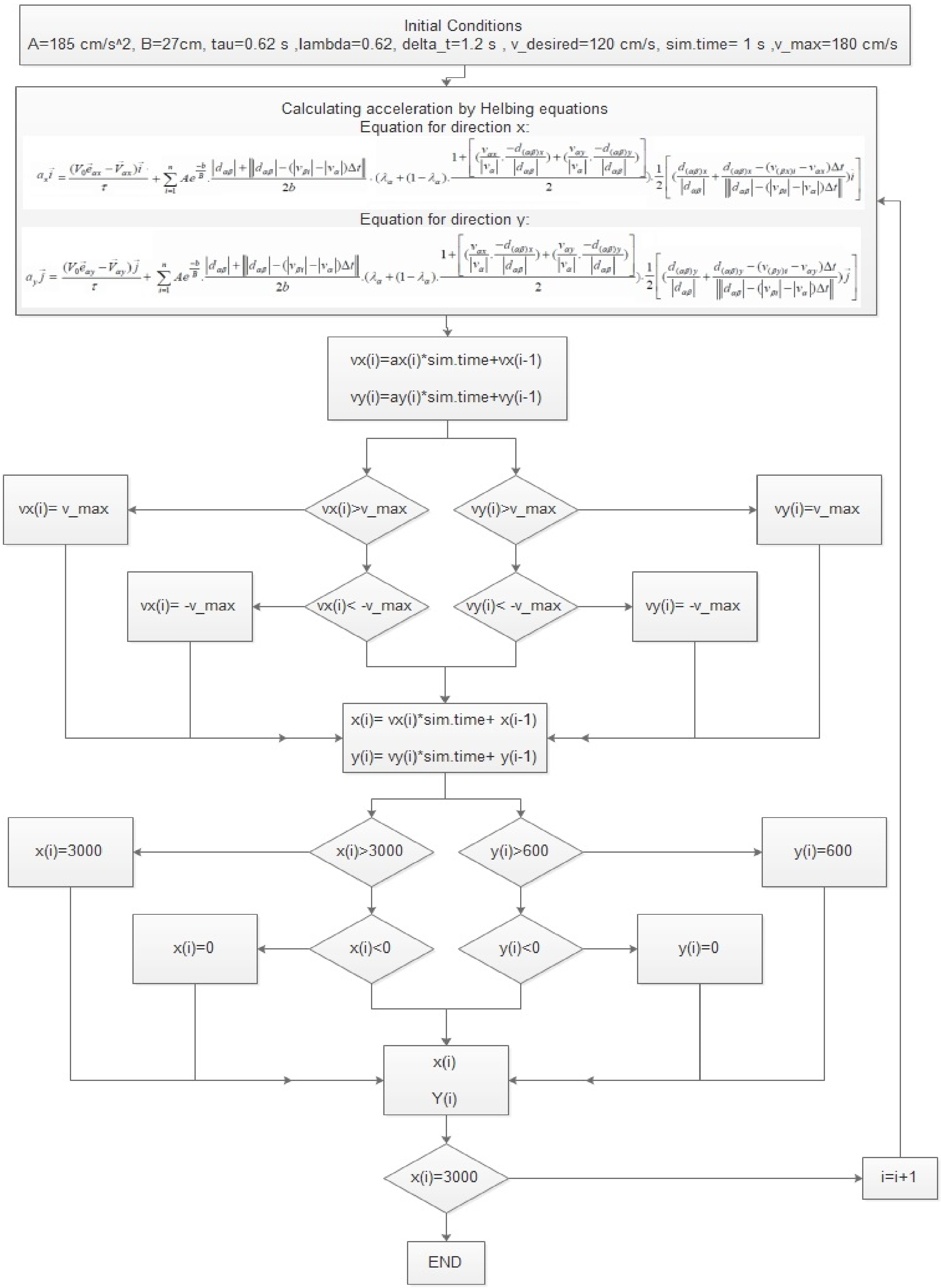

Figure 2. Pedestrian crossing simulation flowchart

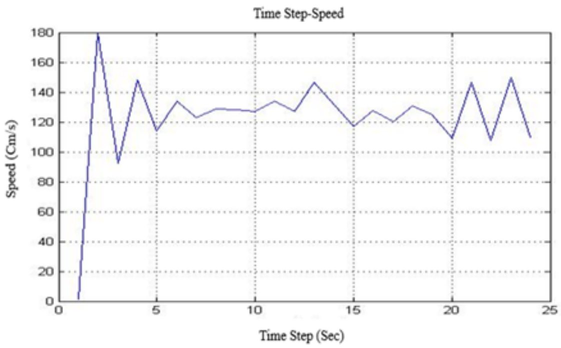

Figure 3. Diagram of speed (in the x direction) for the pedestrian with 30 cm initial distance with the other pedestrian, LLE= 3.7080

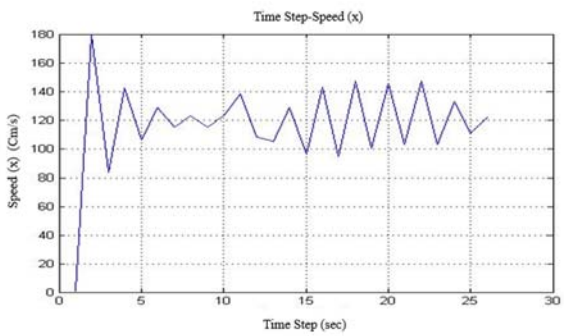

Figure 4. Diagram of speed (in the x direction) for the pedestrian with 68 cm initial distance with the other pedestrian, LLE= 2.3549

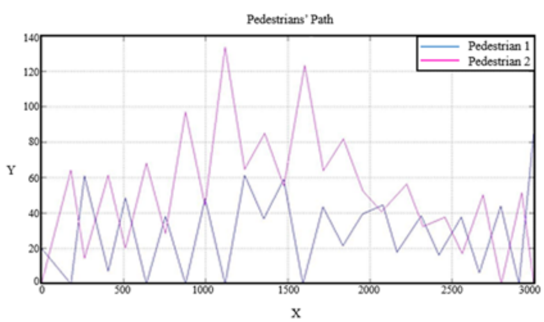

Figure 5. Crossing paths of two pedestrians with 20cm initial distance from each other

LLE for different distances between two pedestrians is shown in Figure 6. As can be seen in this figure, if the distance is less than 100 cm, most of LLEs are positive. Thus, speed has a chaotic behavior in such distances.

Furthermore, the diagram of final speed of the pedestrian with respect to initial distance from the other one is shown in Figures 7. As can be seen in this figure, the pedestrian's speed at a distance less than 100 cm does not converge to a particular value, while in more distant situations pedestrian eventually reaches to a stable speed.

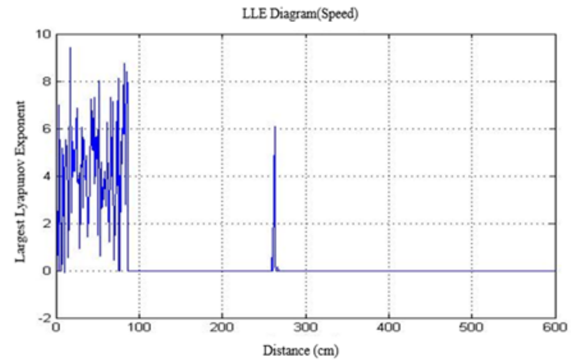

Figure 6. LLE of speed for different distances between pedestrians

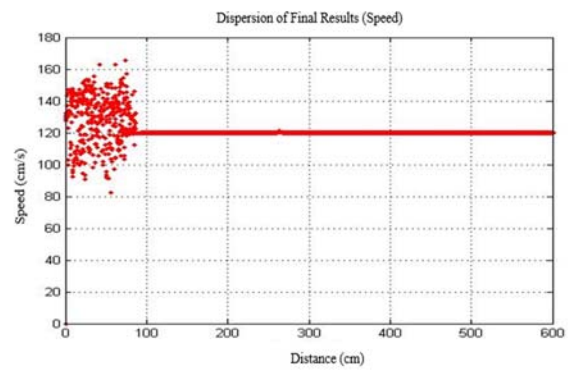

Figure 7. Diagram of dispersion of final speeds in presence of the other pedestrian with different initial distances

In order to verify the results, another Matlab code was written to simulate the walking behavior of more than two pedestrians. The numbers of pedestrians in various runs were considered different. In this code, it was assumed that all of the pedestrians have an equal distance from each other. Again, only some selected results are reported here to demonstrate the approach.

Figure 8 presents the variation of final speed for a pedestrian in presence of 21 other people with equal initial distances from the adjacent ones, and as expected, it shows a chaotic fluctuation. This behavior can be easily observed from another aspect shown in Figure 9. In this figure the crossing paths of 22 pedestrians have been illustrated, and as can be seen, after a while, a complete disorder occurs in pedestrians' crossing paths.

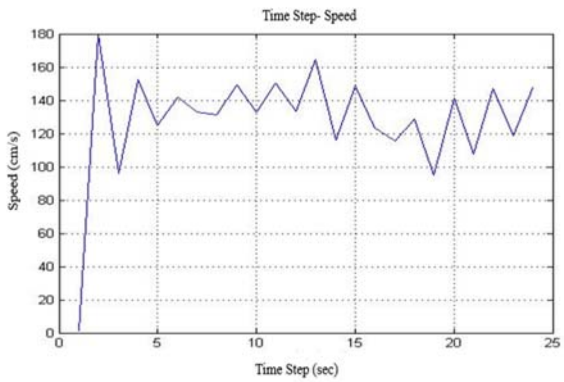

Figure 8. Final speed of a pedestrian in presence of 21 other pedestrians with certain initial distances

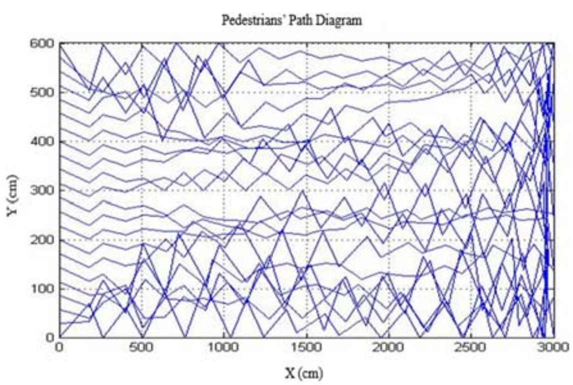

Figure 9. Crossing paths of 22 pedestrians with particular initial distances

Figure 10 Shows the LLE of pedestrian's final speed for different numbers of pedestrians who cross the road at the same time. According to this figure, when the number of the pedestrians is more than 6, LLE is positive and the speed behavior of pedestrians is chaotic. When the number of pedestrians is more than 6, the distance between pedestrians is less than 100 cm. Figure 11 illustrates the LLE of speed with respect to the initial distances between two adjacent pedestrians whom are assumed to be equable for all. Like the state of two pedestrians, speed shows a chaotic pattern in initial distances less than about 100 cm.

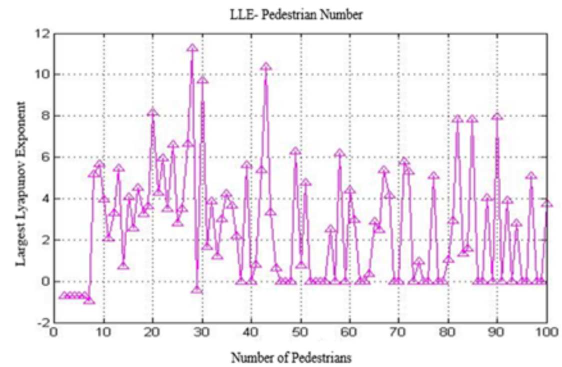

Figure 10. LLE of final speed for different number of pedestrians from 2 to 100

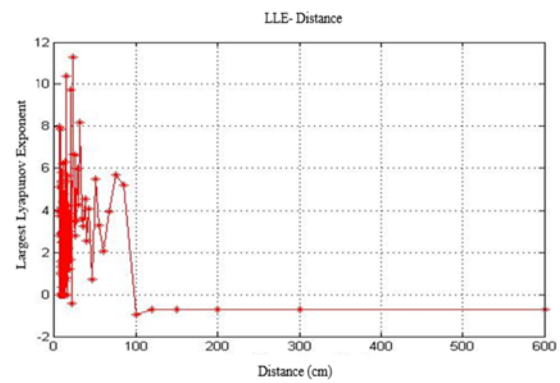

Figure 11. LLE of final speed for distances that correspond to the certain number of pedestrians

Increasing the number of pedestrians or decreasing the distance between pedestrians creates some turbulence in the pedestrian's path. This turbulence causes a longer path and increases the average displacement of pedestrians. On the other hand, by increasing the average displacement, there is no increase in the crossing time. Therefore, the average speed of the pedestrians increases.

As can be seen in Figure 12, the average displacement of pedestrians increases as the number of pedestrians increases and their initial distance decreases. Furthermore, according to Figure 13 when the number of pedestrians increases, their average speed consequently increases too. Increasing average displacement means that pedestrians should walk more and increasing average speed means that pedestrians should walk faster than their desired speed. Longer path in the chaotic situation compared to the normal one and higher speed compared to the desired one decreases the utility of the system. Hence, the system should be taken out of chaos to improve its utility.

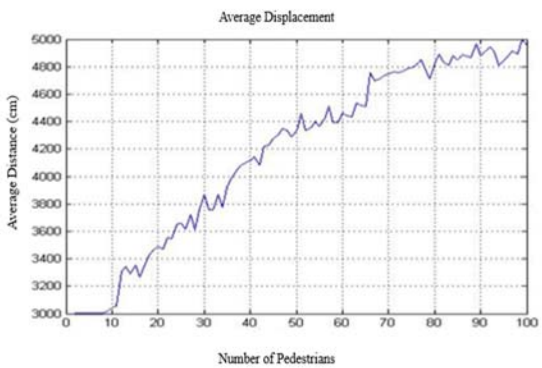

Figure 12. Average displacement with respect to different numbers of pedestrians from 2 to 100

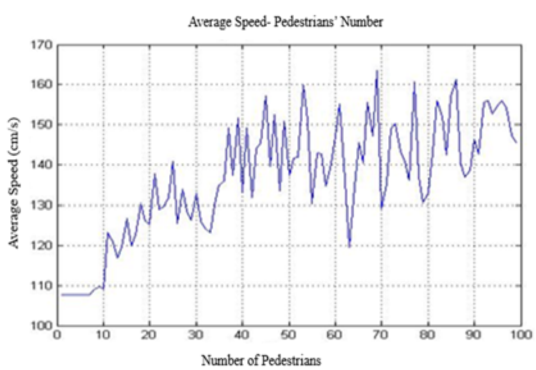

Figure 13. Average speed with respect to different numbers of pedestrians from 2 to 100

The distance between pedestrians is the term that cause a chaotic behavior. This term can be controlled by some parameters. Equation 3 shows these parameters and their mathematical relation with the distance between pedestrians.

$$D = \frac{d}{Q*T} \quad (3)$$

where D denotes the average distance between pedestrians in centimeters, d is the length of pedestrian's crossing lines in centimeters and Q is the pedestrian's flow rate in pedestrians per second. Finally, T is the term that is different with respect to crossing type, as described below:

- In signalized intersections, it denotes red signal time in seconds.
- In unsignalized intersections and midblock crossings, it denotes average time in seconds to obtain the gap acceptance.

After determining D for certain crossing type, Equation 3 can be used as follows to take the system out of chaos.

a- Increasing the length of pedestrian's crossing lines.

b- Decreasing red time in signalized intersections. Using speed bumps or speed humps before crossing lines in unsignalized intersections and mid-block crossings to reduce the average gap acceptance time.

c- Pedestrian's demand management by using another crossing facility.

5. Conclusion

Sensitivity analysis of social force model showed that speed of pedestrians behaved chaotically when the initial distance between pedestrians is less than a certain value. When the initial distance between pedestrians is less than 100 cm, speed of pedestrians behaved chaotically in x direction, y direction and movement direction. Also chaotic behavior observed in few points between the distance 100cm to 300cm too. To detect this chaotic behavior, LLE was calculated and speed dispersion diagram with respect to initial distance was plotted.

This chaotic behavior caused an increase in pedestrian's displacement without any changes in the duration time. Moreover, longer pedestrian's crossing path as well as higher average speed, decreased the utility of the system. As a result, some solutions such as increasing the length of pedestrian crossing lines, decreasing red time in signalized intersections, using speed bumps or speed humps before crossing lines in unsignalized intersections and mid-block crossings and using another crossing facility simultaneously are provided to get the system out of chaos and improve it.

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