Supply Chain Coordination through Lead Time Crashing in a Socially Responsible Supply Chain Considering Transportation Modes and Carbon Emissions Tax

Seyyed-Mahdi Hosseini-Motlagh[1], Samira Ebrahimi[2], Nazanin Nami[3], Joshua Ignatius[4]

Received: 02.01.2018 Accepted: 05.05.2018

Abstract

In this paper, a socially responsible supply chain consisting of one supplier and one retailer is proposed. The supplier decides on replenishment cycle multiplier and the retailer invests in corporate social responsibility (CSR) and decides on the order-up-to-level under a periodic review replenishment policy. The retailer's decisions impact on the supplier's probability as well as the supply chain. Therefore, the supplier proposes a lead time crashing contract to entice the retailer to participate in the coordination model. In this paper, the supplier can reduce the lead time by spending more cost and select a faster transportation mode. Two transportation modes (fast and slow) are considered in the proposed contract. By selecting a fast transportation mode, the retailer's inventory cost and shortage will be decreased. Therefore, the retailers' profit will be increased, while the lead time reduction cost and carbon emissions tax are imposed on the supplier under the lead time crashing contract. Numerical experiments and sensitivity analyses also show that the lead time crashing contract can coordinate the proposed model and the profit of supply chain and both members will be improved. Moreover, according to this contract the extra profit that is achieved from the coordination model is shared between both members of the supply chain fairly.

Keywords: Transportation modes, lead time crashing contract, supply chain coordination, corporate social responsibility, periodic review inventory system

Corresponding author E-mail: motlagh@iust.ac.ir

[1] Assistant Professor, School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran.

[2] M.SC. student , School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

[3] M.SC. student, School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

[4] Associate Professor, WMG, University of Warwick, Coventry, UK. UK

1. Introduction

Lead time crashing is one of the characteristics of agile approach in the supply chain management in order to create a responsive structure to improve customer service [Rushton, Croucher, and Baker, 2010]. One of the ways to reduce the lead time is to use a faster transportation mode. Faster transportation mode will surely cost more, but it reduces the need for holding stock and increases the service level [Rushton, Croucher, and Baker, 2010]. Different transportation modes may be available among the supply chain members with different lead times, costs, fuel consumption and carbon emissions. Each of these factors impacts on the selection of transportation modes by the supplier. In the supply chains, reducing the lead time by transportation modes can decrease the retailer's cost while it is costly to the supplier [Johari, Hosseini-Motlagh and Nematollahi, 2017b]. As an example, two transportation modes such as less than truckload (LTL) and full truckload (FTL), are so popular in the real world [Heydari, Zaabi-Ahmadi and Choi, 2016]. Under LTL mode, the shipper's items do not take up the whole space of the truck. The shipper only pays the cost of the volume occupied by his/her items, and the rest of the space may be occupied with other shippers' items. This transportation mode may increase vehicle stops and increase the delivery time. On the other hand, in FTL mode, the items fill up all the space of the vehicle. In this mode, the lead time reduces and inventory costs decrease while more transportation costs are imposed on the supplier. The lead time can be reduced by adopting a faster transportation mode. According to Corbett, Wang and Winebrake [2009] by increasing the speed, the carbon emissions into the environment will be increased. Transportation is one of the significant sources of carbon emissions [Bazan et al., 2015]; therefore, governments impose some rules such as carbon emissions tax to constrain the carbon emissions of transportation [Hoen et al. 2014]. Several countries such as the United Kingdom, France and Australia impose the carbon emissions tax in the transportation based on fuel consumption, considering that the carbon emissions per unit fuel consumed are constant [Carling et al. 2017]. Thus, in order to reduce the lead time, the supplier has to pay the carbon emissions tax and lead time crashing costs. Lead time reduction leads to fewer safety stocks, less shortage and accordingly fewer costs in the supply chain [Leng and Parlar, 2009]. The lead time crashing not only increases the retailer's profitability but also impacts on the profitability of the supplier and the whole supply chain [Johari, Hosseini-Motlagh and Nematollahi, 2017]. Therefore, there is a trade-off between lead time reduction and its incurred costs for the supplier.

With the increasing trend of globalization, corporate social responsibility (CSR) efforts take more attention from the companies [Panda, 2014]. CSR is defined as an activity influences the environmental protection, carbon emissions restriction, human rights and philanthropy [Ni, Li and Tang, 2010]. As an example in the real world, Starbucks Company commits to make social responsibility activities by providing one million coffee trees to farmers as a partner in Conservation International's Sustainable Coffee Challenge [www.starbucks.com /responsibility]. Nowadays the companies have noticed that investing in the CSR enhances the popularity of the products and increases the profitability [Nematollahi, Hosseini-Motlagh and Heydari, 2017]. For example, some companies such as Nike, Adidas, Walmart and Gap have done the CSR activities in their supply chains which result in customers' willingness to pay more for the products with the CSR efforts [Panda, 2014]. Investments in the CSR efforts by one supply chain member not only affect its own profit, but also influences the other supply chain members [Nematollahi, Hosseini-Motlagh and Heydari, 2017]. Hence, if the supply chain decisions are made in the coordinated model, the profitability of whole supply chain and all members increase [Nematollahi, Hosseini-Motlagh and Heydari, 2017].

Supply chain coordination is one of the most significant issues in the supply chain management. Decisions in the supply chain can be made in two models, the decentralized and centralized models. Under the decentralized model, each member decides to optimize its profitability regardless the other members [Johari, Hosseini-Motlagh and Nematollahi, 2017a]. In this model, the profitability is not optimal from the whole supply chain viewpoint. Under the centralized model, a decision maker decides on all supply chain decisions. In this model, the profitability of the whole supply chain improves but it may lead to the loss for some members in comparison to the decentralized model [Nouri, Hosseini-Motlagh and Nematollahi, 2018]. Therefore, the incentive contracts are used in order to motivate the members to make optimal decisions from the supply chain viewpoint. There are different coordination contracts in the literature such as delay in payment [Hojati, Hosseini-Motlagh, Nematollahi, 2017], revenue sharing [Sarathi, Sarmah, Jenamani, 2014], buyback [Wu, 2013] and quantity discount [Li and Liu, 2006]. Another incentive mechanism is lead time crashing. Under this contract, the supplier reduces the lead time in order to convince the retailer to participate in the coordination model. By applying the lead time crashing contract, the retailer's inventory cost and shortage cost decreases, as a result, his/her profit enhances in comparison to the decentralized model. Thus, the supplier makes a trade-off between lead time reduction and its incurred costs.

In this paper, a two-echelon socially responsible supply chain consisting of one supplier and one retailer is investigated. The retailer uses a periodic review inventory system to replenish its stock and his/her decision variables are the CSR investment and order-up-to-level. The supplier uses a lot-for-lot replenishment policy and decides on the replenishment cycle multiplier. The proposed supply chain is investigated under three models; decentralized, centralized and coordinated models. In the decentralized model, each member of the supply chain takes decisions to optimize his/her profitability individually. In the centralized model, decisions are made from the whole supply chain viewpoint. The retailer's decisions impact on the supplier's profitability, while the supplier's decision doesn't have effects on the retailer's profitability. Therefore, under the centralized model, the decision variables of the retailer change and the retailer's profit may be less than the decentralized model, while the whole supply chain profitability enhances. Thus, the centralized model is not acceptable for the retailer. The supplier proposes the lead time crashing as an incentive contract to convince the retailer to participate in the joint decision-making model. Under the proposed contract, the supplier reduces the lead time by adopting a faster transportation mode. The faster transportation mode uses more fuel and emits more emissions to the environment. In practice, the government tries to control and reduce the carbon emissions via imposing the emissions tax on additional used fuel. Thus, choosing faster mode imposes more cost on the supplier. Consequently, the supplier has to make a trade-off between the imposed cost and reduced lead time. Under lead time crashing contract, not only the whole supply chain profitability improves, but also the profits of all members increase in comparison to those under the decentralized model. To the best of our knowledge, simultaneous coordination of CSR investment and replenishment decision under the periodic review inventory system has not yet been investigated. In this paper, we coordinate the CSR investment and periodic review inventory decisions by lead time crashing contract. Under this contract, the lead time reduces by a faster transportation mode; however, it imposes the cost of carbon emissions tax on the supplier.

The sections of this paper are organized as follows. Section 2 contains the literature review and related former papers. Notations and assumptions are presented in section 3. The decentralized, centralized and coordinated models are proposed in section4. Numerical examples and sensitivity analysis are experimented in section 5. Finally, the conclusion is proposed in section 6.

2. Literature Review

In this section, a brief review of related literature is provided. To coordinate the decisions in the supply chain, different coordination contracts are developed. For example, Li, Wang, and Dai, [2016] investigated a supply chain consisting of one supplier, one retailer and one carrier with price sensitive market demand and stochastic lead time. In their study, quantity discount contract is used to coordinate pricing and inventory decisions. Jaber and Osman, [2006] coordinated a two-echelon supply chain consisting of one supplier and one retailer by a delay in payment contract. Bai, Chen, and Xu, [2017] used a revenue and promotional cost-sharing contract and a two-part tariff contract to coordinate a two-echelon supply chain consisting of a manufacturer and a retailer. Another contract that can be used to coordinate the supply chain is based on controlling the lead time. For example, Leng and Parlar, [2009] considered a lead time crashing contract to coordinate a two-echelon supply chain under EOQ inventory system. In their model, the manufacturer's lead time included three elements: production time, set up time and shipping time. Heydari, [2014] proposed a mechanism based on controlling lead time in order to motivate the retailer to participate in the coordination decisions. They considered a decentralized supply chain consisting of one supplier and one retailer. Heydari, Zaabi-Ahmadi, and Choi, [2016] coordinated a two-echelon supply chain by lead time crashing contract under EOQ inventory system. The lead time reduces by transportation modes (fast and slow). Johari, Hosseini-Motlagh, and Nematollahi, [2017] developed a lead time crashing contract using different modes of transportation to coordinate the pricing and replenishment decisions in a two-echelon supply chain. In this paper, we developed a lead time crashing contract to coordinate the CSR investment and replenishment decisions under a periodic review inventory system. In the proposed contract, the selection of transportation modes is considered through the carbon emissions tax.

Recently, the CSR investment has become an important issue to companies and this activity enhances the demand and profitability of the supply chain. Ni, Li, and Tang, [2010] studied a two-echelon supply chain, in which the upstream invests in the CSR and the downstream pays a fraction of the CSR cost as the wholesale price. Panda [2014] investigated the coordination of the CSR in a two-echelon supply chain with a revenue sharing contract. The manufacturer's and retailer's CSR activities are considered in this paper. Modak et al. [2014] presented a two-echelon dual-channel supply chain considering CSR investment and coordinated the supply chain by quantity discount contract. Panda et al., [2015] investigated a manufacturer-distributor-retailer supply chain, in which the manufacturer implements the CSR policies. They used revenue sharing contract to coordinate the supply chain. Hsueh, [2014] coordinated a two-echelon supply chain with a revenue sharing contract. In this study, the manufacturer invests in the CSR. Chen et al. [2016] investigated the CSR in the competitive supply chain that consisting of two competing downstream retailers and two upstream firms. Panda, Modak and Barron [2017] studied the effect of CSR and coordination in a closed-loop supply chain with one manufacturer and one retailer. Nematollahi, Hosseini-Motlagh, and Heydari, [2017] considered a two-echelon supply chain in which the supplier invested in the CSR to increase its sale. They proposed a novel model to coordinate the supply chain and to optimize the CSR investment level as well as order quantity. The CSR investment and periodic review replenishment decisions are not coordinated in any of the above studies.

In the recent decades, the government imposes some regulation on the supply chain members in order to control and reduce the carbon emissions. Bazan, Jaber, and Zanoni, [2015] coordinated a two-echelon supply chain with energy usage and greenhouse gas emissions of production and transportation. Tang et al. [2017] investigated the control of the emissions in inventory management a transportation with different carbon emissions reduction approaches. Dong et al. [2016] considered a two-echelon supply chain consisting of one manufacturer and one retailer under cap-and-trade regulation. The cap-and-trade regulation limits the carbon emissions of the manufacturer. Bai et al. (2017) coordinated a manufacturer-retailer sustainable supply chain under the carbon reduction policy. In this paper, we consider the carbon emissions tax in the coordination model. According to the carbon policy, the carbon emissions of the transportation is controlled.

In this paper, we investigate the CSR investment under a periodic review inventory system. Periodic review is one of the inventory control systems. Annadurai, and Uthayakumar, [2010] considered a periodic review inventory system with controllable lead time, in which the lost sale, review period and lead time are decision variables. Kouki and Jouini, [2015] developed a periodic review perishable inventory system. In this study, the order quantity, reorder level and review period are considered as decision variables. Mallidis et al. [2014] studied the impacts of joint optimization of network design and periodic review inventory decisions in the three-echelon supply chain. Nematollahi, Hosseini-Motlagh, and Heydari, [2017] proposed a two-echelon pharmaceutical supply chain in which the pharma-retailer uses a periodic review inventory system. They coordinated the proposed supply chain by collaboration model. Hojati, [2017] used a delay in payment contract to coordinate a supplier-retailer supply chain under the periodic review inventory system. The CSR investment is not considered in above studies.

According to the reviewed literature, simultaneous coordination of CSR investment and replenishment decisions under periodic review inventory system has not been studied yet. In this paper coordination of CSR investment and replenishment decisions under periodic review inventory system is investigated. Moreover, the lead time crashing contract through the transportation modes has been investigated less in the coordination literature. To the best of our knowledge, two papers used from the lead time crashing contract. The current study like Heydari, Zaabi-Ahmadi, and Choi, [2016] and Johari, Hosseini-Motlagh, and Nematollahi, [2017] coordinate the two-echelon supply chain with the lead time crashing contract through the transportation modes. However, both studies ignore the corporate social responsibility in their studies. In both papers, only the lead time reduction cost is considered. While using faster transportation mode increases fuel consumption and carbon emissions. In some countries, governments impose the carbon emissions tax for per unit carbon emissions in order to control the carbon emissions of the transportations. In this paper, the carbon policy is considered to control the carbon emissions. Therefore, there is a trade-off between the lead time reduction and its incurred costs for the supplier.

3. Problem Definition

In this paper, a two-echelon supply chain consisting of one supplier and one retailer is considered. The retailer invests in the CSR in order to increase demand. The CSR investment of the retailer affects not only on the profit of retailer, but also on the supplier's profit. The retailer faces a stochastic CSR-sensitive demand with a normal distribution. The retailer uses a periodic review inventory system in order to replenish inventories. The lead time is considered constant and deterministic. Moreover, the retailer may face shortage because of the stochastic market demand and it is supposed that the shortage will be partially lost. The order-up-to-level that is determined by the retailer influence the supplier's profitability. On the other hand, the supplier applies a lot-for-lot strategy for replenishing its inventory. The replenishment cycle multiplier (n) is the supplier's decision variable. The supplier replenishes his/her inventory every (nT) time units. Three decision-making structure are investigated; (1) decentralized model (2) centralized model (3) coordination model. Under the decentralized model, each member individually determines his/her decisions. In this model, the profitability of the whole supply chain is not maximized because the decisions of the retailer on the CSR and order-up-to-level influence in the supplier's profit. Under the centralized model, all supply chain decisions (i.e. order-up-to-level, CSR, replenishment cycle) are made from the whole supply chain viewpoint. Under such a case, the retailer may lose some profit. In this paper, we aim to develop an incentive mechanism that can be able to coordinate the members of the supply chain. For this purpose, a lead time crashing contract is used to convince the supply chain members to participate in the joint-decision making structure. Under the proposed contract, the supplier can reduce the lead time through different transportation modes (fast and slow). Under the coordination model, the government imposes the carbon tax on additional used fuel for reducing the lead time. Thus, under the proposed contract, the lead time reduction cost and carbon emissions tax are inflicted on the supplier. Under the carbon emissions tax, we investigate which of the transportation modes (fast and slow) are better to choose from profitability viewpoint. Figure 1 shows the supply chain structure.

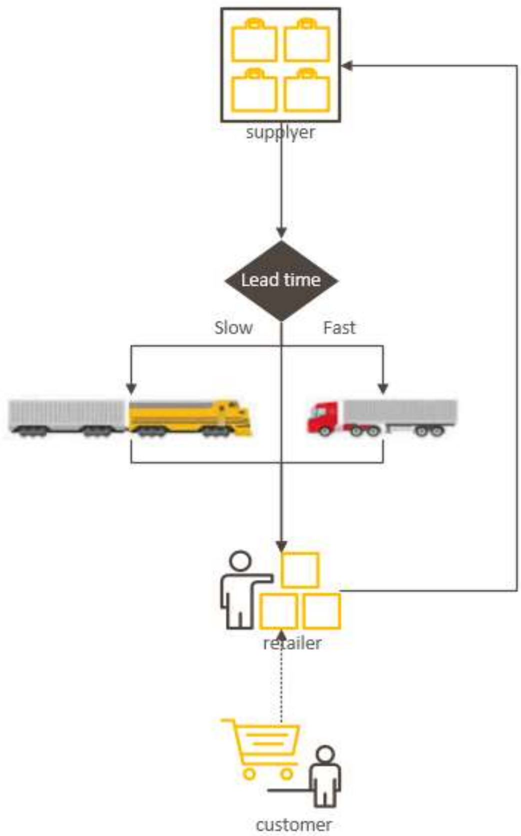

Figure 1. The investigate supply chain model

3.1 Notations

The decision variables and parameters used in this study are presented in Table 1.

Table 1. The decision variables and parameters used in this study

Parameters and Variables	
For the retailer	
x	Cost of investment in CSR per unit of product (Decision variable)
R	Order-up-to-level (Decision variable)
k	Safety factor of the retailer
$D(x)$	Expected demand rate per year at CSR investment x
a	Initial market demand
b	Coefficient of the retailer's CSR investment on increasing the demand
P	The retail price per item
T	Length of a review period
L	Length of the lead time
X^{+}	Maximum value of x and 0, that is X+= max {x,0}
X	Protection interval demand, which follows a normal distribution with finite
σ	Standard deviation of the demand per unit time
A_r	Retailer's fixed ordering cost per order
h_r	Retailer's inventory holding cost per item
β	Fraction of the demand during the stock-out period that will be lost, 0<β <1
π	Shortage cost per item short
For the supplier	
n	Supplier's replenishment cycle multiplier (Decision variable)
w	Wholesale price per item
e	Supplier's purchase cost per item
A_s	Supplier's fixed ordering cost per order
h_s	Supplier's inventory holding cost per item
In coordination model	
θ	Amount of CO2 emissions from fuel per gallon consumed (ton/gallon)
C_t	Carbon emissions tax
C_s	Cost of each percent of lead time reduction in slow transportation mode
C_f	Cost of each percent of lead time reduction in fast transportation mode
g_s	fuel volume required per trip (gallons) in slow mode
g_f	fuel volume required per trip (gallons) in fast mode
A	Point at which more lead time reduction requires shifting to the fast
F	Maximum possible lead time crashing
LR	Percentage of lead time crashing
α	Bargaining power of the supplier
Note: the superscripts * and ** in each variable mark the decentralized and centralized models, respectively.	

3.2 Assumption

- The market demand is assumed stochastic and follows a normal distribution $((a+bx), \sigma^2)$ where the mean of demand is a linear function of CSR investment. In the literature, such demand is considered (e.g. Hsueh (2014); Nematollahi et al. (2017)).
- The inventory level is reviewed every T units of time. A sufficient quantity is ordered up to the level R, and the ordering quantity is got after L units of time.
- The length of the lead time L is less than the length of review period T. Therefore, there is never more than a single order outstanding in each cycle.
- The order-up-to-level is calculated as $R = D(T+L) + k\sigma\sqrt{T+L}$, in which $D(T+L)$ denotes expected demand during protection interval $(T+L)$, and $k\sigma\sqrt{T+L}$ is the safety stock.
- The retailer faces with partially backordered shortages.
- The supplier can reduce the lead time through transportation modes by spending more cost. The lead time reduction cost and carbon emissions tax are imposed on the supplier under the proposed contract. According to

Heydari, Zaabi-Ahmadi, and Choi, [2016], transportation cost is considered as C_sLR and according to Bazan, Jaber, and Zanoni, [2015], carbon emissions tax is considered as $(g_s\theta)C_t$.

4. Mathematical Model

In this section, we investigate three different decision-making models: (1) Decentralized decision-making model, (2) Centralized decision-making model, (3) Coordinated model.

4.1 Decentralized Decision-Making Model

Under the decentralized model, each member attempts to optimize its own profit regardless of the other supply chain members. In the proposed supply chain, the retailer decides on the CSR investment (x) and order-up-to-level (R). The supplier decides on replenishment cycle multiplier (n). In the following, we investigate the decentralized model for the retailer and supplier.

4.1.1 Retailer Profit Function

The retailer uses a periodic review inventory system in order to replenish its stock. The length of the review period has been supposed to be fixed. The retailer replenishes its stock at a specified time. Under the periodic review inventory system, similar to Montgomery et al., (1973) the holding cost per year is calculated as $h_r\left[R - DL - \frac{DT}{2} + \beta E(X-R)^+\right]$ and the expected stock-out cost is calculated as $\frac{\pi+\beta(P-w)}{T}E(X-R)^+$. Let $\pi_r(R,x)$ shows the retailer's annual profit function. Hence, the profit function of the retailer is as follows:

$$\begin{aligned}\pi_r(R,x) &\\ &= (P-w)(a+bx) - \frac{A_r}{T} \\ &- h_r\left[R - (a+bx)L - \frac{(a+bx)T}{2} \right.\\ &\left. + \beta E(X-R)^+\right] \\ &- \frac{\pi+\beta(P-w-x)}{T}E(X-R)^+ \\ &- x(a+bx)\end{aligned} \tag{1}$$

In which the first term calculates the retailer's annual revenue. The second term is ordering cost. The third and fourth terms denote holding cost and shortage cost, respectively. The last term is the CSR cost. The order-up-to-level is calculated as $R = D(T+L) + k\sigma\sqrt{T+L}$ where the order-up-to-level is a function of k (safety factor). The expected shortage quantity $E(X-R)^+$ can be defined as:

$$\begin{aligned}E(X-R)^+ &= \int_R^\infty (X-R)f_x dx \\ &= \sigma\sqrt{T+L}\,\psi(k) > 0\end{aligned} \tag{2}$$

In which $\psi(k) = \varphi(k) - k[1-\Phi(k)]$, $\varphi(k)$ and $\Phi(k)$ indicate the standard normal and cumulative distribution function, respectively. For the simplicity, we consider the safety factor k as a decision variable instead of the order-up-to-level R. The retailer's profit function can be transferred to:

$$\begin{aligned}\pi_r(k,x) &\\ &= (P-w-x)(a+bx) - \frac{A_r}{T} \\ &- h_r\left[\frac{(a+bx)T}{2} + k\sigma\sqrt{T+L} \right.\\ &\left. + \beta\sigma\sqrt{T+L}\psi(k)\right] \\ &- \frac{\pi+\beta(P-w-x)}{T}\sigma\sqrt{T+L}\psi(k)\end{aligned} \tag{3}$$

Proposition 1. *The retailer's profit function is concave with respect to x and k, and the optimal value of these decision variables are computed as below:*

$$1-\Phi(k^*) = \frac{h_r}{h_r\beta + \frac{1}{T}(\pi + \beta(P-w-x))} \quad (4)$$

$$x^* = \frac{p-w}{2} - \frac{a}{2b} - \frac{Th_r}{4} + \frac{\beta}{2bT}\sigma\sqrt{T+L}\psi(k) \quad (5)$$

Proof. See "Appendix A".

To finding the optimal values of CSR investment and safety factor, a solution procedure is provided as follows:

Locally optimal algorithm

Step 1: Set the lowest possible value to x.

Step 2: Calculate k from Eq. (4).

Step 3: Calculate x from Eq. (5) based on obtained k from Step 2.

Step 4: If two successive values of x, k are equal, go to step 5; Otherwise, go step 2.

Step 5: Calculate the retailer's profit from Eq. (3) using obtained optimal values of x and k.

4.1.2 Supplier Profit Function

Under the decentralized model, the supplier uses a lot-for-lot policy and the supplier's replenishment period is nT. According to Rosenblatt and Lee [1985], the supplier's order multiplier n must be a positive integer to optimize replenishment policy. Moreover, the total demand received by the supplier is equal the demand received by the retailer minus lost sales. Let $\pi_s(n)$ shows the supplier's annual profit function. Therefore, the supplier's profit function is calculated as:

$$\pi_s(n) = (w-e)\left[(a+bx) - \frac{\beta}{T}\sigma\sqrt{T+L}\psi(k)\right] - \frac{A_s}{nT} - h_s\left[\frac{(n-1)\left((a+bx)T - \beta\sigma\sqrt{T+L}\psi(k)\right)}{2}\right] \quad (6)$$

In which the first term is the supplier's annual revenue. The second term presents ordering cost. The third term denotes holding cost.

Proposition 2. *The supplier's profit function is concave with respect to n, and the optimal value of this variable is calculated as follows:*

$$n^* = \sqrt{\frac{2A_s}{h_sT\left((a+bx)T - \beta\sigma\sqrt{T+L}\psi(k)\right)}} \quad (7)$$

Proof. See "Appendix B".

Since n should be an integer, therefore either the smallest following integer or the largest previous integer of n whichever results in the larger value of $\pi_s(n)$ will be the optimum value of n.

4.2 Centralized Decision-Making Model

Under the centralized model, it is supposed that a decision maker aims to optimize the whole supply chain profit. In this case, the CSR investment decisions and replenishment policies are determined from the whole SC perspective. Let $\pi_{sc}(k,x,n)$ be the annual profit function of the supply chain that is the sum of the retailer and the supplier profit function. The supply chain profit function is calculated as:

$$\pi_{sc}(k,x,n) = \pi_r(k,x) + \pi_s(n) = (P-x-e)(a+bx) - \frac{1}{T}\left[A_r + \frac{A_s}{n}\right] - \frac{\sigma\sqrt{T+L}\psi(k)}{T}\left[\left(\pi + \beta(P-x-e)\right) - \frac{h_s(n-1)\beta T}{2}\right] - h_r\left[k\sigma\sqrt{T+L} + \beta\sigma\sqrt{T+L}\psi(k)\right] - \frac{(a+bx)T}{2}\left(h_r + h_s(n-1)\right) \quad (8)$$

Proposition 3. *The supply chain profit function is concave with respect to k, x, n and the optimal values of these decision variables are calculated as follows:*

$$1-\Phi(k^{**}) = h_r / [h_r\beta + \frac{1}{T}\left[\left(\pi + \beta(P-x-e)\right) - \frac{h_s(n-1)\beta T}{2}\right]] \quad (9)$$

$$x^{**} = \frac{\beta\sigma\sqrt{T+L}\psi(k)}{2bT} + \frac{P-e}{2} - \frac{T}{4}\left(h_r + h_s(n-1)\right) - \frac{a}{2b} \quad (10)$$

$$n^{**} = \sqrt{\frac{2A_s}{h_s T\left[\left((a+bx)T\right) - \beta\sigma\sqrt{T+L}\psi(k)\right]}} \quad (11)$$

Proof. See "Appendix C".

To finding the optimal values of CSR investment, safety factor and replenishment cycle multiplier, a solution procedure is provided as follows:

Globally optimal algorithm

Step 1: Set the lowest possible value to x.

Step 2: Set *n=1.* (lowest possible value)

Step 3: Calculate k from Eq. (9).

Step 4: Calculate x from Eq. (10) based on obtained k.

Step 5: Calculate n from Eq. (11) based on obtained k and x.

Step 6: If two successive values of x, k, n are equal, go to step 7. Otherwise, go to step 3.

Step 7: Calculate the supply chain profit from Eq. (8) using obtained optimal values of $x, k,$ and n.

4.3 Coordination Model

Under the centralized model, the whole supply chain profit increases in comparison to the decentralized model. While the retailer incurs losses in the centralized model. Hence, the retailer does not accept the centralized model. So, the supplier proposes a lead time crashing contract as an incentive mechanism to motivate the retailer to shift its decisions from local decisions to global decisions. Under his contract, the supplier can reduce the lead time using two transportation modes (fast and slow) by spending more cost. Therefore, the lead time crashing is profitable to the retailer but it is costly to the supplier. The lead time reduction cost and carbon emissions tax are inflicted on the supplier. This indicates that there has to be a trade-off between lead time reduction and its incurred costs. Under the proposed contract, LR is used as a factor which is considered between 0 and 1. Under this contract, the lead time is reduced from L to L_{new} , which is calculated as follows:

$$L_{new} = (1-LR)L \quad (12)$$

In this equation, if LR takes the value near one, it means that the supplier reduces more lead time to enhance the profit of the retailer. If this value becomes close to zero, it means that the supplier reduces the lead time slightly. Under the coordination model, the retailer's expected profit function is as follows:

$$\pi_r(k^{**}, x^{**}, L_{new}) = (P - w - x^{**})(a + bx^{**}) - \frac{A_r}{T} - h_r \left[\frac{(a + bx^{**})T}{2} + k^{**}\sigma\sqrt{T + L_{new}} + \beta\sigma\sqrt{T + L_{new}}\psi(k^{**})\right] - \frac{\pi + \beta(P - w - x^{**})}{T}\sigma\sqrt{T + L_{new}}\psi(k^{**}) \tag{13}$$

The LR must be acceptable for both retailer and supplier to participate in the coordination model. In other words, the retailer participates in the coordination model if his/her profit is more than the decentralized model. Hence, the following condition must be satisfied to convince the retailer to participate in the coordination model:

$$\pi_r(k^{**}, x^{**}, L_{new}) > \pi_r(k^*, x^*) \tag{14}$$

The minimum value of LR to convince the retailer to participate in the joint decision making can be calculated as follows:

$$LR_{min} = 1 - (\Big[(x^{**}\big((P - w - x^{**})b - a\big) - x\big((P - w - x)b - a\big)/\sigma) - \frac{Th_r\, b}{2\sigma}(x^{**} - x) + \sqrt{T + L}\left(h_r\, k + h_r\, \beta\psi(k) + \frac{\pi + \beta(P - w - x)}{T}\psi(k)\right) / \Big[h_r\, k^{**} + h_r\, \beta\sigma\psi(k^{**}) + \frac{\pi + \beta(P - w - x^{**})}{T}\psi(k^{**})\Big]\Big]^2 - T)^{\frac{1}{L}} \tag{15}$$

The supplier can reduce the lead time using a faster transportation mode. By choosing a faster transportation mode, the fuel consumption and carbon emissions increase. Thus, in order to limit the carbon emissions, the government imposes a carbon tax on the supplier. Under this carbon policy, the supplier has to pay tax for per additional fuel consumption. Thus, the lead time crashing cost and carbon emissions tax are imposed on the supplier. Under the coordination model, the supplier's expected profit function is as follows:

$$\pi_s(n^{**}, L_{new}) = (w - e)\left[(a + bx) - \frac{\beta}{T}\sigma\sqrt{T + L_{new}}\psi(k)\right] - \frac{A_s}{n^{**}T} - h_s\left[\frac{(n^{**} - 1)\Big((a + bx)T - \beta\sigma\sqrt{T + L_{new}}\psi(k)\Big)}{2}\right] - \frac{LRC}{T} \tag{16}$$

The last term in Eq. (16) is costs of carbon tax and lead time crashing that are imposed to the supplier. The supplier can reduce the lead time through two transportation modes (fast and slow). Therefore, the lead time reduction cost function (LRC) is a two-criterion function. When the slow mode is selected, the costs are calculated from the first criterion in which the first term is lead time reduction cost and the second term is carbon emissions tax. When the fast mode is adopted, the costs are determined from the second criterion. The fuel consumption in each transportation mode is different as well as the cost of reducing lead time. Accordingly, the whole lead time reduction cost for the supplier is calculated as follows:

$$LRC = \begin{cases} C_s LR + (g_s\theta)LRC_t & 0 < LR \le A \\ C_f LR + (g_f\theta)LRC_t & A < LR \le M \end{cases} \tag{17}$$

LR should take value in the interval $(0, M)$ and M is lower than one. If LR is in the interval $(0, A)$, the shipment is through slow transportation mode. If LR is in the interval (A, M), the supplier uses the fast transportation mode. The supplier participates in the coordination model if his/her profit is more than the decentralized model. Hence, the following condition must be satisfied to convince the supplier to participate in the coordination model:

$$\pi_s(k^{**}, x^{**}, n^{**}, L_{new}) > \pi_s(k^*, x^*, n^*) \tag{18}$$

A closed-form equation for the maximum value of LR which is appropriate for the supplier to implement the coordination mechanism cannot be calculated. Therefore, we use an algorithm in order to calculate LR_{max} from the supplier's perspective.

Calculating LR_{max} algorithm

Step1: Set $LR = M$.

Step2: Calculate the supplier's profit using Eq. (16).

Step3: Calculate $\pi_s(k^{**}, x^{**}, n^{**}, L_{new}) - \pi_s(k^{*}, x^{*}, n^{*})$.

Step4: If the obtained value in Step 3 is greater than zero, LR_{max} is equal to LR; otherwise let $LR = LR - \varepsilon$, where ε is a small positive quantity, and go to step 2.

The coordination mechanism can be implemented with any LR in the interval $[LR_{min}, LR_{max}]$. If $LR = LR_{max}$, the retailer gains the whole extra profit of the coordination model. If $LR = LR_{min}$ the supplier gains the whole extra profit of the coordination model. In this paper, we determine the proper LR based on the members' bargaining power. The bargaining power of the supplier is considered α, so the bargaining power of the retailer would be $(1 - \alpha)$. Therefore, LR_{co} can be calculated as follows:

$$LR_{co} = \alpha . LR_{min} + (1 - \alpha) . LR_{max} \tag{19}$$

5. Numerical Experiments

In this section, we consider the three test problems to investigate the performance of the proposed models. Table 2 indicates data for the three test problems. According to Bazan et al., (2015), in all three examples, carbon emissions from fuel per gallon consumed is considered $\theta = 0.01008414\ tons/gallon$.

The results of decentralized, centralized and coordination model to the three test problems are denoted in Table 3. As shown in Table 3, in the centralized model, service level and CSR investment improve and by increasing the CSR investment, the market demand grows. The centralized model in all three test problems optimizes the whole supply chain profit and the supplier's profit while the centralized model reduces the retailer's profit in comparison to the decentralized model. Therefore, the retailer does not participate in the centralized model. The proposed lead time crashing contract not only improves the profitability of the whole supply chain but also improves the profitability of both supplier and retailer in comparison to the decentralized model. Therefore, the members' participation are guaranteed by the lead time crashing contract. In examples 1 and 3, the lead time crashing can coordinate the supply chain. Under the coordination model, the profitability of supply chain is more than the centralized model. In fact, the supplier's expenses to reduce the lead time are less than the extra profit gained from the coordination model. In example 2, as it is shown in Table 3, the profitability of supply chain under the coordination model is less than the centralized model while it is more than the decentralized model. Thus, the lead time crashing cost is less than the extra profit gained from the coordination model and it can coordinate the supply chain.

Table 2. The three investigated test problems

Parameter	Test Problem 1	Test Problem 2	Test Problem 3
P	110	150	130
w	80	93	74
e	75	80	58
a	1000	2000	4000
b	90	55	110
T (Day)	50	20	40

A_r	150	250	500
A_s	350	300	450
σ	300	700	1200
h_r	24	35	45
h_s	12	20	30
π	3	1	5
β	0.9	0.6	0.8
L (Day)	35	15	30
A	0.7	0.4	0.5
M	0.9	0.85	0.9
C_s	110	180	150
C_f	140	220	180
g_s	15	20	30
g_f	95	115	170
c_t	50	60	70
α	0.6	0.4	0.5

Table 3. The results from running decentralized, centralized and coordinated model

	Test Problem 1	**Test Problem 2**	**Test Problem 3**
Decentralized structure			
k^*	1.1163	1.5241	1.2435
x^*	8.9735	10.4371	9.4793
n^*	1	2	1
π_r	28315.6926	97960.0207	176837.1957
π_s	6167.1876	28494.9652	73430.3722
π_{sc}	34482.8802	126454.9859	250267.5679
$D(x)$	1807.6171	2574.0424	5042.7280
Centralized structure			
k^{**}	1.1698	1.5837	1.3186
x^{**}	11.4377	16.5857	17.3481
n^{**}	1	2	1
π_r	27745.4155	95802.7049	169687.6981
π_s	7308.2838	32811.9464	87741.1573
π_{sc}	35053.6993	128614.6514	257428.8555
$D(x)$	2029.4011	2912.2167	5908.2922
Coordinated structure			
k	1.1698	1.5837	1.3186
x	11.4377	16.5857	17.3481
n	1	2	1
π_r	28517.7182	98404.4468	177530.5552
π_s	6803.3638	28920.7684	85991.4381
π_{sc}	35321.0821	127325.2153	263521.9933
LR_{min}	47.80%	64.11%	76.73%
LR_{max}	87%	84%	90%
LR_{co}	63.48%	76.04%	83.36%
Transportation mode	Slow	Fast	Fast

In order to evaluate the applicability of the proposed models, some sensitivity analyses are provided. The data for sensitivity analyses are taken from test problem 1. Figure 2 shows the changes in the supply chain profit by increasing b. According to Figure 2, by increasing b, the supply chain profit improves under the decentralized, centralized and coordinated models. In the coordination model, the supply chain profit is more than the decentralized model under all values of b, which indicates the applicability of the proposed lead time crashing contract. As shown in Figure 2, under high values of b, the supply chain profit in the coordination model is less than the centralized model but it is still more than the decentralized model. Therefore, the lead time crashing can coordinate the supply chain in all values of b. It shows the capability of the coordination model. As a result, by increasing the consumers' social awareness, the whole supply chain profitability improves.

Figure 3 demonstrates the changes in the supplier's profit under the decentralized, centralized and coordination models by increasing b. According to figure 3, by b increases, the supplier's profit increases in all three models. Also under various values of b, the supplier's profit in the centralized model is more than two other models. But the retailer does not accept the centralized model. Proposing the lead time crashing contract reduces the supplier's profit in comparison to the centralized model, but his/her profit in the coordination model is more than the decentralized model. Therefore, participation in the coordination model is profitable for the supplier. On the other hand, Figure 3 indicates that the consumer's social awareness and the retailer's CSR investment impact on the supplier's profitability.

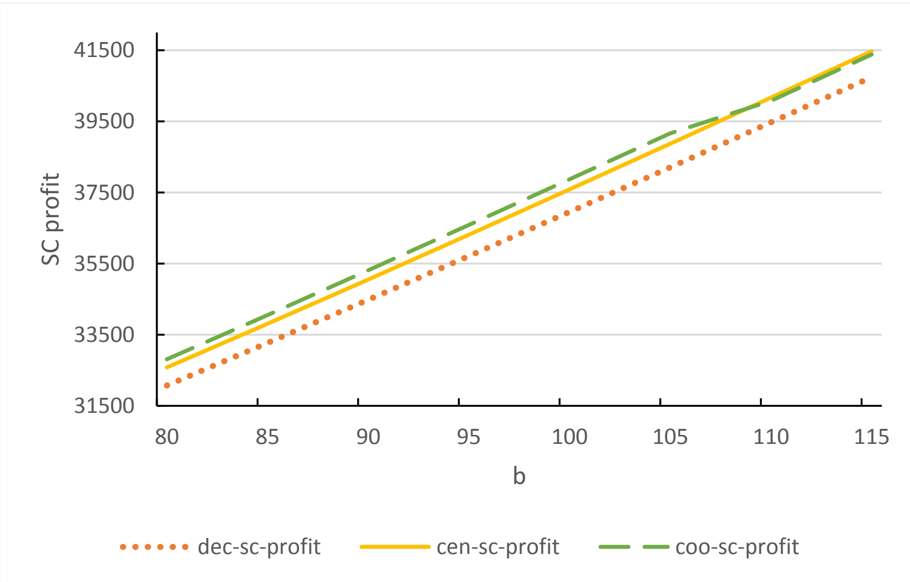

Figure 2. Changes in the supply chain profit by increasing *b*

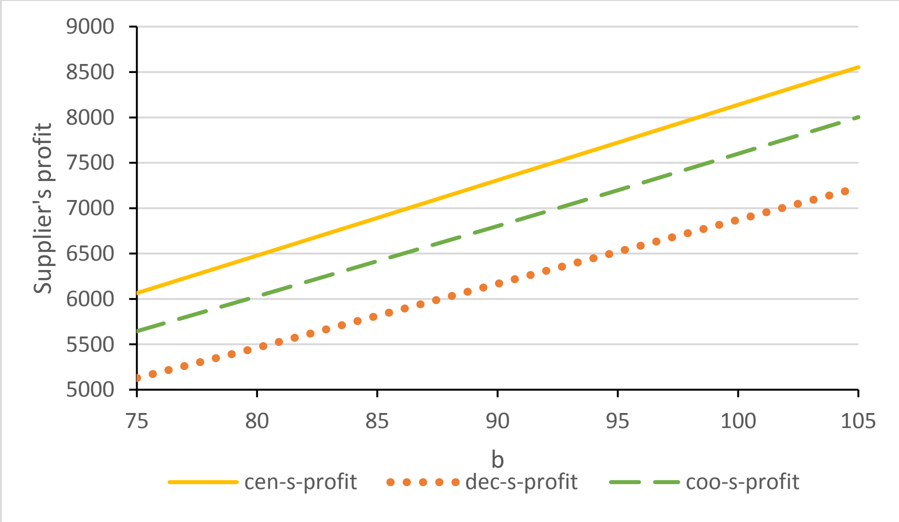

Figure 3. Changes in the Supplier's profit by increasing *b*

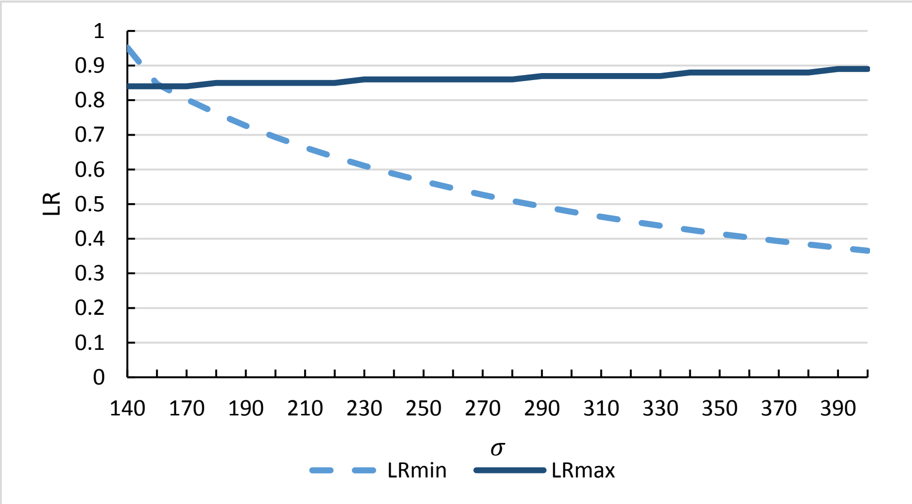

Figure 4. Values of LR_{max} and LR_{min} by increasing the demand uncertainty

The effect of demand uncertainty on LR_{min} and LR_{max} is shown in Figure 4. As indicated in figure 4, before the intersection point between LR_{min} and LR_{max}, the interval $[LR_{min}, LR_{max}]$ is empty and the lead time crashing contract cannot coordinate the supply chain. In other words, the lead time crashing cost is more than the extra profit gained from the coordination model. After the intersection point, the interval $[LR_{min}, LR_{max}]$ is not empty and the contract is capable to coordinate the supply chain. In this interval, the extra profit of the coordinated model is more than the lead time reduction cost. As shown in figure 4, the lead time crashing contract is appropriate and has capability to coordinate the supply chain members under the high uncertainty of demand.

Values of LR_{max} and LR_{min} by increasing the lost sale is illustrated in figure 5. As shown in figure 5 before the intersection point between LR_{max} and LR_{min} , coordination cannot be implemented. But by increasing β, after the intersection point, LR_{max} is greater than LR_{min} and coordination model is applicable. As β increases, LR_{min} which is determined by the retailer decreases and the interval between LR_{min} and LR_{max} becomes larger. Therefore, in high values of β, the retailer prefers to participate in the coordination model even with the lower reduction.

Figure 6 shows the effects of carbon emissions tax on LR_{min} and LR_{max}. As indicated in figure 6, as the carbon emissions tax increases, the interval between LR_{min} and LR_{max} becomes close and LR_{max} takes smaller values by increasing the carbon emissions tax. By increasing C_t, the costs of lead time crashing increase for the supplier. Therefore, the government can increase the limitation of reducing the lead time by increasing the carbon emissions tax.

Values of LR_{max} and LR_{min} by changing C_f is denoted in figure 7. According to figure 7, as C_f increases LR_{max} reduces. In the high value of the C_f, the interval between LR_{max} and LR_{min} becomes close. In the high value of C_f, using fast mode is not economical for the supplier and lead time reduction cost is more than the supplier's benefit. Therefore, the supplier shifts from the fast transportation mode to the slow transportation mode and LR_{max} is calculated by the slow mode costs.

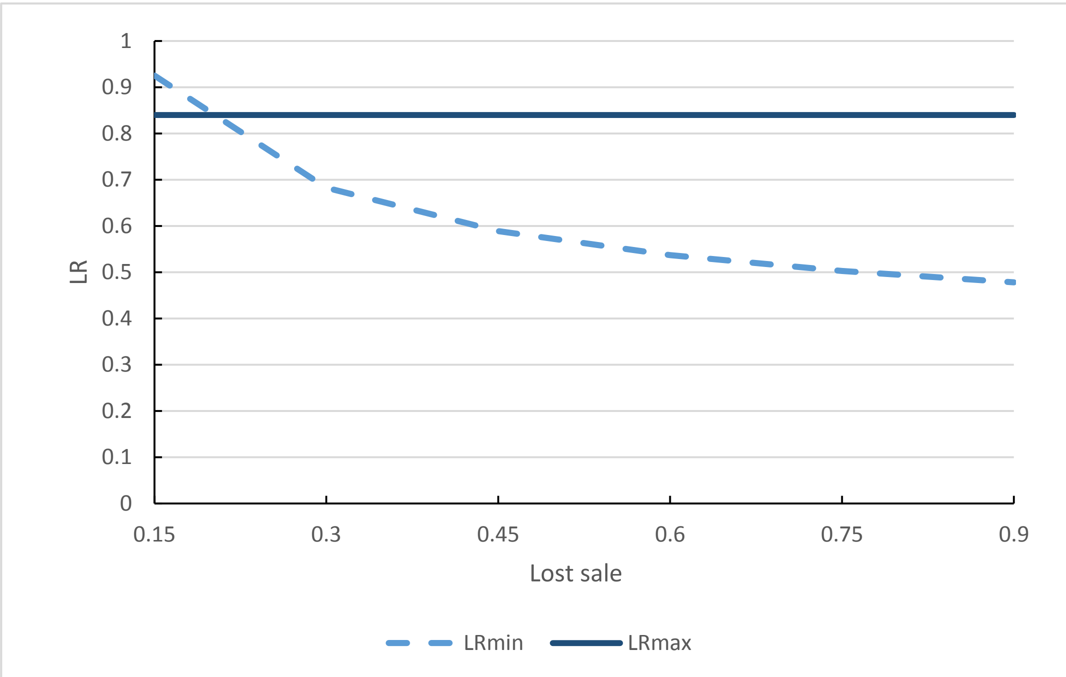

Figure 5. Values of LR_{max} and LR_{min} by increasing the lost sale

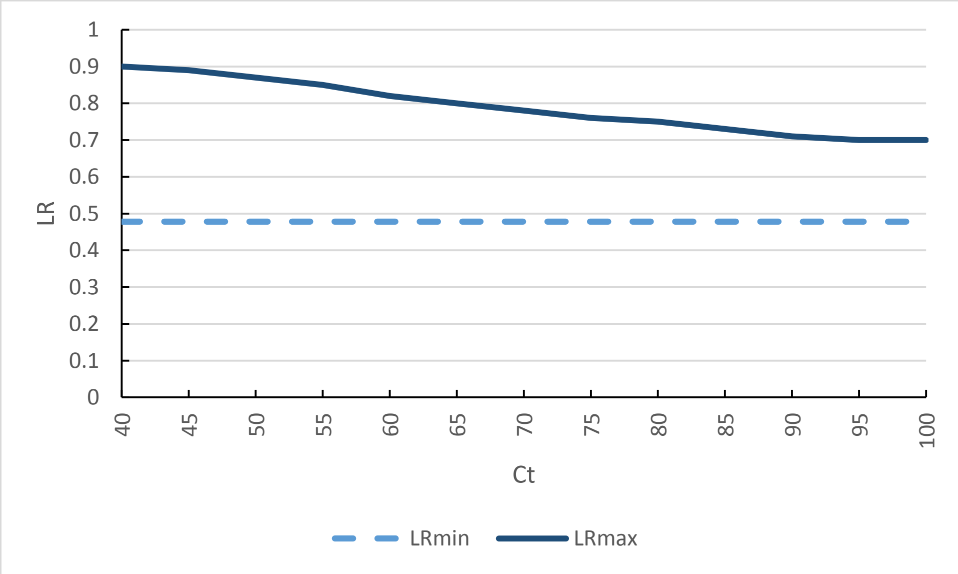

Figure 6. Values of LR_{max} and LR_{min} by increasing the carbon emissions tax

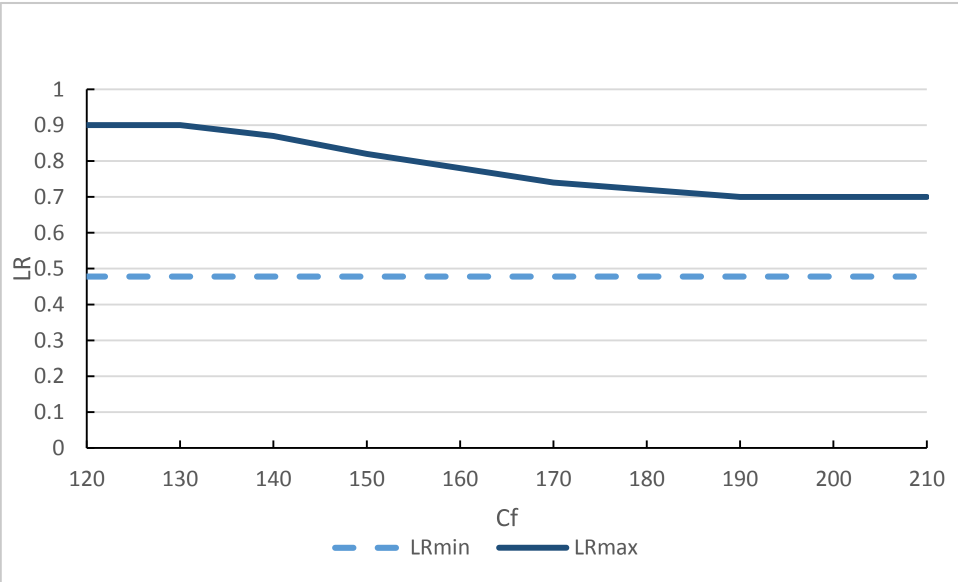

Figure 7. Values of LR_{max} and LR_{min} by increasing C_f

6. Conclusion

In this paper, we coordinated a two-echelon socially responsible supply chain with a lead time crashing contract. In the proposed supply chain, the retailer decided on the CSR investment and replenishment inventory decisions under a periodic review inventory system. The retailer faced with a stochastic CSR-sensitive demand. The retailer's CSR investment impacted on the demand. The supplier decided on the replenishment cycle multiplier under a lot-for-lot inventory system. The decentralized and centralized and coordination models were investigated. The centralized model was profitable from the supply chain viewpoint in comparison to the decentralized model. While the retailer incurred losses in the centralized model. Therefore, the supplier proposed a lead time crashing contract to motivate the retailer to participate in the coordination model. In order to reduce the lead time, two transportation modes (fast and slow) were considered in this model. The lead time reduction could reduce the retailer's safety stock and shortage, thus it reduced the retailer's costs. On the other hand, the lead time reduction cost and carbon emissions tax were inflicted on the supplier. This illustrates that there has to be a trade-off between lead time reduction and its incurred costs. Numerical examples denoted the applicability of the developed coordination model. The main managerial insights are as follow:

- This investigation can assist managers to improve the supply chain efficiency in both economic and social viewpoints. The results show that the lead time crashing contract not only enhances the profitability but also improves the social responsibility efforts. Thus, the lead time crashing contract is able to coordinate the supply chain members. Moreover, the surplus profit can be shared between the supplier and retailer based on their bargaining power.
- In the recent decades, by increasing the customers' awareness about social issues, retailer's investment in the CSR will be beneficial. In the proposed supply chain, the retailer's decisions on the CSR investment and order-up-to-level and the supplier's decision on replenishment cycle impact on the profitability and social issues of the entire supply chain. Current paper can help the members of the supply chain to achieve a better understanding of these issues.
- It is observed that even in high demand uncertainty, the lead time crashing contract is able to coordinate the proposed supply chain. This shows the efficiency and capability of the coordination contract.
- Under the coordination model, the supplier reduces the lead time by choosing a faster transportation mode in order to more profitability. While a faster transportation mode uses more fuel, consequently increases carbon emissions. The government imposes a carbon policy to control carbon emissions. According to the observations, this policy restricts the supplier's reducing lead time and decreases carbon footprint. Moreover, by increasing the carbon tax, the lead time reduction is limited more.

To extend the proposed model, two retailers can be considered that compete on the CSR investment and coordinate the supply chain by lead time crashing. Moreover, loading or unloading lead time, production lead time and set up lead time can be considered as the lead time elements.

7. References

-Annadurai, K. and Uthayakumar, R. (2010) "Reducing lost-sales rate in (T, R, L) inventory model with controllable lead time", Applied Mathematical Modelling, Vol. 34, No. 11, pp. 3465-3477.

-Bai, Q., Chen, M. and Xu, L. (2017) "Revenue and promotional cost-sharing contract versus two-part tariff contract in coordinating sustainable supply chain systems with deteriorating items" International Journal of Production Economics, Vol. 187, pp. 85-101.

-Bazan, E., Jaber, M. Y., & Zanoni, S. (2015) "Supply chain models with greenhouse gases emissions, energy usage and different coordination decisions", Applied Mathematical Modelling, Vol. 39, No. 17, pp. 5131-5151.

-Carling, K., Håkansson, J., Meng, X. and Rudholm, N. (2017) "The effect on CO 2 emissions of taxing truck distance in retail transports", Transportation Research Part A: Policy and Practice, Vol. 97, pp.47-54.

-Chen, C. L., Liu, Q., Li, J. and Wang, L. F. (2016) "Corporate social responsibility and downstream price competition with retailer's effort, International Review of Economics & Finance, Vol. 46, pp. 36-54.

-Corbett, J. J., Wang, H. and Winebrake, J. J. (2009) "The effectiveness and costs of speed reductions on emissions from international shipping", Transportation Research Part D: Transport and Environment, Vol. 14, No. 8, pp. 593-598.

-Dong, C., Shen, B., Chow, P. S., Yang, L. and Ng, C. T. (2016) "Sustainability investment under cap-and-trade regulation. Annals of Operations Research", Vol. 240, No. 2, pp. 509-531.

-Kouki, Ch. and Jouini, O. (2015) "On the effect of life time variability on the performance of inventory systems", International Journal of Production Economics, Vol. 167, pp. 23–34.

-Heydari, J. (2014) "Lead time variation control using reliable shipment equipment: An incentive scheme for supply chain coordination", Transportation Research Part E: Logistics and Transportation Review, Vol. 63, pp. 44-58.

-Heydari, J., Zaabi-Ahmadi, P. and Choi, T. M. (2016) "Coordinating supply chains with stochastic demand by crashing lead times", Computers & Operations Research, Available on line October 2016.

-Hoen, K. M. R., Tan, T., Fransoo, J. C. and Van Houtum, G. J. (2014) "Effect of carbon emission regulations on transport mode selection under stochastic demand", Flexible Services and Manufacturing Journal, Vol. 26, No. 1-2, pp. 170-195.

-Hojati, S., Seyedhosseini, S. M., Hosseini-Motlagh, S. M. and Nematollahi, M. (2017) "Coordination and profit sharing in a two-level supply chain under periodic review inventory policy with delay in payments contract", Journal of Industrial and Systems Engineering, Vol. 10, pp.109-131.

-Hsueh, C. F. (2014) "Improving corporate social responsibility in a supply chain through a new revenue sharing contract", International Journal of Production Economics, Vol. 151, pp. 214-222.

-Jaber, M. Y. and Osman, I. H. (2006) "Coordinating a two-level supply chain with delay in payments and profit sharing", Computers & Industrial Engineering, Vol. 50, No. 4, pp. 385-400.

-Johari, M., Hosseini-Motlagh, S. M. and Nematollahi, M. (2017a) "Simultaneous coordination of review period and order-up-to-level in a manufacturer-retailer chain", Journal of Industrial and Systems Engineering, Vol. 10, pp.1-17.

-Johari, M., Hosseini-Motlagh, S. M. and Nematollahi, M. (2017b) "Supply chain coordination using different modes of transportation considering stochastic price-dependent demand and periodic review replenishment policy", International Journal of Transportation Engineering, Vol. 5, No. 2, pp. 137-165.

-Leng, M. and Parlar, M. (2009) "Lead-time reduction in a two-level supply chain: Non-cooperative equilibria vs. coordination with a profit-sharing contract", International Journal of Production Economics, Vol. 118, No. 2, pp. 521-544.

-Li, J. and Liu, L. (2006) Supply chain coordination with quantity discount policy. International Journal of Production Economics, Vol.101, No. 1, pp. 89-98.

-Li, L., Wang, Y. and Dai, W. (2016) "Coordinating supplier retailer and carrier with price discount policy", Applied Mathematical Modelling, Vol. 40, No. 1, pp. 646-657.

-Mallidis, I., Vlachos, D., Iakovou, E. and Dekker, R. (2014) "Design and planning for green global supply chains under periodic review replenishment policies", Transportation Research Part E: Logistics and Transportation Review, Vol. 72, pp. 210-235.

-Modak, N. M., Panda, S., Sana, S. S. and Basu, M. (2014) "Corporate social responsibility, coordination and profit distribution in a dual-channel supply chain", Pacific Science Review, Vol. 16, No. 4, pp. 235-249.

-Montgomery, D. C., Bazaraa, M. S. and Keswani, A. K. (1973) "Inventory models with a mixture of backorders and lost sales", Naval Research Logistics Quarterly, Vol. 20, No. 2, pp.255-263.

-Nematollahi, M., Hosseini-Motlagh, S. M. and Heydari, J. (2017) "Coordination of social responsibility and order quantity in a two-echelon supply chain: A collaborative decision-making perspective", International Journal of Production Economics, Vol. 184, pp. 107-121.

-Nematollahi, M., Hosseini-Motlagh, S. M. and Heydari, J. (2017) "Economic and social collaborative decision-making on visit interval and service level in a two-echelon pharmaceutical supply chain", Journal of Cleaner Production, Vol. 142, pp.3956-3969.

-Ni, D., Li, K. W. and Tang, X. (2010) "Social responsibility allocation in two-echelon supply chains: Insights from wholesale price contracts", European Journal of Operational Research, Vol. 207, No. 3, pp. 1269-1279.

-Nouri, M., Hosseini-Motlagh, S.-M., Nematollahi, M. and Sarker, B. R. (2018) "Coordinating manufacturer's innovation and retailer's promotion and replenishment using a compensation-based wholesale price contract", International Journal of Production Economics, Vol. 198, pp.11–24.

-Panda, S. (2014) "Coordination of a socially responsible supply chain using revenue sharing contract", Transportation Research Part E: Logistics and Transportation Review, Vol. 67, pp. 92-104.

-Panda, S., Modak, N. M. and Cárdenas-Barrón, L. E. (2017) "Coordinating a socially responsible closed-loop supply chain with product recycling", International Journal of Production Economics, Vol. 188, pp.11-21.

-Panda, S., Modak, N. M., Basu, M. and Goyal, S. K. (2015) "Channel coordination and profit distribution in a social responsible three-layer supply chain", International Journal of Production Economics, Vol.168, pp.224-233.

-Rosenblatt, M. J. and Lee, H. L. (1985) "Improving profitability with quantity discounts under fixed demand", IIE Transactions, Vol. 17, pp.388–395.

-Rushton, A., Croucher, P. and Baker, P. (2010) "The handbook of logistics and distribution management: Understanding the supply chain", UK: Kogan Page Publishers.

-Sarathi, G. P., Sarmah, S. P. and Jenamani, M. (2014) "An integrated revenue sharing and quantity discounts contract for coordinating a supply chain dealing with short life-cycle products", Applied Mathematical Modelling, Vol. 38, No. 15, pp. 4120-4136.

-Tang, S., Wang, W., Cho, S. and Yan, H. (2017) "Reducing emissions in transportation and inventory management: (R, Q) Policy with considerations of carbon reduction", European Journal of Operational Research., Vol. 269. Issue 1, pp. 327-340.

-Wu, D. (2013) "Coordination of competing supply chains with news-vendor and buyback contract", International Journal of Production Economics, Vol. 144, No. 1, pp. 1-13.

Appendix

Appendix A: To prove concavity of the retailer's profit function with respect to $k, x,$ the Hessian matrix should be computed.

$$H(\pi_r) = \begin{bmatrix} \frac{\partial^2\pi(k,x)}{\partial k^2} & \frac{\partial^2\pi(k,x)}{\partial k \partial x} \\ \frac{\partial^2\pi(k,x)}{\partial k \partial x} & \frac{\partial^2\pi(k,x)}{\partial x^2} \end{bmatrix} \quad \text{(A-1)}$$

$$\frac{\partial \pi_r(k,x)}{\partial k} = -h_r\,\beta\sigma\sqrt{T+L}(\Phi(k)-1) - h_r\,\sigma\sqrt{T+L} - \frac{\pi+\beta(P-w-x)}{T}\sigma\sqrt{T+L}(\Phi(k)-1) \quad \text{(A-2)}$$

$$H_{11} = \frac{\partial^2\pi(k,x)}{\partial k^2} = -h_r\beta\sigma\sqrt{T+L}\varphi(k) - \frac{\pi+\beta(P-w-x)}{T}\sigma\sqrt{T+L}\varphi(k) \quad \text{(A-3)}$$

The first principal minor is negative ($H_{11} < 0$).

$$\frac{\partial \pi_r(k,x)}{\partial x} = (p-w)b - a - 2bx - \frac{Th_r\,b}{2} + \frac{\beta}{T}\sigma\sqrt{T+L}\psi(k) \quad \text{(A-4)}$$

$$\frac{\partial^2\pi(k,x)}{\partial x^2} = -2b \quad \text{(A-5)}$$

$$\frac{\partial^2\pi(k,x)}{\partial k \partial x} = \frac{\partial^2\pi(k,x)}{\partial x \partial k} = \frac{\sigma\beta\sqrt{T+L}(\Phi(k)-1)}{T} \quad \text{(A-6)}$$

$$H_{22} = \frac{\partial^2\pi(k,x)}{\partial k^2} \times \frac{\partial^2\pi(k,x)}{\partial x^2} - \left(\frac{\partial^2\pi(k,x)}{\partial k\partial x}\right)^2 = -\left(h_r\beta\sigma\sqrt{T+L}\varphi(k) + \frac{1}{T}\big(\pi+\beta(P-w)\big)\sigma\sqrt{T+L}\varphi(k)\right) \times (-2b) - \left(\frac{\sigma\beta\sqrt{T+L}(\Phi(k)-1)}{T}\right)^2 \quad \text{(A-7)}$$

The second principle minor is positive under the following condition:

$$2b\left(\sigma\sqrt{T+L}\varphi(k)\right)\left(h_r\beta + \frac{1}{T}\big(\pi+\beta(P-w)\big)\right) > \left(\frac{\sigma\beta\sqrt{T+L}(\Phi(k)-1)}{T}\right)^2 \quad \text{(A-8)}$$

This condition is tested numerically and observed that it would be satisfied for reasonable parameter values. Hence, the Hessian matrix is negative and the retailer's profit function is concave. The optimal values are calculated as follow:

$$1 - \Phi(k^*) = \frac{h_r}{h_r\,\beta + \frac{1}{T}(\pi + \beta(P-w-x^*))} \quad \text{(A-9)}$$

$$x^* = \frac{p-w}{2} - \frac{a}{2b} - \frac{Th_r}{4} + \frac{\beta}{2bT}\sigma\sqrt{T+L}\psi(k^*) \quad \text{(A-10)}$$

Appendix B: To prove concavity of the supplier's profit function, It is enough to show that its second-order derivative of $\pi_s(n)$ with respect to n is negative.

$$\frac{\partial \pi_s(n)}{\partial n} = \frac{A_s}{n^2 T} - \frac{h_s\left((a+bx)T - \beta\sigma\sqrt{T+L}\psi(k)\right)}{2} \tag{B-1}$$

$$\frac{\partial^2 \pi_s(n)}{\partial n^2} = -\frac{2nTA_s}{n^4T^2} = -\frac{2A_s}{n^3 T} < 0 \tag{B-2}$$

The supplier's profit function is concave with respect to n and the optimal value of n is calculated as follows:

$$n^* = \sqrt{\frac{2A_s}{h_s T\left((a+bx)T - \beta\sigma\sqrt{T+L}\psi(k)\right)}} \tag{B-3}$$

Appendix C: To prove concavity of the supply chain profit function with respect to *k*, *x*, *n*, the Hessian matrix should be calculated.

$$H(\pi_{sc}) = \begin{bmatrix} \frac{\partial^2 \pi(k,x,n)}{\partial k^2} & \frac{\partial^2 \pi(k,x,n)}{\partial k \partial x} & \frac{\partial^2 \pi(k,x,n)}{\partial k \partial n} \\ \frac{\partial^2 \pi(k,x,n)}{\partial k \partial x} & \frac{\partial^2 \pi(k,x,n)}{\partial x^2} & \frac{\partial^2 \pi(k,x,n)}{\partial x \partial n} \\ \frac{\partial^2 \pi(k,x,n)}{\partial k \partial n} & \frac{\partial^2 \pi(k,x,n)}{\partial x \partial n} & \frac{\partial^2 \pi(k,x,n)}{\partial n^2} \end{bmatrix} \tag{C-1}$$

$$\frac{\partial \pi_{sc}(k,x,n)}{\partial k} = (\Phi(k) - 1)\left(\sigma\sqrt{T+L}\right)\left[\frac{h_s(n-1)\beta}{2} - \frac{\left(\pi + \beta(P-x-e)\right)}{T} - h_r\,\beta\right] - h_r\,\sigma\sqrt{T+L} \tag{C-2}$$

$$H_{11} = \frac{\partial^2 \pi_{sc}(k,x,n)}{\partial k^2} = \varphi(k)\left(\sigma\sqrt{T+L}\right)\left[\frac{h_s(n-1)\beta}{2} - \frac{\left(\pi + \beta(P-x-e)\right)}{T} - h_r\,\beta\right] \tag{C-3}$$

The first principle minor is negative under the following condition.

$$\frac{\left(\pi + \beta(P-x-e)\right)}{T} + h_r\,\beta > \frac{h_s(n-1)\beta}{2} \tag{C-4}$$

$$\frac{\partial \pi_{sc}(k,x,n)}{\partial x} = (P-e)b - a - 2bx + \frac{\beta\sigma\sqrt{T+L}\psi(k)}{T} - \frac{\left(h_r + h_s(n-1)\right)bT}{2} \tag{C-5}$$

$$\frac{\partial^2 \pi_{sc}(k,x,n)}{\partial x^2} = -2b \tag{C-6}$$

$$\frac{\partial^2 \pi_{sc}(k,x,n)}{\partial k \partial x} = \frac{\partial^2 \pi_{sc}(k,x,n)}{\partial x \partial k} = \frac{(\Phi(k)-1)\left(\sigma\sqrt{T+L}\right)\beta}{T} \tag{C-7}$$

$$H_{22} = \frac{\partial^2 \pi(k,x,n)}{\partial k^2} \times \frac{\partial^2 \pi(k,x,n)}{\partial x^2} - \left(\frac{\partial^2 \pi(k,x,n)}{\partial k \partial x}\right)^2 = \varphi(k)\left(\sigma\sqrt{T+L}\right)\left[\frac{h_s(n-1)\beta}{2} - \frac{(\pi+\beta(P-x-e))}{T} - h_r\,\beta\right] \times (-2b) - \left(\frac{\sigma\beta\sqrt{T+L}(\Phi(k)-1)}{T}\right)^2 \tag{C-8}$$

The second principle minor is positive under the following condition:

$$2b\varphi(k)\left(\sigma\sqrt{T+L}\right)\left[\frac{(\pi+\beta(P-x-e))}{T} + h_r\,\beta - \frac{h_s(n-1)\beta}{2}\right] > \left(\frac{\sigma\beta\sqrt{T+L}(\Phi(k)-1)}{T}\right)^2 \tag{C-9}$$

$$\frac{\partial \pi_{sc}(k,x,n)}{\partial n} = \frac{A_s}{Tn^2} + \frac{\sigma\sqrt{T+L}\psi(k)h_s\beta}{2} - \frac{(a+bx)Th_s}{2} \tag{C-10}$$

$$\frac{\partial^2 \pi_{sc}(k,x,n)}{\partial n^2} = -\frac{2nTA_s}{n^4T^2} = -\frac{2A_s}{n^3T} \tag{C-11}$$

$$\frac{\partial^2 \pi_{sc}(k,x,n)}{\partial k \partial n} = \frac{\partial^2 \pi_{sc}(k,x,n)}{\partial n \partial k} = \frac{(\Phi(k)-1)\left(\sigma\sqrt{T+L}\right)h_s\beta}{2} \tag{C-12}$$

$$\frac{\partial^2 \pi_{sc}(k,x,n)}{\partial n \partial x} = \frac{\partial^2 \pi_{sc}(k,x,n)}{\partial x \partial n} = -\frac{Th_sb}{2} \tag{C-13}$$

$$\begin{aligned} H_{33} = \frac{\partial^2 \pi(k,x,n)}{\partial k^2} &\times \left[\left(\frac{\partial^2 \pi(k,x,n)}{\partial x^2} \times \frac{\partial^2 \pi(k,x,n)}{\partial n^2}\right) - \left(\frac{\partial^2 \pi(k,x,n)}{\partial x \partial n}\right)^2\right] \\ &- \frac{\partial^2 \pi(k,x,n)}{\partial k \partial x}\left[\left(\frac{\partial^2 \pi(k,x,n)}{\partial k \partial x} \times \frac{\partial^2 \pi(k,x,n)}{\partial n^2}\right) - \left(\frac{\partial^2 \pi(k,x,n)}{\partial x \partial n} \times \frac{\partial^2 \pi(k,x,n)}{\partial k \partial n}\right)\right] \\ &+ \frac{\partial^2 \pi(k,x,n)}{\partial k \partial n}\left[\left(\frac{\partial^2 \pi(k,x,n)}{\partial k \partial x} \times \frac{\partial^2 \pi(k,x,n)}{\partial x \partial n}\right) - \left(\frac{\partial^2 \pi(k,x,n)}{\partial x^2} \times \frac{\partial^2 \pi(k,x,n)}{\partial k \partial n}\right)\right] \end{aligned} \tag{C-14}$$

$$H_{33} = \varphi(k)\left(\sigma\sqrt{T+L}\right)\left[\frac{h_s(n-1)\beta}{2} - \frac{\left(\pi+\beta(P-x-e)\right)}{T} - h_r\,\beta\right] \times \left[\left(-2b \times -\frac{2A_s}{n^3T}\right) - \left(-\frac{Th_sb}{2}\right)^2\right]$$
$$- \frac{(\Phi(k)-1)\left(\sigma\sqrt{T+L}\right)\beta}{T}\left[\left(\frac{(\Phi(k)-1)\left(\sigma\sqrt{T+L}\right)\beta}{T} \times \frac{-2A_s}{n^3T}\right)\right.$$
$$\left.- \left(-\frac{Th_sb}{2} \times \frac{(\Phi(k)-1)\left(\sigma\sqrt{T+L}\right)h_s\beta}{2}\right)\right]$$
$$+ \frac{(\Phi(k)-1)\left(\sigma\sqrt{T+L}\right)h_s\beta}{2}\left[\left(\frac{(\Phi(k)-1)\left(\sigma\sqrt{T+L}\right)\beta}{T} \times -\frac{Th_sb}{2}\right)\right.$$
$$\left.- \left(-2b \times \frac{(\Phi(k)-1)\left(\sigma\sqrt{T+L}\right)h_s\beta}{2}\right)\right] \tag{C-15}$$

The third principle minor is negative under following condition:

$$\varphi(k)\left(\sigma\sqrt{T+L}\right)\left[\frac{h_s(n-1)\beta}{2} - \frac{\left(\pi+\beta(P-x-e)\right)}{T} - h_r\,\beta\right] \times \left[\left(\frac{4bA_s}{n^3T}\right) - \left(\frac{Th_sb}{2}\right)^2\right]$$
$$< \left((\Phi(k)-1)\left(\sigma\sqrt{T+L}\right)h_s\beta\right)^2\left[\left(\frac{-2A_s}{{h_s}^2n^3T^3}\right)\right] \tag{C-16}$$

All the conditions are tested numerically and observed that it would be satisfied for reasonable parameter values. Then, Hessian matrix is negative definite and the supply chain function is concave with respect to k, x, n. The optimal values are calculated as follows:

$$1 - \Phi(k^{**}) = \frac{h_r}{h_r\,\beta + \frac{1}{T}\left[\left(\pi+\beta(P-x-e)\right) - \frac{h_s(n-1)\beta T}{2}\right]} \tag{C-17}$$

$$x^{**} = \frac{\beta\sigma\sqrt{T+L}\psi(k)}{2bT} + \frac{P-e}{2} - \frac{T}{4}\left(h_r + h_s(n-1)\right) - \frac{a}{2b} \tag{C-18}$$

$$n^{**} = \sqrt{\frac{2A_s}{h_sT\left[\left((a+bx)T\right) - \beta\sigma\sqrt{T+L}\psi(k)\right]}} \tag{C-19}$$