Identification of Hazardous Situations using Kernel Density Estimation Method Based on Time to Collision, Case study: Left-turn on Unsignalized Intersection

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Abstract

The first step in improving traffic safety is identifying hazardous situations. Based on traffic accidents' data, identifying hazardous situations in roads and the network is possible. However, in small areas such as intersections, especially in maneuvers resolution, identifying hazardous situations is impossible using accident's data. In this paper, time-to-collision (TTC) as a traffic conflict indicator and kernel density estimation (KDE) method have been used to identify hazardous situations. KDE applies smooth function on critical TTC value events, this surface indicates risk changes. The maximum quantity of this function represents the hazardous situations. To assess and implement the presented method, left-turn on unsignalized intersection has been chosen. TTC data are determined by automated video analysis and coordinating TTC smaller than threshold value was used as input data in KDE method. Hazardous situations have been identified and the factors that caused them have been recognized using these results and performing safety audit. Two countermeasures are proposed to improve safety of left-turn in study location.

Keywords: KDE method, time-to-collision, hazardous situations, traffic conflict

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1. Introduction

Social and economic costs of traffic accidents have grown in the last decades. According to the World Health Organization (WHO) report, 1.24 million people die annually in traffic accidents around the world and between 20 to 50 million suffer non-fatal injuries [WHO, 2013]. Thus, there is an urgent necessity for improving traffic safety to reduce accidents.

For this purpose it is important to understand how and where traffic accidents occurr. Spatial patterns of traffic accidents can distinguish hazardous situations and make accident preventive efforts more effective [Xie and Yan, 2008]. Traffic accident data have numerous problems. For example not all accident are reported, behavioural aspects are rarely available and accidents are very exceptional and rare events [Laureshyn et al., 2010]. Because of the rarity of accidents at individual and specially small sites (accidents could be zero), traffic safety and spatial pattern analysis are difficult or impossible. Hence, it is necessary to use a method based on accident surrogate for determining hazardous situation and risk of different locations.

Objective of this paper is to present a method based on time-to-collision indicator for identifying hazardous situations and risk change surface. The important issue is how to recognize the hazardous situations and risk changes based on chosen TTC threshold in the study area. Kernel Density Estimation (KDE) is one of the methods for point pattern analysis. Traffic conflicts such as accidents have Sphere of influence that is considered by KDE. KDE is used for identification of hazardous road situations [Erdogan et al., 2008, Xie and Yan, 2008, Anderson, 2009, Yu et al., 2012]. As a case study, an unsignalized intersection has been chosen and left-turn safety has been assessed. Parameters required for calculation of TTC were obtained using image processing and camera calibration.

Video analysis has been widely used in traffic safety evaluation, such as traffic accident prediction [Weiming et al., 2004, Sayed and Saunier, 2008, Oh and Kim, 2010], before-after studies [Autey et al., 2012, Sayed et al., 2012] and behavioral studies [Laureshyn et al., 2009a, Laureshyn et al., 2009b]. In some cases, an estimation on the number of conflicts in different regions of studied location is important for traffic safety evaluation. For instance, Guido et al. [2011] have evaluated safety performance from the perspective of “rear-end” vehicle interactions using measures obtained experimentally from a videotaping at an urban roundabout. Vehicle paths have been segmented into equal parts and a number of critical conflicts have been obtained in each segment (histogram) for comparing safety measures. The construction of a bivariate histogram requires specification of size of bins, origin of the system of bins and orientation of the bins [Silverman, 1986]. KDE method can overcome the above restrictions. On the other hand, bivariate density estimates obtained using continuous kernels are much easier to comprehend.

Section 2 presents literature review of traffic conflict, video analysis and camera calibration. The proposed methodology for determining hazardous situations is presented in section 3 that includes 4 modules. The case study is introduced in section 4 and finally results and discussion are described in section 5.

1.Literature Review

1.1 Traffic Conflict

Traffic conflict indicators as an accident surrogate have been widely used to assess traffic safety [Autey et al., 2012, El-Basyouny and Sayed, 2013].Traffic conflict is an active approach because it occurs more frequently than accident and desired sample data is obtained over shorter time [Sayed et al., 2012]. Many indicators are applied in literature review such as Time Gap (TG) [Vogel, 2003], Time Advantage (TAdv) [Laureshyn et al., 2010], Proportion Stopping Distance (PSD) [Allen et al., 1978], Tim-to-Collision (TTC) [Hayward, 1971], Time Integrated Time-to-Collision (TIT) and Time Exposed Time-to-collision indicator (TET) as extended time-to-collision measure [Minderhoud and Bovy, 2001], Maximum Deceleration Rate to Avoid a Crash (DRAC) [Almquist et al., 1991] and Crash Potential Index (CPI) [Cunto and Saccomanno, 2007]. Among these indicators, TTC is one of the most widely used indicators that validated for estimating safety and being used in advanced driver assistance system (ADAS) for collision warning and obstacles avoidance.

TTC has been defined as “the time that remains until a collision between two vehicles would have occurred if the collision course and speed difference are maintained ” [Hayward, 1971]. TTC is a time-based continuous parameter and as long as the vehicles are on a collision course, is measurable. TTC inversely is related with the severity of accidents [Jin et al., 2013]. TTC is calculated between two vehicles during a maneuver and determined minimum value is selected for safety analyses. Suggested TTC threshold value vary according to area, facility and driver support status in a range between 1.5 to 5 seconds [Jin et al., 2011].

1.2 Video Analysis and Camera Calibration

The collected conflicting data can be feasible from different ways such as field observers [Perkins and Harris, 1967, Sayed and Zein, 1999], simulation models [Mehmood et al., 2001, Sayed et al., 1994] and video analysis[Autey et al., 2012, Sayed et al., 2013]. The conflicting data in many previous studies have been subjectively collected by trained observers. In addition to possible errors in severity estimation of conflict, to coverage all maneuvers in particular locations is impossible or difficult for observer. Affecting traffic conflict measures in simulation models can result in observer variability challenge. To address major limitations of simulation models and field observer, especially in behavioral studies, video analysis is useful [El-Basyouny and Sayed, 2013]. Video analysis allows collecting data over a long period of time so that it could be re-examined. The time spent on extraction of speed and position data for video analysis is much less than manual mode [Laureshyn et al., 2009a].

Obtaining real 3D coordinate of vehicles from 2D images and dimension changes by projection in surface plane are main shortcoming of using video analysis. These problems can be explained as follows," The use of video-analysis also limits the quality and scope of a safety study, where safety critical events can be difficult to detect in two-dimensional imagery, and subject to problems related to the relative positioning of the camera and the coverage this provides " [Ismail et al.]. Camera calibration is used to overcome this shortcoming. Camera calibration's purpose is to estimate camera parameters for projecting objects onto the image plane. Real world coordinate also can be obtained by back projecting objects [Ismail et al., 2009a].

2. Methodology

The purpose of this study is to identify hazardous situations based on conflicting data using point pattern analysis. For collecting the conflicting data, video analysis is used to track vehicles from one image to the next and save their positions as pixel. To analyze the position of vehicles, real coordinates are obtained by camera calibration. TTC is calculated for each two vehicles in a maneuver. The coordinates of vehicles with TTC lower than threshold is used as critical conflict coordinates. To obtain hazardous situation, KDE method is used. Figure 1 shows flow of hazardous situation identification process.

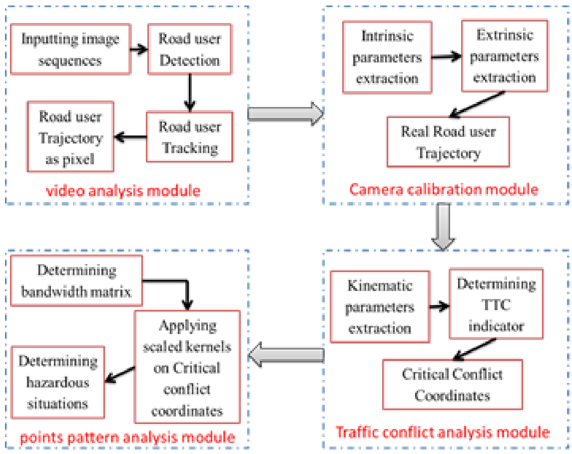

Figure 1. Hazardous situation identification block diagram

2.2 Video Analysis

In this paper, automated video analysis is used for conflicting data collection and objectively evaluating safety. After reviewing video in terms of image processing, challenges such as change the vehicle angle to the camera, reduction in dimensionality of vehicle because of perspective, changes in the light reflected from vehicles during tracking interval, in some cases hiding the vehicle because of physical effects and high speed, and overlapping the vehicles in images have been detected. Selective processing system to eliminate the challenges proposed by this study uses production and adaptive model in spars area.

After detection of vehicles, they are tracked in image sequences by a rate of 20 frames per second, and their coordinates are extracted as pixel (Figure 2). To obtain the actual coordinates of the vehicles, the camera must be calibrated. After obtaining coordinates of the vehicles, their paths are retrieved.

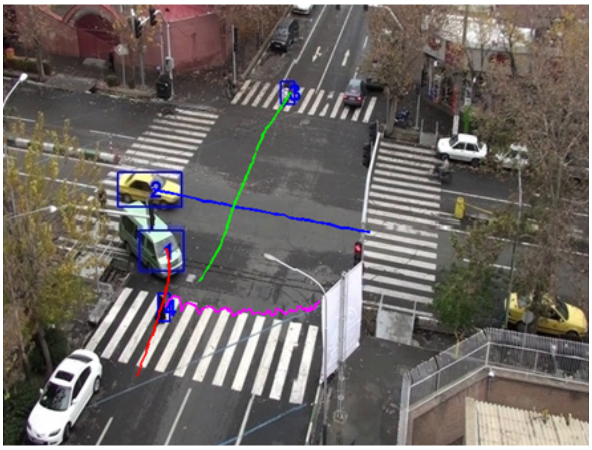

Figure 2. The retrieved vehicle paths using automated video analysis

2.3 Camera Calibration

Camera calibration is a necessary step to recover the three-dimensional coordinate of vehicles using two-dimensional images. The camera calibration parameters are divided into two categories of intrinsic and extrinsic parameters. The intrinsic parameters are a set of parameters for projecting vehicle coordinate to image plane. The extrinsic parameters denote the coordinate system transformations from 3D world coordinates to 3D camera coordinates. The Camera calibration parameters are defined in detailed by [Karim Ismail et al., 2012]. For extracting the camera calibration parameters, the camera calibration Toolbox for MATLAB [Caltec, 2013] has been used.

2.4 Traffic Conflict Analysis

The trajectory of vehicle has been extracted from video analysis and camera calibration module. In each moment, it is supposed that vehicles do not change their movements and obtain future positions based on this assumption; if these positions coincide temporally and spatially with other vehicles (collision course), then TTC will be determined. TTC is equal to the time that remains until two vehicles arrive at a common spatial zone and collide to each other.

Parameters needed for calculating TTC are velocity, and coordinates of vehicles corners. These parameters have been achieved from real coordinates that obtained from image sequence by rate of 20 frames per second. For considering vehicle dimensions in the calculation of conflict indicators, vehicles have been modeled as two-dimensional. Figure 3 shows an example of a two-dimensional modeling for four different times with intervals of one second. In addition to that the two vehicles can approach each other at any angle, different collision types are also possible for the same angle. For all the possible types of collision, a corner of one of the vehicles meets a side of the other one. Based on procedure that has been presented by Laureshyn et al. [2010], TTC is calculated for a moving line section (side of vehicle) and a point (corner of vehicle) and the lowest TTC value found among all the possible corner-side combinations is to be used. According to the findings of Sayed and Zein [1999], threshold value of TTC is 1.5s in intersections. Based on this threshold value, coordinates of left-turn vehicles (critical conflict coordinates) from minor to major street are extracted.

2.5 Point Pattern Analysis of Critical Conflicts

Kernel density estimation (KDE) is a non-parametric method for applying a smooth density function on spatial data. In some fields such as signal processing and econometrics, it is also termed as Parzen–Rosenblatt window method. KDE method has a simple concept and compare to other density estimation methods such as nearest neighbor and maximum penalized likelihood is most common [Silverman, 1986, Xie and Yan, 2008]. KDE method includes applying a kernel (symmetric surface with a mathematical function) to points and then summing the height of kernels in the reference locations. Obtained value for reference locations is a measure of density for that location. The general form of a kernel density estimator for a bivariate sample is defined by Eq.(4).

$$\hat{f}(x;\mathbf{H}) = n^{-1}\sum_{i=1}^{n} K_{\mathbf{H}}(x - X_i) \qquad (4)$$

Where:

$x = (x_1, x_2)^T$: coordinate of points that function value is desired (reference locations),

$X_i = (X_{i1}, X_{i2}), i = 1,2,\ldots,n$: point events (critical conflict coordinates),

$K(x)$: is the bivariate kernel function that

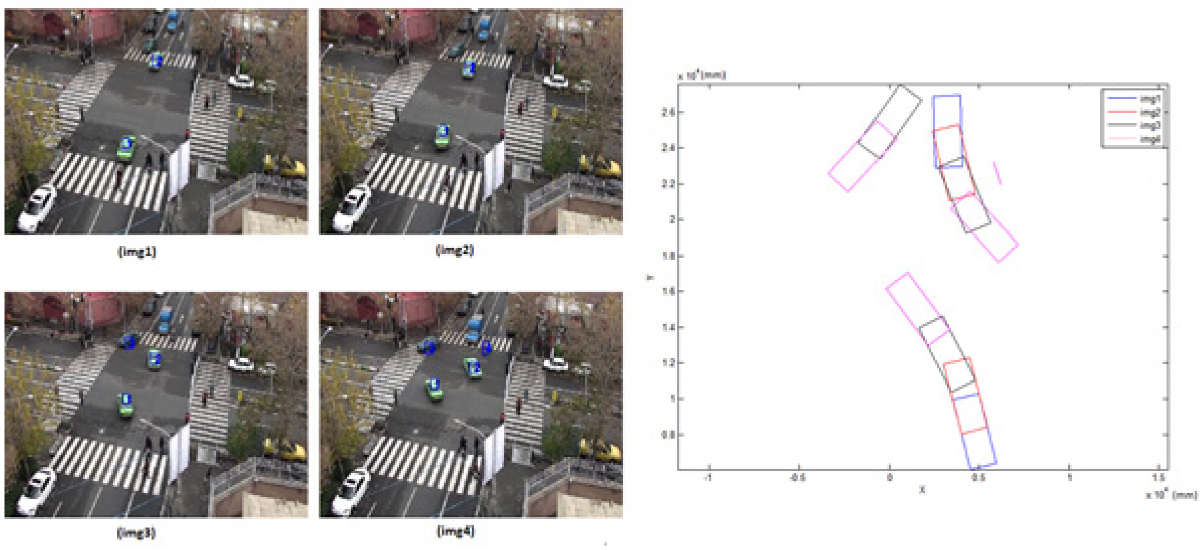

Figure 3. Two-dimensional modeling of vehicles

used as a weight function,

H: bandwidth matrix which is a positive-definite matrix, $K_H(x) = |\mathbf{H}|^{-1/2} K(\mathbf{H}^{-1/2} x)$: the scaled kernel function, where $|\mathbf{H}|$ is The determinant of the bandwidth matrix.

Several kernel functions such as uniform, triangle, Gaussian, epanechnikov, negative exponential and conic can be used as a weight function[Bil et al., 2013, Horová et al., 2012, Xie and Yan, 2008]. Bandwidth selection is more important than function type in kernel smoothing [Duong and Hazelton, 2003, Duong and Hazelton, 2005]. By using matrix bandwidth, sphere of influence of a conflict is being considered; this means that a conflict can occur in surrounding area with lower probability according to a mathematic function. Hence, Gaussian kernel has been used as a common kernel function and defined as following for univariate sample data.

$$K(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2) \quad (5)$$

For creating bivariate kernel from symmetric univariate kernels, product kernel and spherically symmetric kernel methods can be used [Horová et al., 2012]. In this paper product kernel method with Eq.(6) has been used. Figure 4 shows Gaussian kernel.

$$K^P(\mathbf{x}) = k(x_1)k(x_2) \quad (6)$$

Where K^p is product kernel.

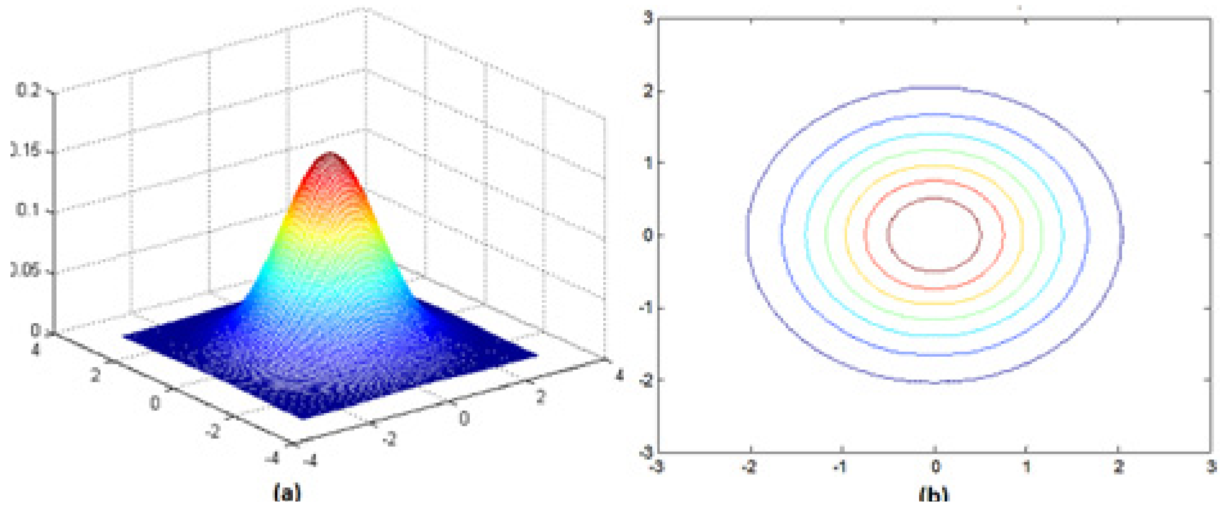

Figure 4. Gaussian kernel, (a) 3D plot, (b) contour plot

Due to significant influence of bandwidth matrix on the estimation, two following methods for determining full matrix have been used. Eq.(4,5) present d-dimensional multivariate estimation of bandwidth. In bivariate case, d should be equal to 2.

1) Reference rule

$$\hat{\mathbf{H}}_{REF} = \left(\frac{4}{n(d+2)} \right)^{2/d+4} \hat{\Sigma} \qquad (7)$$

Where:

$\hat{\mathbf{H}}_{REF}$: Reference rule bandwidth matrix,

n: Number of observation,

d: Number of dimensions that equal 2 in bivariate case,

$\hat{\Sigma}$: The empirical estimate of covariance matrix,

2) Maximal smoothing principle

$$\hat{\mathbf{H}}_{MS} = \left(\frac{(d+8)^{(d+6)/2} \pi^{2/d} V(K)}{16n\Gamma((d+8)/2)\,(d+2)} \right)^{2/(d+4)} \hat{\Sigma} \qquad (8)$$

Where:

$\hat{\mathbf{H}}_{MS}$: maximal smoothing principle bandwidth matrix,

$V(K)$: variance of kernel estimator.

To obtain a more practical method and significant height of in the created surface, after summing the kernels, height of surface is divided on height of scaled kernel to have an estimation of the number of critical conflicts.

3. Case Study

Four-leg unsignalized intersection located at the crossroad of Vessaleshirazi-Bozorgmehr in Tehran has been selected as study location (Figure 5). Because this intersection is unsignalized and has high traffic volume, high rate of conflicts between vehicles is expected which makes it to be a suitable study location. Major street is divided by refuge with three lanes in each direction. Minor street is undivided with two lanes in each direction. In each direction, one lane is used as on-street parking. The light pattern is flashing yellow light on the major street and flashing red light on minor street. Video data has been collected on a working day at night. The video camera was mounted on a six-story building near the intersection (Figure 5). As shown in Figure 5, video camera cover the entire intersection. Left-turn movement is the most dangerous maneuvers especially at unsignalized intersections. This movement has the most overlap with other movements. To implement the proposed method, kernel smoothing Toolbox for MATLAB [Horová et al., 2012] has been used. KDE method applied smooth surface on coordinate of critical conflicts based on threshold value for left-turning vehicles.

4.Results and Discussion

4.2 Camera Calibration Results

For determining the vehicle coordinates, camera calibration has been performed and Intrinsic parameters are obtained as follows.

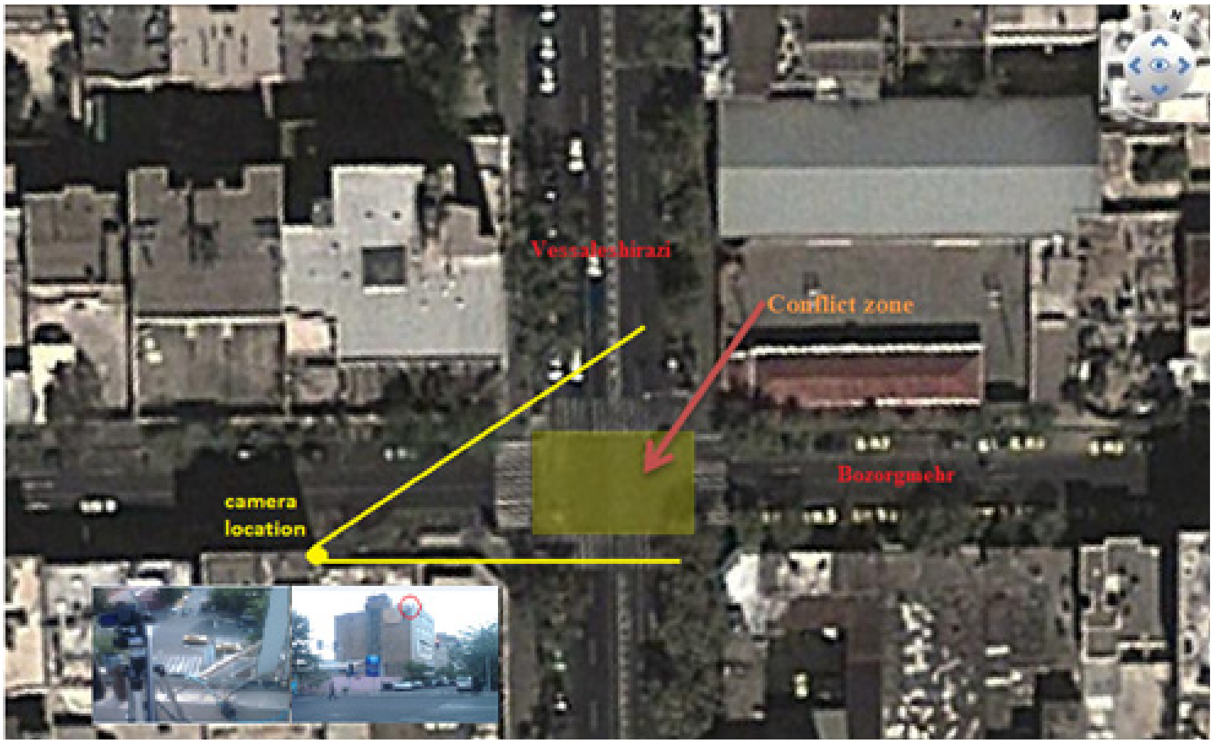

Figure 5. Camera location conflict zone at study location

$$\begin{cases} \text{Focal Length:} & fc = [\ 943.64865 \quad 1262.55192\ ] \pm [\ 6.86981 \quad 9.33886\ ]; \\ \text{Principal point:} & cc = [\ 337.72600 \quad 189.95787\ ] \pm [\ 6.51470 \quad 6.59243\ ]; \\ \text{Skew:} & \alpha_c = [\ 0.00000\ ] \pm [\ 0.00000\ ] \Rightarrow \text{angle of pixel axes} = 90.00000 \pm 0.00000 \text{ degrees}; \\ \text{Distortion:} & kc = [\ 0.27017 \quad -1.41352 \quad -0.01821 \quad 0.02551 \quad 0.00000\ ] \pm [\ 0.04538 \quad 0.43608 \quad 0.00217 \quad 0.00403 \quad 0.00000\ ]; \\ \text{back projection error:} & err=[0.19887 \quad 0.41764]. \end{cases} \quad (1)$$

err is the standard deviation of the back projection error (in pixel) in both directions of coordinate system. The uncertainties of calibration parameters are also estimated. The numerical values are approximately three times the standard deviations.

Computed Extrinsic parameters including rotation matrix (R) and translation vector (T) are shown below.

$$T = [\ -11834.504091 \quad 6232.403874 \quad 46082.588471] \quad (2)$$

$$R = \begin{bmatrix} 0.970734 & 0.240034 & 0.007680 \\ 0.117360 & -0.446238 & -0.887186 \\ -0.209528 & 0.862123 & -0.461349 \end{bmatrix} \quad (3)$$

Since there is not any algebraic method for back projection that recovers 3D real world coordinate of vehicles in image sequences, the try and error method has been used. Figure 6 shows calibration results and world coordinate system. Locating plus signs in circles is indicated by camera calibration accuracy, the distance of adjacent points is one meter.

4.3 Analysis of Critical Traffic Conflict Pattern

Figure 7 presents a zoning of left-turn movement based on number of the overlapping expected paths. Movement volumes, vehicle speed and driver behaviors (by using be-

Figure 6. Camera calibration results

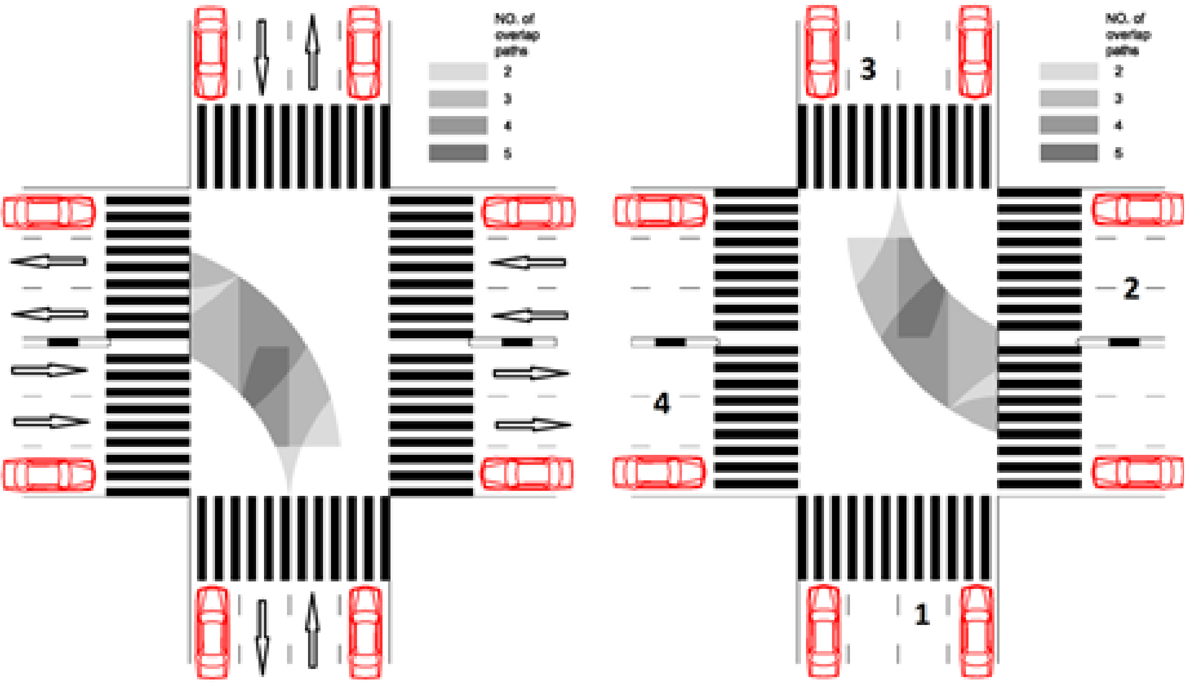

Figure 7. Zoning of left turn movement

havioral indicators) are important factors for recognizing hazardous situations. TTC as a micro-behavioral indicator is used to detect dangerous point events and KDE method has been used to make continuous risk surface from point events.

Figure 8 and 9 show risk surface of left-turn for two bandwidth matrixes, maximal smoothing principle and reference. The height of surfaces indicates an estimation of the number of critical conflicts in different locations. In Figure 8-a and Figure 9-a, risk surface has been created by reference rule method, while maximal smoothing principle method has been used for creating risk surface in Figure 8-b and Figure 9-b. In maximal smoothing principle the selected bandwidth is bigger than reference rule method, thus each kernel has broader impact area that causes a surface with smoother and greater height compared to reference rule method. Both bandwidth selection methods are well indicating hazardous situations and risk changes. If data dispersion is high and more continuous surface is desired, maximal smoothing principle is proposed. On the other hand, if the local maximum is important, performance of reference rule method will be better. The proposed method makes critical point events to continuous surface. Hence, in addition to detecting hazardous situations, risk changes will also be viewable.

Figure 10 shows the traffic volume for the period of analysis. Since right and left turning traffic volume respectively from approach 1 and 2 is low, risk values in common spatial zone of these movement with left-turning from approach 3 is lower than other zones (Figure 8). For left-turning from approach 1, risk surface is more continuous because traffic movement is more balanced. Direct traffic on major street compared to other movements is high. Therefore, it is expected that common spatial zones of these movements with left-turning from minor street be hazardous; output result of KDE method confirms this hypothesis (Figure 8 and 9).

These surfaces could be used as an initial step in safety programs. For instance, after putting the expected left-turn area with the risk surfaces from KDE method (Figure 11), it was found that hazardous situations are not located in expected left-turn area. After checking videos and performing safety audit, two principal problems were detected. The first is that sight distances for left-turning vehicle is less than the recommended sight distances and drivers of approaching vehicles have an obstructed view of the intersection. The second is that vehicles before entering the intersection, begin to turn (Figure 12) or vehicles use opposite direction for turning when their lane is occupied (Figure 13).

5. Recommended Countermeasures

Analysis of results obtained from proposed method and performing safety audit led to proposal of two recommendations that can be used as safety countermeasures. The first

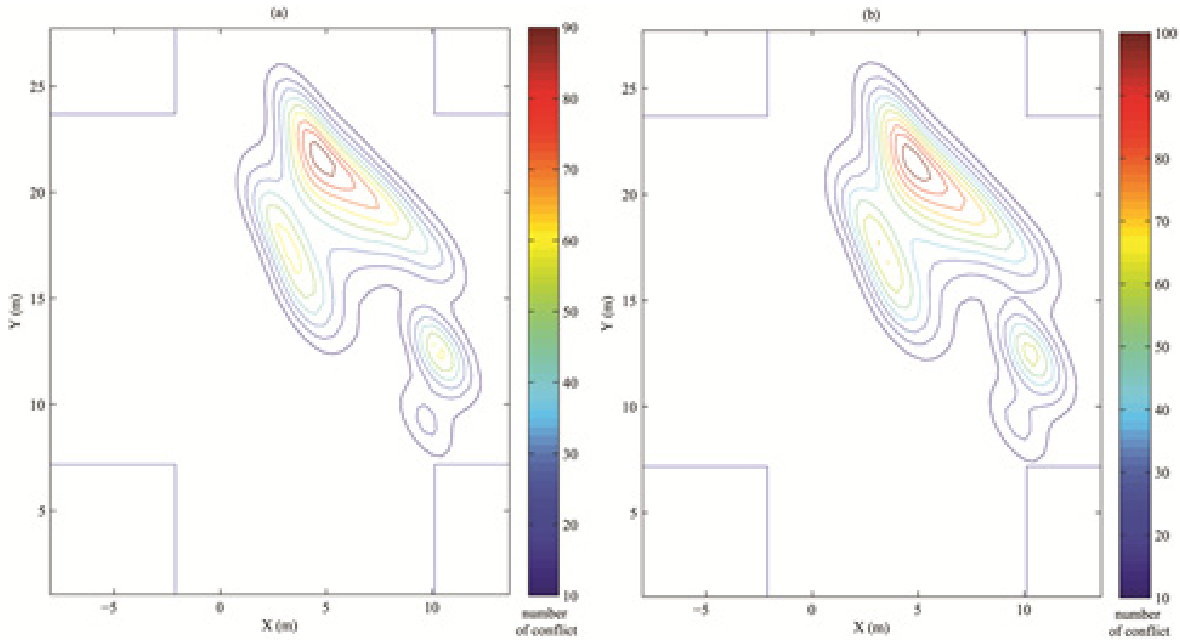

Figure 8. Risk surface, (a) : Reference rule, (b) : Maximal smoothing principle

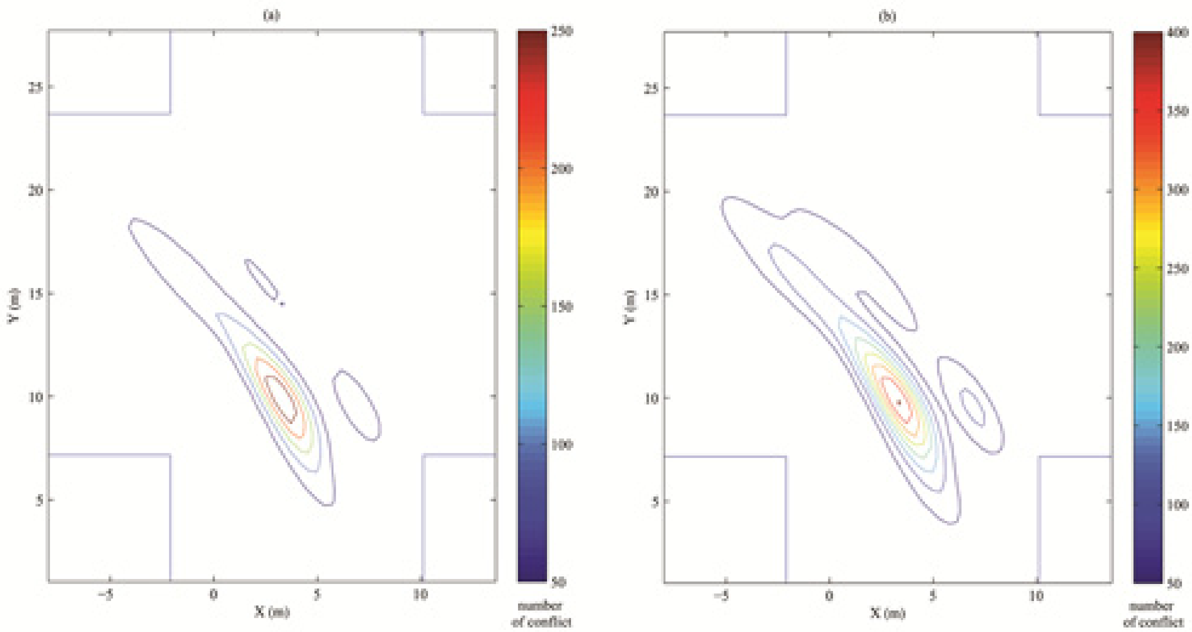

Figure 9. Risk surface, (a) : Reference rule, (b) : Maximal smoothing principle

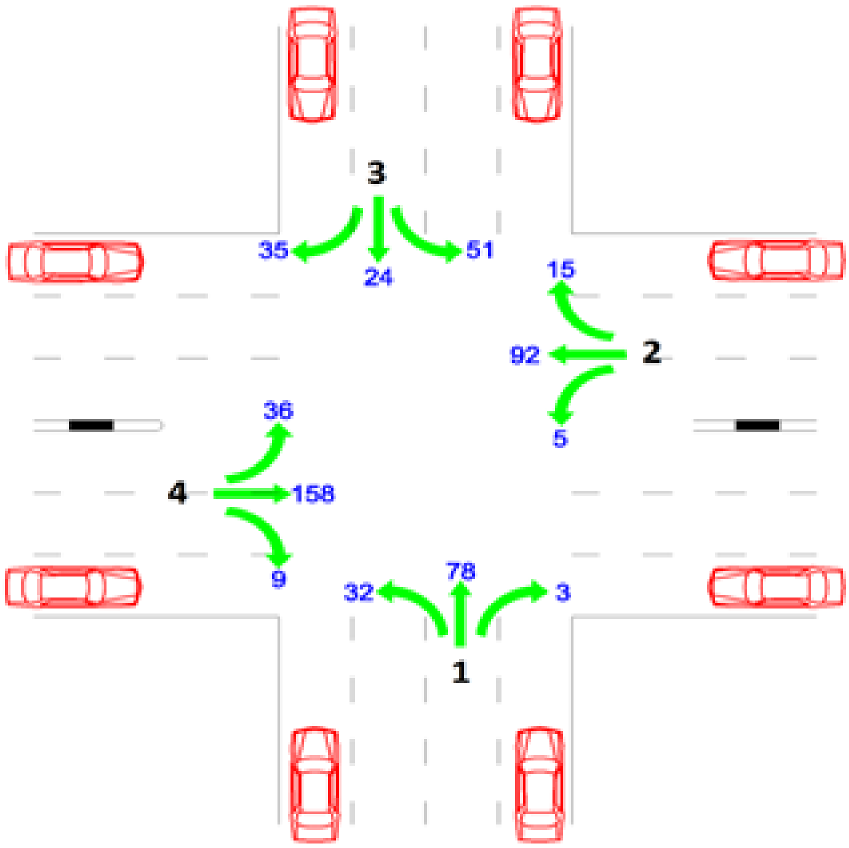

Figure 10. Traffic volume of movements

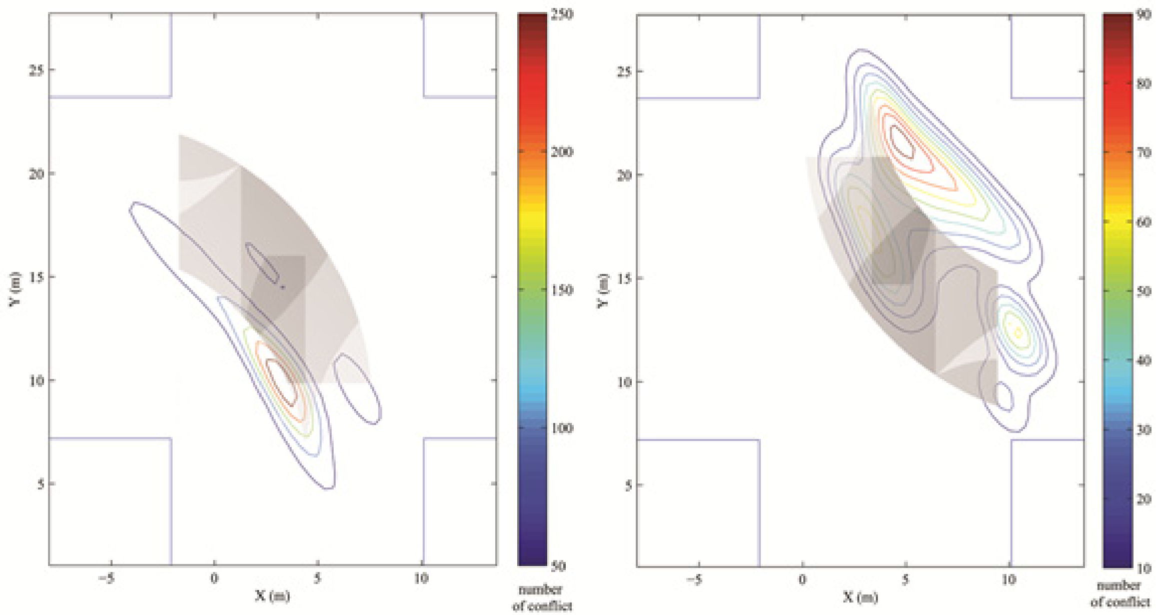

Figure 11. KDE and naive zoning for left-turn

Figure 12. Vehicles left-turning before entering the intersection

Figure 13. Vehicles is using opposite direction for left-tuning

countermeasure includes a geometric realignment as shown in Figure 14-a. Divisional island at the end of minor approaches is used to control left-turn vehicles at the initial of maneuver. This island configuration fits a wide range of volumes. As third countermeasure, extending refuge at major street can be used (Figure 14-b). This realignment control left-turn vehicles at the end of maneuver.

6. Conclusion

This paper presents a methodology to identify hazardous situations and risk changes based on conflict indicators, especially for locations where frequency accident data is insufficient for detailed analysis. Proposed methodology leads to an overarching visualization and manipulation of the critical conflicts. Moreover, this method will make us to become able to have an estimation of the critical conflicts in each location. Using KDE method, hazardous smooth surface has been applied on point

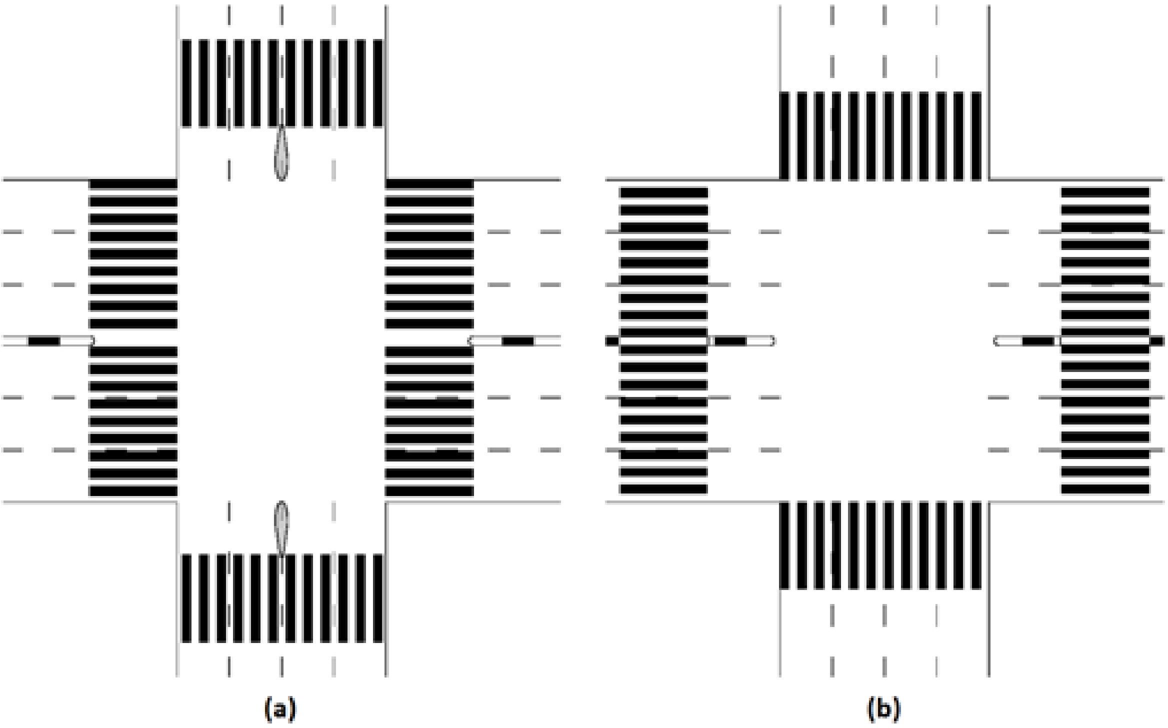

Figure 14. Proposed countermeasures to improve safety of left-turning vehicles

events and makes discrete events to continuous surface. The risk of having a critical conflict similar to accidents will geographically occur not just at a single point but over a given area. Hence, the proposed methodology takes into account spread of critical conflict risk. Estimating the number of critical conflicts using maximal smoothing principle bandwidth matrix is more than reference rule bandwidth matrix and its surface is smoother. These results could help traffic safety experts to have a better understanding of safety problems. In case study due to obstacles in the view and turning earlier or in opposite direction, hazardous situations come into existence out of the expected left-turn area. To solve these safety problems, two countermeasures are proposed. In this study, the critical conflicts have been just used in analysis and all scaled kernels have the same dimension. Using all conflicts with kernel dimension proportional to indicator value could be investigated in future studies.

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