On Calibration and Application of Logit-Based Stochastic Traffic Assignment Models

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Abstract

There is a growing recognition that discrete choice models are capable of providing a more realistic picture of route choice behavior. In particular, influential factors other than travel time that are found to affect the choice of route trigger the application of random utility models in the route choice literature.

This paper focuses on path-based, logit-type stochastic route choice models, in which several forms of logit-family models have been calibrated using practical data and examined on an illustrative network. For each type of the logit-family models, two modeling approaches have been implemented in stochastic traffic assignment (STA). The first approach is a univariate route choice model. Challenges in the estimation of path utility are discussed and a heuristic estimation algorithm for univariate models is proposed. As the proposed approximate calibration method does not require resorting to choice data, it can be regarded as a more practical method than the traditional approach and can overcome many inherent difficulties in calibration of route choice models in univariate case. The second one includes a multi-criteria path utility function considering travel time and monetary cost along with travelers' income to determine the equilibrium network flow. This model has been calibrated based on a stated preferences data set.

This study showed that estimation of the utility could have remarkable impacts on the equilibrium flow and thereby on policy assessments, while the impact of model specification is far less severe. The importance of this achievement arises from the fact that most of the efforts made in stochastic assignment literature have been dedicated to apply theoretically appealing choice models, and model calibration by comparison, have not received considerable attention.

Keywords: Stochastic traffic assignment, route choice, logit models, stated preference data, model specification and calibration

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1. Introduction

In the conventional four-step transportation planning, traffic assignment is the last step in which the equilibrium traffic flow is determined in a network, given the travel demand matrix and the network characteristics. Traffic assignment is a fundamental part of quantifying network performance and directly affects policy assessment outcomes. Travel time is the only determinant in the conventional route choice models, while there are other factors affecting route choice, such as out-of-pocket cost and even demographics of travelers [Dial, 2000]. Having a handful number of socio-economic variables for each traveler in the emerging activity-based models, behavioral and realistic route choice models are becoming even more acknowledged in the network policy assessments.

Deterministic traffic assignment is the state of the practice and has been established based on the Wardrop's user equilibrium (UE) criterion, stating that "the journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route" [Wardrop, 1952]. Route choice procedure is a critical inner problem in traffic assignment models, as it contains the assumptions of the analyst about user's behavior. In deterministic UE (DUE), for example, this inner problem is an all-or-nothing assignment which allocates all trips to quickest paths. However, it is argued that DUE is not behaviorally flexible and realistic, since it utilizes some implicit restrictive assumptions. DUE assumes, for instance, that (I) all the trip makers have a perfect knowledge about the network; (II) they follow a similar and consistent choice procedure; and (III) all the passengers are perfect optimizers. Stochastic traffic assignment (STA) models, on the other hand, have been developed to relax these assumptions to some extent. Daganzo and Sheffi [1977] introduced the concept of perceived travel time, and thereby defining stochastic user equilibrium (SUE) stating that "in a SUE network, no user believes he can improve his travel time by unilaterally changing routes".

Random utility models are applied in SUE to account for the randomness in route choice decisions. Logit-family models are commonly used, as they have a closed form formula for the route choice probability that ease the estimation and interpretation of the results. SUE models, however, are predominantly applied having travel time as the only determinant of path utilities, although there is no theoretical limitation on the number of explanatory variables in random utility modeling. Moreover, according to the literature, parameters of path utility functions are generally specified in a predetermined basis and were typically set to rounded numbers such as the unity rather than being estimated based on the observed data [Chen et al. 2003; Prashker and Bekhor, 1998], whereas the parameters convey important behavioral information and their value affect the predicted traffic flow.

Motivated by the above discussion, this study is to estimate and apply a single-criteria as well as a multi-criteria path utility function containing travel time and costs divided by income, both calibrated based on experimental data. The behavioral parameter in all the univariate logit-family models of SUE known as the dispersion parameter that is the only behavioral parameter directly related to trip maker's perception is calibrated using a heuristic method proposed in this study. This parameter is the coefficient of travel time in the path utility function. Changing the interpretation of this parameter to an equivalent standpoint, a heuristic algorithm for estimation of this parameter is introduced that can be applied in the univariate path utility situation. Performance of the calibrated models is compared with different types of logit-family route choice models in the pedagogical network of Nguyen and Dupuis [1984].

It is also worth mentioning that the different choice models applied in this study have all been previously developed in the literature, however to the best of our knowledge, no comprehensive comparison has been made formerly on the difference of the result of application of these models in a unified study. Moreover, most of the previous efforts have been dedicated to development of the more advanced choice models. This study, however, was aimed to consider both application and estimation phases equally, as it has been shown in this study that the estimation is not only a subordinate issue in network analysis but also can much more affect the prediction result than specification of the model. Further, the proposed approximate method of calibration for univariate models is originally introduced in this study for the first time with the aim of alleviating the intrinsic challenges in calibration of SUE models. Finally, application of different logit-family models in a multi-criteria basis, while including demographic aspects of travelers to TA phase, is considered in this study for the first time.

The remaining parts of the paper are organized as follows. In section 2, the theoretical background for different types of logit-family STA models are provided. The examined models include a broad range of models introduced in the literature, from the simplest multinomial logit and its modifications to the state-of-the-art generalized extreme value models. In section 3, the data sets and data collection methods are explained. Two types of data have been collected for this study, one for univariate models and the other for multi-criteria models. The heuristic approach for calibration of univariate SUE models, as well as the results of estimations (for both univariate and Multi-criteria models) are elaborated in this section. Section 4 is dedicated to examining the results of applying the estimated SUE models to an illustrative network. The equilibrium algorithm, adapted and applied to this study, is introduced in this section. Furthermore, a comparison on the influence of calibration and specification of the choice model on prediction of flow is made, the effect of congestion on similarity of UE and SUE flows previously studied by Sheffi and Powell [1981] are reconsidered and the potential capability of the proposed multi-criteria model in assessment of monetary transportation supply management policies, such as road or fuel pricing, are illustrated. Section 5 concludes the study and summarizes the findings. Some directions for future researches are also put forward in this section.

2. Theoretical Background

Consider trip-maker n facing $\mathcal{K}$ paths in the set of $\mathbb{K}$ for traveling from zone r to s. The utility that n perceives from path k is denoted by U_{nk} in Eq. 1, and is decomposed into a deterministic (V_{nk}), and a random component (ε_{nk}). The latter encompasses all the unobserved factors, and highly influences the structure and properties of the route choice model.

$$U_{nk} = V_{nk} + \varepsilon_{nk} \quad \forall n = 1, \dots, N;\ \forall k \in \mathbb{K} \tag{1}$$

N is the total number of trip makers between origin r and destination s. Superscript rs is omitted for the sake of ease of reference. V_{nk} is a function of explanatory variables that is typically specified as $-\theta\, t_k$ in univariate models, where t_k denotes the travel time of path k. θ is termed the “dispersion parameter”.

According to the random utility theory postulation, each traveler chooses the path that he believes is the most desirable. Therefore, the probability of taking path k, by traveler n is defined in Eq. 2. Denoting the vector of error terms for person n as $\boldsymbol{\varepsilon}_n = \langle \varepsilon_{1n}, \varepsilon_{2n}, \dots, \varepsilon_{\mathcal{K}n} \rangle$, and the joint probability density function of $\boldsymbol{\varepsilon}_n$ as $f(\boldsymbol{\varepsilon}_n)$, P_{nk} can be restated as in Eq. 3, in which *I* is an indicator function which is equal to 1 when the event in the parenthesis occurs and is zero otherwise [Train, 2009].

$$P_{nk} = Prob\big(U_{nk} > U_{nj}\ ; \forall j \neq k\big) = Prob\big(V_{nk} + \varepsilon_{nk} > V_{nj} + \varepsilon_{nj}\ ; \forall j \neq k\big) = Prob\big(\varepsilon_{nj} - \varepsilon_{nk} < V_{nk} - V_{nj}\ ; \forall j \neq k\big) \tag{2}$$

$$P_{nk} = \int_{\varepsilon_n} I(\varepsilon_{nj} - \varepsilon_{nk} < V_{nk} - V_{nj}, \forall j \neq k)\, f(\varepsilon_n)\, d\varepsilon_n \tag{3}$$

For logit-family models, *f* is specified such that the above integral can analytically be solved and stated as a closed form. Tractability of the estimated probabilities makes this class of models the most widely used in the choice modeling context. ¬

According to the literature, there is a broad range of logit-type models developed and applied in the literature of transportation networks modeling. Figure 1 provides a general overview of these models that their theoretical frameworks are outlined in the following subsections.

2.1. Multinomial Logit (MNL)

MNL model is derived by assuming independently and identically extreme value distribution for the error terms. This results in a diagonal homoskedastic covariance matrix of the errors, and the following choice probability:

$$P_k = \frac{e^{V_k}}{\sum_{j\in\mathbb{K}} e^{V_j}} \quad (4)$$

The well-known Dial's algorithm [Dial, 1971], also called STOCH, loads network according to the above equation without a need to generate paths explicitly. This link-based algorithm also reduces the set of all paths to a subset of paths as choice set of travelers. Paths attribute and, as a result, their systematic utilities are flow-dependent and vary due to congestion effects. Therefore, it must be determined simultaneously with path choice probabilities, which indicates the concept of equilibrium. Multinomial Logit-based SUE was first formulated by Fisk [Fisk, 1980] as a convex optimization problem. Evaluation of the objective function and its derivatives is more difficult than DUE, as path flows enter the objective function as decision variables. Therefore, SUE flows are usually obtained by the Method of Successive Averages (MSA), a convex combinations method with predetermined sequences of step sizes. Powell and Sheffi (1982) discussed the conditions under which MSA converges, and showed that these conditions are satisfied in logit-based SUE, when path-sets are considered as fixed sets. There are also heuristic methods which optimizes step sizes for solving Fisk's formulation [Chen and Alfa, 1991]. The equivalent mathematical formulation of MNL-SUE, formulated by Fisk (1980), in the notation of single-criterion case is as follows:

$$\text{MIN} \quad Z = \sum_a \int_0^{x_a} t_a(\omega)d\omega + \frac{1}{\theta}\sum_{rs}\sum_{k\in\mathbb{K}^{rs}} f_k^{rs}\ln(f_k^{rs}) \quad (5)$$

$$\textit{Subject to} \quad \sum_{k\in\mathbb{K}^{rs}} f_k^{rs} = q^{rs} \quad \forall\, rs$$

$$f_k^{rs} \geq 0 \quad \forall\, rs, \forall k \in \mathbb{K}^{rs}$$

Where a is an index of links, x_a is the flow on link a, f_k^{rs} denotes flow on path k between OD pair rs, and q^{rs} is total travel demand between OD pair rs. The definitional path-link incidence relationship must also be hold implicitly:

$$x_a = \sum_{rs}\sum_{k\in\mathbb{K}^{rs}} f_k^{rs}\delta_{ak}^{rs} \quad (6)$$

where δ_{ak}^{rs} is a dummy equal to one if link a is on path k between OD pair rs, and zero otherwise.

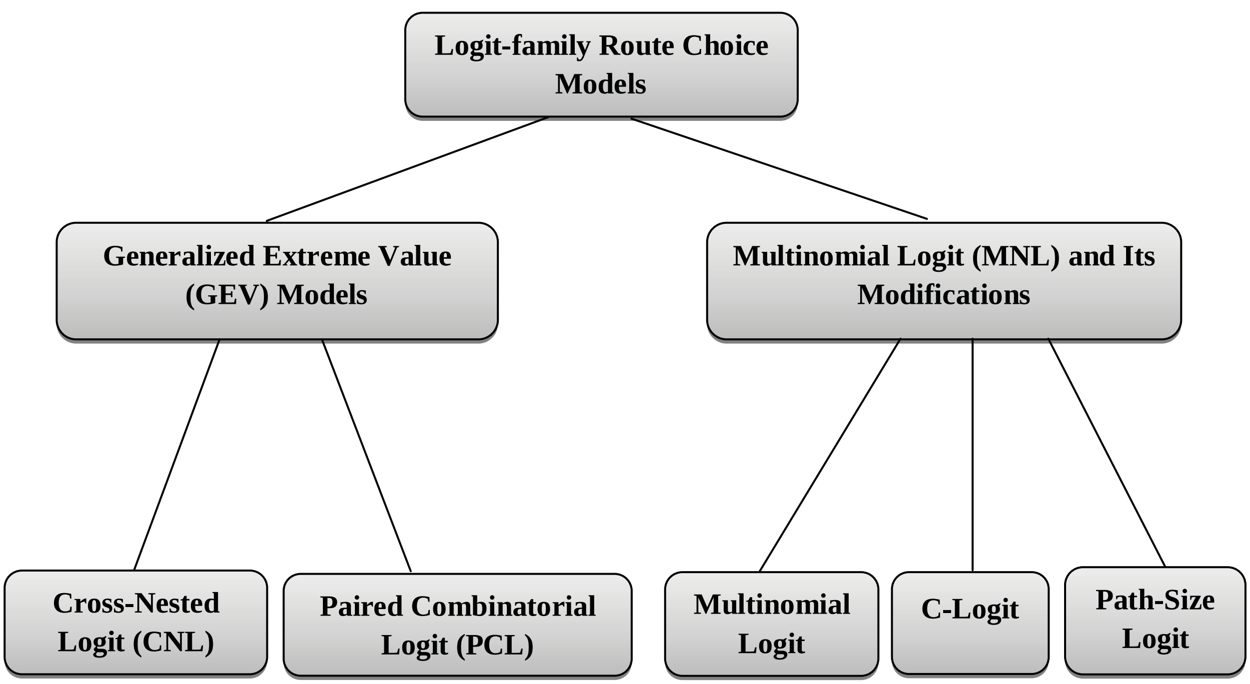


Figure 1. An overview on logit-type models applied in transportation network analysis.

2.2. C-Logit

The main deficiency of application of MNL model in network modeling is that this model simply assumes that all the paths connecting each O-D pair are regarded by decision makers as independent alternatives. Accordingly, this approach unrealistically ignores that fact that in a typical network many paths have shared segments and in this sense they cannot be considered as independent alternatives. Therefore, further research was conducted to overcome the standard logit drawbacks in the context, specifically the problem of path overlapping. Cascetta et.al [1996] introduced the concept of *commonality factor* for each path and proposed C-logit, as a modification of the standard MNL. Choice probability in this model is given by Eq. 7 where cf_k, the commonality factor, is an overall measure of commonality of path k with other paths. This can be viewed as a utility correction factor due to the overlapping problem and is calculated by Eq. 8. In this equation, l_k and l_j denote lengths of paths *k* and *j* respectively and l_{kj} is the length of common parts of these paths. β and γ are calibration parameters that are set to one in this research, in accordance with Prashker and Bekhor [1998].

$$P_k = \frac{e^{V_k - cf_k}}{\sum_{j\in \mathbb{K}} e^{V_j - cf_j}} \quad (7)$$

$$cf_k = \beta \ln \sum_{j\in \mathbb{K}} \left(\frac{l_{kj}}{\sqrt{l_k l_j}}\right)^{\gamma} \quad (8)$$

2.3. Path-Size Logit

Similar to C-logit model, Ben-Akiva and Bierlaire [1999] developed path-size logit (PSL), as another modification of MNL. PSL modifies each utility by adding the log of *path size*, instead of subtracting the commonality factor from each utility. The choice probability is formulated in Eq. 9. The correction factor PS_k is called the "size" of path *k* and is calculated in Eq. 10, in which L_a is the length of link a and Γ_k is the set of links in route *k*.

$$P_k = \frac{e^{V_k + \ln PS_k}}{\sum_{j\in \mathbb{K}} e^{V_j + \ln PS_j}} = \frac{PS_k e^{V_k}}{\sum_{j\in \mathbb{K}} PS_j e^{V_j}} \quad (9)$$

$$PS_k = \sum_{a\in \Gamma_k} \left(\frac{L_a}{l_k}\right) \left(\sum_{j\in \mathbb{K}} \left(\frac{l_k}{l_j}\right)^{\gamma} \delta_{aj}\right)^{-1} \quad (10)$$

2.4. Cross-nested Logit

Cross-nested logit (CNL) and paired combinatorial logit (PCL) are special cases of a more general class of choice models called generalized extreme value (GEV) models, derived by McFadden [1978]. CNL model provides a flexible nesting structure allowing each alternative to belong to any nest with different degrees of membership. Overlapping structure of the nests allows releasing the main theoretical drawback of MNL model in route choice. In GEV, the probability that a decision maker chooses alternative k among $\mathcal{K}$ alternatives is given in Eq. 11.

$$P_k = \frac{y_k G_k(y_1, \ldots, y_{\mathcal{K}})}{\rho G(y_1, \ldots, y_{\mathcal{K}})} \quad (11)$$

G is a generating function of $\mathcal{K}$ non-negative variables, G_k is the first-order derivative of G with respect to y_k, and y_k is the exponent of V_k. The generating function must meet certain conditions [McFadden, 1978] as follows:

i. is homogenous of degree $\rho > 0$; that is: $G(\alpha y_1, \ldots, \alpha y_{\mathcal{K}}) = \alpha^{\rho} G(y_1, \ldots, y_{\mathcal{K}})$

\.(McFadden discussed the condition of homogeneity of degree one and Ben-Akiva and Lerman [1985] extended it to homogeneity of any degree)

ii. $\lim_{y_i \to +\infty} G(y_1, \ldots, y_k) = +\infty \quad \forall i = 1, \ldots, \mathcal{K}$

iii. The *m*th cross-partial derivative of $G(y_1, \ldots, y_k)$ with respect to any combination of *m* distinct y_i□s is non-negative for odd m and non-positive for even *m*. Further, the joint cumulative distribution function of ε_i's is defined as $F(\varepsilon_1, \ldots, \varepsilon_{\mathcal{K}}) = e^{-G(e^{-y_1}, \ldots, e^{-y_{\mathcal{K}}})}$.

CNL model has an overlapping structure and was developed by Vovsha [1997]. Vovsha [1997] and Prashker and Bekhor [1999] used similar generating function (Eq. 12) that leads to the choice probability in Eq. 13.

$$G(y_1, \ldots, y_{\mathcal{K}}) = \sum_a \left(\sum_k \alpha_{ak} y_k\right)^{\mu} \quad (12)$$

$$P_k = \frac{\exp\left[V_k + ln\sum_a \alpha_{ak}\left(\sum_{j\in \mathbb{K}} \alpha_{aj} e^{V_j}\right)^{\mu-1}\right]}{\sum_{i\in \mathbb{K}} \exp\left[V_i + ln\sum_a \alpha_{ai}\left(\sum_{j\in \mathbb{K}} \alpha_{aj} e^{V_j}\right)^{\mu-1}\right]} = \sum_a P_{(a)} \times P_{(k|a)} \tag{13}$$

$$= \sum_a \left(\frac{\left(\sum_{j\in \mathbb{K}} \alpha_{aj} e^{V_j}\right)^{\mu}}{\sum_b \left(\sum_{j\in \mathbb{K}} \alpha_{bj} e^{V_j}\right)^{\mu}}\right) \times \left(\frac{\alpha_{ak} e^{V_k}}{\sum_{j\in \mathbb{K}} \alpha_{aj} e^{V_j}}\right)$$

$$G(y_1, \dots, y_{\mathcal{K}}) = \sum_{j=1}^{\mathcal{K}-1} \sum_{i=j+1}^{\mathcal{K}} \left(1-\sigma_{ij}\right)\left(y_i^{\frac{1}{1-\sigma_{ij}}} + y_j^{\frac{1}{1-\sigma_{ij}}}\right)^{1-\sigma_{ij}} \tag{14}$$

$$P_k = \frac{\sum_{j\neq k} e^{\frac{V_k}{1-\sigma_{kj}}}\left(1-\sigma_{kj}\right)\left(e^{\frac{V_k}{1-\sigma_{kj}}} + e^{\frac{V_j}{1-\sigma_{kj}}}\right)^{-\sigma_{kj}}}{\sum_{j=1}^{\mathcal{K}-1}\sum_{i=j+1}^{\mathcal{K}}\left(1-\sigma_{ij}\right)\left(e^{\frac{V_k}{1-\sigma_{kj}}} + e^{\frac{V_j}{1-\sigma_{kj}}}\right)^{1-\sigma_{kj}}} = \sum_{j\neq k} P_{(kj)}.P_{(k|kj)} \tag{15}$$

$$= \sum_{j\neq k}\left(\frac{e^{\frac{V_k}{1-\sigma_{kj}}}}{e^{\frac{V_k}{1-\sigma_{kj}}} + e^{\frac{V_j}{1-\sigma_{kj}}}}\right)\left(\frac{\left(1-\sigma_{kj}\right)\left(e^{\frac{V_k}{1-\sigma_{kj}}} + e^{\frac{V_j}{1-\sigma_{kj}}}\right)^{1-\sigma_{kj}}}{\sum_{j=1}^{\mathcal{K}-1}\sum_{i=j+1}^{\mathcal{K}}\left(1-\sigma_{ij}\right)\left(e^{\frac{V_k}{1-\sigma_{kj}}} + e^{\frac{V_j}{1-\sigma_{kj}}}\right)^{1-\sigma_{kj}}}\right)$$

μ is a non-negative calibration parameter that should be less than one in order for the choice model to be consistent with utility maximization axiom. A typical value of 0.5 is used for this parameter in this research, when the estimation stage is skipped.

2.5. Paired Combinatorial Logit

Chu [1989] proposed PCL, in which each pair of alternatives constitute their own nest and there is a separate similarity index (σ_{ij}) for each pair of paths. The generating function and choice probability is as follow:

The adaptation of PCL for route choice was first done by Prashker and Bekhor [1998] by attributingσ_{kj}'s directly to the network topology:

$$\sigma_{ij} = \frac{l_{ij}}{(l_i l_j)^{0.5}} \tag{16}$$

3. Data and Estimation

To calibrate a logit path utility function, a set of stated preference data as well as a set of perception error data was collected, in April 2012, in Tehran, the capital of Iran.

The reason why we designed SP experiments is that a major challenge in estimating route choice models with revealed preference data is the large number of alternatives. Enumerating all possible paths in real networks is neither practical nor the estimation of choice models based on that is well-established. Although logit models allow estimation on a subset of paths without disturbing consistency of estimates, most sampling approaches do not reproduce satisfactory results. For this purpose, paths could have either equal or unequal chance of selection in the choice set. Although the first option comes with a simple estimation procedure, unreasonable paths may be reproduced that are never even considered by travelers. This leads to inefficient estimates, since "comparing a person's chosen alternative with a group of highly undesirable alternatives provide little information about the reasons for person's choice" [Train, 2009]. Therefore, the second option which is based on a systematic selection procedure is usually preferred to obtain unbiased estimates. The path utilities, however, should be corrected according to the sampling procedure. The estimation is challenging, but some correction methods are discussed in the literature [Frejinger et al. 2009; Bovy et al. 2009; Prato and Bekhor 2007]. Moreover, the actual equilibrium condition existing in reality determines alternative routes such that they don't have enough variability that is essential for precise estimations.

Paper-based survey questionnaires were distributed in three high schools, in which students from different geographical zones are registered. 460 students were asked to pass the questionnaires to one of their family members who makes daily work trip. 220 completed questionnaires were returned to the school officials, making a response rate of 48 percent. Each respondent was asked to consider 8 experiments, making a total of 1760 choice situations. Each experiment was a choice between two hypothetical entirely disjointed routes that was unlabeled, but their attributes such as time (in minutes) and cost (in 10 Rials) were given. This data allows one to apply the standard logit, since paths are entirely non-overlapping. Further, correlation of observations over time, which is a critical issue in standard logit models dealing with repeated choice data, is considerably mitigated through the use of unlabeled alternatives. This is because respondents' choice is, arguably, unaffected by other factors that are not stated in the question. This method of survey design allows one to consider the error terms close to white (pure) noises, although there are unobserved personal factors that could bring a level of correlation among choices of each respondent.

Furthermore, family monthly income (in 10 million Rials) and average fuel consumption of respondent's personal car in the AM peak (in liters per 100 kilometers) were asked. Out-of-pocket cost of travel was, then, estimated based on the fuel cost, path tolls and path lengths. Additional questions were also asked to explore traveler's inaccuracy in perceiving travel times for three well-known OD pairs in Tehran. The respondents were requested to write their estimate for travel time in each path during the AM peak, and also indicate their level of familiarity to each route. Actual travel times, on the other hand, were measured three times and were averaged for each path in different working days.

This section elaborates estimation of two route choice models to be implemented in a stochastic traffic assignment. First, a univariate logit model is estimated, having the travel time as the only route choice determinant. Dispersion coefficient of this model is estimated based on the collected experimental data and compared with a typical value that is commonly used in the literature. Second, a multi-criteria logit utility is estimated having travel time and cost as the route choice explanatory variables. Travel cost is normalized by income to represent a systematic taste variation among travelers. Four income groups are distinguished and a utility function is associated to each class in the traffic assignment phase, assuming an exponential probability density function for income across people of each origin.

3.1. Univariate Route Choice Model Estimation

Behavioral aspects of route choice decisions are reflected in the dispersion coefficient, in a univariate SUE assignment. Estimation of this parameter has yet received little attention, due to special issues in the estimation procedure. This is elucidated below, followed by a heuristic method that is proposed to approximate the dispersion coefficient.

Random disturbances are assumed to be independently and identically distributed in a standard logit model. The density function of Gumbel distribution with scale parameter of $\mu > 0$ and location parameter of η is $f(\varepsilon) = \mu e^{-\mu(\varepsilon-\eta)} e^{-e^{-\mu(\varepsilon-\eta)}}$, with a mean and variance of $\eta + \frac{\gamma}{\mu}$ and $\frac{\pi^2}{6\mu^2}$, respectively. $\gamma \cong 0.577$ is Euler's constant. Assuming a Gumbel distribution with a normalized-to-zero location parameter and an arbitrary scale parameter, the logit formula becomes $P_k = \frac{e^{\mu V_k}}{\sum_j e^{\mu V_j}}$. μ cannot be separately identified from the coefficients included in the utility, as the scale of utility is irrelevant to behavior [Train, 2009]. Traditionally, in estimation of logit models, the scale parameter, μ is normalized to unity (or equivalently, the variance is normalized to $\frac{\pi^2}{6}$) and the partial coefficients of utility function are estimated due to this normalization. Nonetheless, one can specify path utility as $U_k = -t_k + \varepsilon_k \;\; \forall k$ in a univariate route choice model, and consider θ as the scale parameter of error distribution and refer to errors for estimation instead of choices. Although the error terms are not directly observable, one can accept the assumption that misestimating the travel times dominates other unobserved factors and thus differences between perceived and measured travel times serves as a proxy of the error term. One may, however, argue that error terms contain more elements and, therefore, this is a restrictive assumption. We accept that the assumption is restrictive for special cases (e.g. recreational routes), but seems an acceptable simplifier assumption for regular daily work trips.

A heuristic approach is introduced based on the above-mentioned supposition, to approximate coefficient of dispersion. Normalizing the location parameter to zero, the log- likelihood function may be stated as in Eq. 17, in which n is the number of observations. Dispersion coefficient must be estimated such that the first-order derivative of the log-likelihood function with respect to θ be equal to zero (Eq. 18).

$$LL(\theta) = \sum_{i=1}^{n} \left(\ln \theta - \theta \varepsilon_i - e^{-\theta \varepsilon_i} \right) \qquad (17)$$

$$\frac{n}{\theta} - \sum_{i=1}^{n} \varepsilon_i + \sum_{i=1}^{n} \varepsilon_i e^{-\theta \varepsilon_i} = 0 \qquad (18)$$

Having the respondents' perceived travel time and the actual travel time for some OD pairs, the error term and, consequently, the dispersion coefficient was estimate from Eq. 18. The respondents were also asked whether they are familiar with the specified route. Based on 96 observations from travelers who were familiar with the routes, a value of 0.1519 was estimated for θ. Including unfamiliar users, the estimated dispersion coefficient reduces to 0.1343, based on 110 observations. A reduction in the value of the dispersion coefficient conveys that a larger share of travelers choose longer paths, when they are not familiar with the routes. This method is capable of considering different coefficients of dispersion for different OD pairs, taking the fact into account that far distant OD pairs tend to have lower values of the dispersion parameter.

As stated, dispersion coefficient has been set to typical pre-specified rounded values in most of previous studies, instead of calibrating with experimental data. Let us compare this estimated value with a typical value of one [Chen et. al, 2003] in a route choice situation between two paths with five minutes difference in travel time. The model with a typical dispersion coefficient predicts a portion of around 0.67 percent for the longer route, while the calibrated model ($\theta = 0.1519$) assigns 32 percent of travelers to the longer path. This significant difference highlights the determining role that this coefficient plays in a STA model.

Another analysis is performed to illustrate how sensitive the dispersion coefficient is to the inaccuracy level of the travelers in estimating travel time of a route with a free flow time of, for example, 38 minutes. For this purpose, θ is estimated based on 110 synthetic observations, among which X travelers have 1, 2, and 3 minutes of prediction error. For instance X = 4, indicates 4 travelers with 1 minutes, 4 travelers with 2 minutes, and 4 travelers with 3 minutes error and the other 98 travelers with an exact estimate. For this hypothetical situation, the value of estimated θ would be 0.91. A value of 10 for X yields to a dispersion coefficient of 0.71. In this case, almost 73 percent of travelers are perfectly predicting the travel time and the maximum perception error for the others are less than only 8 percent. This situation does not seem realistic, at least for Tehran's network, and indicates invalidity of the assumption of unity value for the dispersion coefficient. Further, Figure 2 illustrates three situations that lead to θ = 0.15, 0.3, and 0.6. In this figure, θ = 0.15 corresponds to the observed distribution of perception error for 96 travelers who indicated are familiar with the given route.

3.2. Multi-Criteria Route Choice Model Estimation

This section discusses a logit path utility function that is calibrated considering travel time and normalized out-of-pocket cost. Path utility is defined as $V_{nk} = \alpha t_k + \beta \frac{\ln c_k}{I_n}$, in which I_n is the family-income for person n, and c and t are travel cost and time, respectively. Minimizing the likelihood function, estimated values of α and β turned out to be -0.0891 and -1.1594, respectively. A standard statistical t-test confirmed that both estimated coefficients are statistically significant at a 99 percent confidence interval. The McFadden pseudo rho squared was obtained 17 percent at convergence.

4. Numerical Examples

The proposed logit route choice models are applied in the pedagogical network of Nguyen and Dupuis [1984] with 13 nodes, 19 links, and 4 OD pairs (Figure 3). OD pairs of (1,2), (1,3), (4,2) and (4,3) have, respectively, travel demand of 400, 800, 600 and 200 units. The results are discussed separately for the univariate and multi-criteria route choice models.

4.1. Univariate Model

The univariate route choice model was applied for a typical dispersion parameter of θ=1, and the estimated parameter of θ=0.1519. The volume-delay function of $T_a(x_a) = \alpha_a + \beta_a x_a$, in which T_a is travel time, and x_a is flow on link a, is utilized. The values of parameters α_a and β_a, however, are presented in Table 1. Table 2 and 3 provides equilibrium traffic flow of the univariate route choice model in terms of link and path flows respectively, for the deterministic, MNL, C-logit, PSL, CNL, and PCL traffic assignment models. The outputs reveal that logit-family SUE traffic assignment models with a dispersion coefficient of 1 is by and large similar to the deterministic assignment. Figure 4 illustrates this similarity in terms of link flows predicted by DUE and PCL model. Therefore, pre-specification of the dispersion parameter could even question the primary purpose of the stochastic traffic assignment models which is accounting for the randomness in route choice decisions.

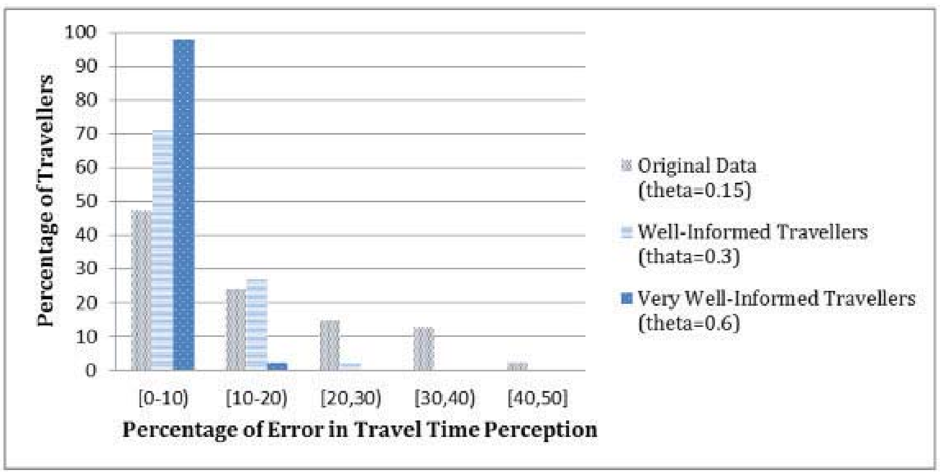


Figure 2. Distribution pattern of travel time perception error data.

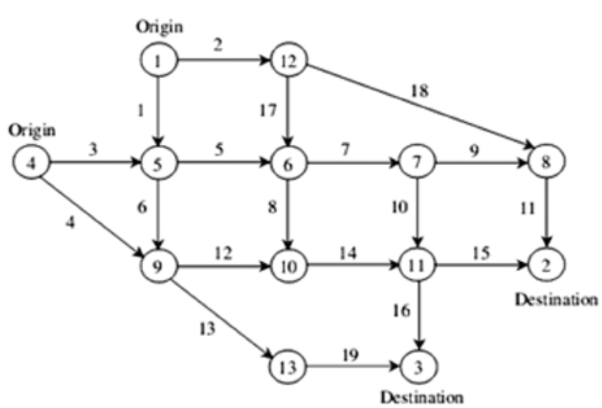


Figure 3. Test network [Nguyen and Dupuis, 1984]

Table 1. Parameters of link volume-delay functions for test network

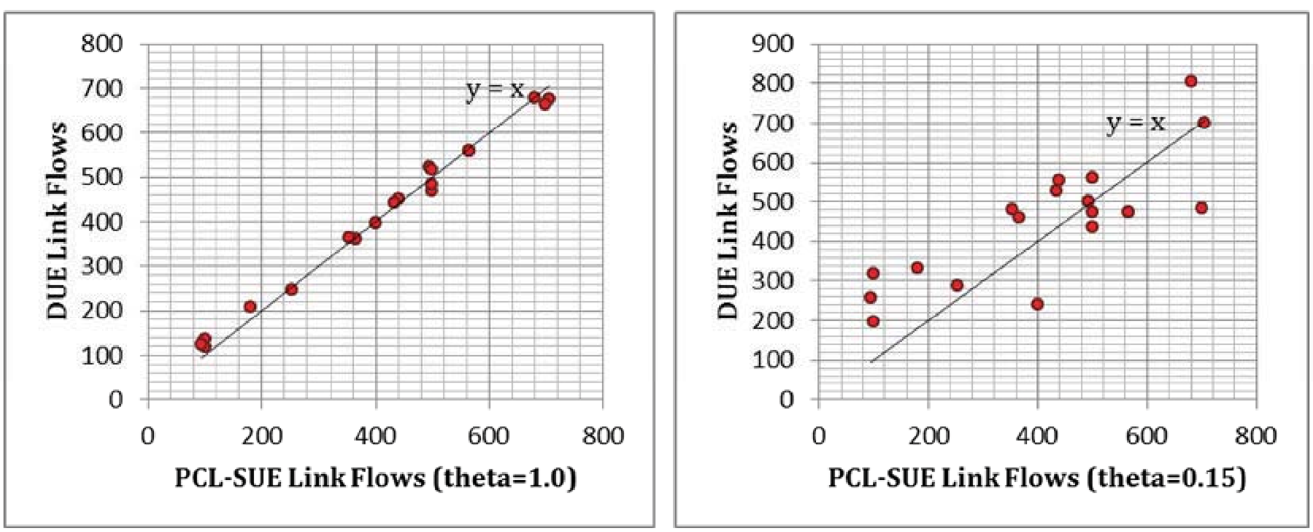
Link	From	To	a_a	β_a
1	1	5	7	0.0125
2	1	12	9	0.01
3	4	5	9	0.01
4	4	9	12	0.005
5	5	6	3	0.0075
6	5	9	9	0.0075
7	6	7	5	0.0125
8	6	10	13	0.005
10	7	11	9	0.0125
11	8	2	9	0.0125
12	9	10	10	0.005
13	9	13	9	0.005
14	10	11	6	0.0025
15	11	2	9	0.005
16	11	3	8	0.01
17	12	6	7	0.0125
18	12	8	14	0.001
19	13	3	11	0.001

Table 2. Equilibrium link flow for univariate models

Link	DUE	MNL		C-logit		PSL		CNL		PCL	
		$\theta = 1.0$	$\theta = 0.18$	$\theta = 1.0$	$\theta = 0.18$	$\theta = 1.0$	$\theta = 0.18$	$\theta = 1.0$	$\theta = 0.18$	$\theta = 1.0$	$\theta = 0.18$
1	706	676	706	675	703	677	707	684	705	677	700
2	494	524	494	525	497	523	493	516	495	523	500
3	100	143	362	143	349	144	357	132	311	135	317
4	700	657	438	657	451	656	443	668	489	665	483
5	440	461	598	451	566	451	562	457	570	450	556
6	366	358	470	367	486	371	501	360	445	362	461
7	354	364	498	364	487	364	476	365	470	365	482
8	180	223	372	214	337	211	324	209	334	209	331
9	100	112	184	115	189	115	182	117	179	118	196
10	254	253	313	249	297	249	294	248	290	247	287
11	500	509	407	513	429	513	437	515	441	516	438
12	500	465	483	464	470	464	474	472	468	469	472
13	566	550	424	560	467	562	470	556	467	559	472
14	680	688	856	678	807	675	799	681	802	678	803
15	500	491	593	487	571	487	563	485	559	484	562
16	434	450	576	440	533	438	530	444	533	441	528
17	94	127	272	127	257	125	238	117	234	125	257
18	400	397	222	398	240	398	255	399	262	398	242
19	566	550	424	560	467	562	470	556	467	559	472

Table 3. Equilibrium path flows for univariate models

OD	Path	Link Sequence	DUE	MNL		C-logit		PSL		CNL		PCL	
				θ=1	θ=0.15	θ=1	θ=0.15	θ=1	θ=0.15	θ=1	θ=0.15	θ=1	θ=0.15
1-2	1	2-18-11	400	397	222	398	240	398	255	399	262	398	242
	2	1-5-7-9-11	0	1	38	1	35	1	30	1	31	1	34
	3	1-5-7-10-15	0	0	22	0	20	0	18	0	20	0	21
	4	1-5-8-14-15	0	0	26	0	25	0	22	0	25	0	27
	5	1-6-12-14-15	0	0	18	0	18	0	22	0	17	0	18
	6	2-17-7-9-11	0	1	32	0	27	0	23	0	18	0	26
	7	2-17-7-10-15	0	0	19	0	16	0	14	0	12	0	14
	8	2-17-8-14-15	0	0	23	0	19	0	17	0	15	0	18
1-3	9	1-6-13-19	366	353	284	363	325	366	335	358	323	361	330
	10	1-5-7-10-16	203	168	106	165	97	165	96	174	107	170	97
	11	1-5-8-14-16	136	148	126	142	109	140	103	150	121	144	112
	12	1-6-12-14-16	0	5	85	4	74	5	81	2	61	1	61
	13	2-17-7-10-16	50	67	91	68	93	68	90	65	88	67	94
	14	2-17-8-14-16	44	59	108	58	102	56	94	51	100	57	105
4-2	15	4-12-14-15	500	457	285	457	294	456	291	468	325	465	325
	16	3-5-7-9-11	100	110	114	113	127	114	129	116	130	117	136
	17	3-5-7-10-15	0	17	67	15	63	15	63	8	55	10	51
	18	3-5-8-14-15	0	15	80	14	72	15	75	8	63	8	59
	19	3-6-12-14-15	0	0	54	0	45	0	41	0	27	0	29
4-3	20	4-13-19	200	197	118	197	123	197	118	198	129	197	123
	21	4-12-14-16	0	3	35	3	34	3	34	2	35	2	35
	22	3-6-13-19	0	0	22	0	18	0	17	0	15	0	19
	23	3-5-7-10-16	0	0	8	0	9	0	13	0	9	0	9
	24	3-5-8-14-16	0	0	10	0	10	0	13	0	9	0	10
	25	3-6-12-14-16	0	0	7	0	6	0	5	0	2	0	4



(a) Estimated dispersion coefficient. (b) Typical dispersion coefficient.

Figure 4. spersion coefficient effect on similarity between DUE and SUE flows.

Table 2 provides equilibrium link flows based on the estimated and typical dispersion parameters for five different model specifications, namely MNL, C-logit, PSL, CNL, and PCL. A comparison of the link flows with DUE reveals that stochastic traffic assignment is more sensitive to calibration of the dispersion parameter than to the route choice model specification. To further illustration of this fact, a PCL model with a calibrated θ is set as the comparison basis in Figure 5, and its predicted link flows are presented versus a MNL model with calibrated θ and also a PCL with typical θ. As shown in this figure, calibration of the dispersion parameter makes more tangible changes compared to adopting a more robust model specification.

Figures 6 and 7 provide a graphical sensitivity analysis on the importance of specification of θ on predicted equilibrium flow, for some certain links and paths of our illustrative network, respectively. Two models were selected as the basis for comparison: MNL model as the simplest model and PCL model as one of the most advanced models which appropriately addresses path correlation problem. Two primary achievements can be gained from the figures. First, according to both figures, one can easily observe that the degree of sensitivity of the flow to the value of θ is maximal in an interval about θ=0 to θ=0.4, in which according to our data we expect the actual value of θ for most real networks locate. This finding in fact emphasizes the importance of paying sufficient attention to the estimation of θ in a precise enough fashion in planning. Second, as can be seen from Figure 7, when θ approaches to zero (or equivalently, the variance of perception errors becomes infinite), MNL path flows for a particular OD pair do tend to equal values, but corresponding PCL path flows do not. This fact indeed contradicts the proposition stated in some previous studies [Prashker and Bekhor, 2000] claiming that for probabilistic models, as perception error variance tends to infinity, the path flows connecting each particular OD pair approaches to same equal values. As the figure obviously shows this statement is only valid for MNL model, and for the models other than MNL which are able to represent path overlapping is not true. For this class of models, it is the covariance structure that determines path flows when the variance becomes infinite.

It is argued that equilibrium traffic flows of the stochastic and deterministic traffic assignments become very similar as a network becomes more congested [Daganzo and Sheffi, 1977; Sheffi and Powell, 1981].

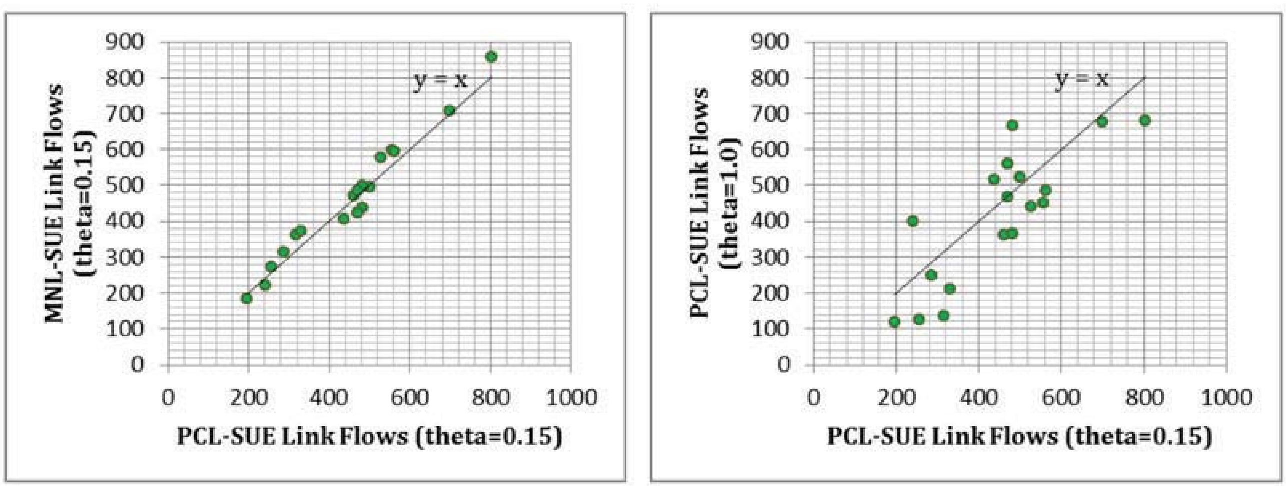
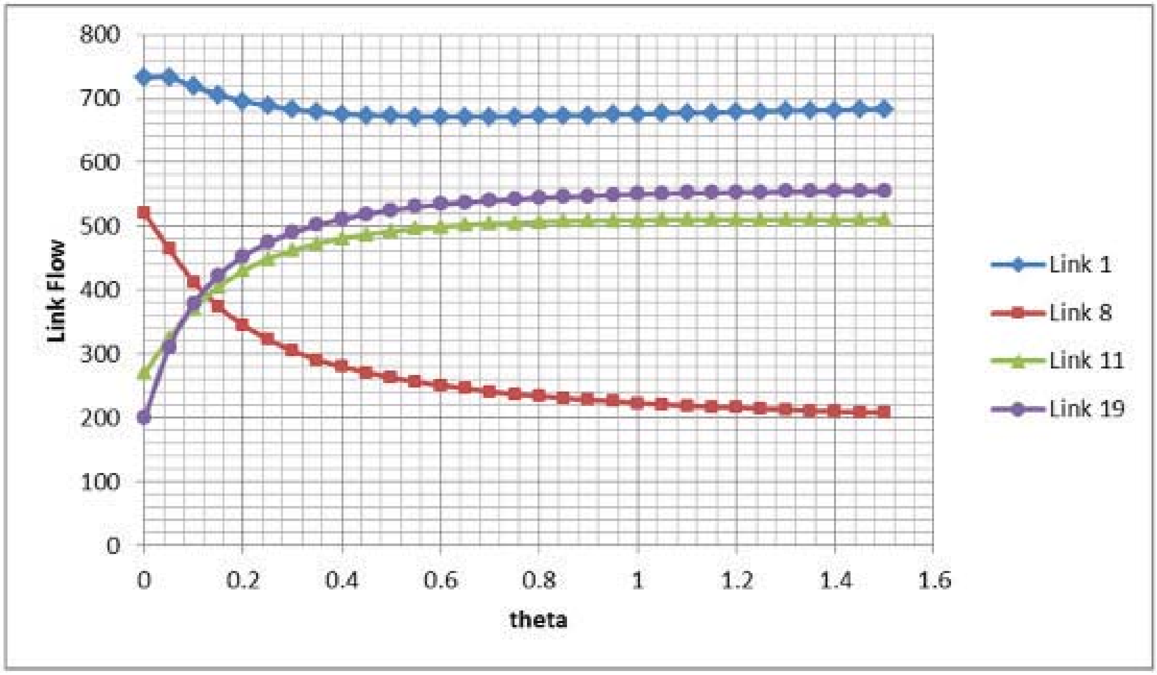
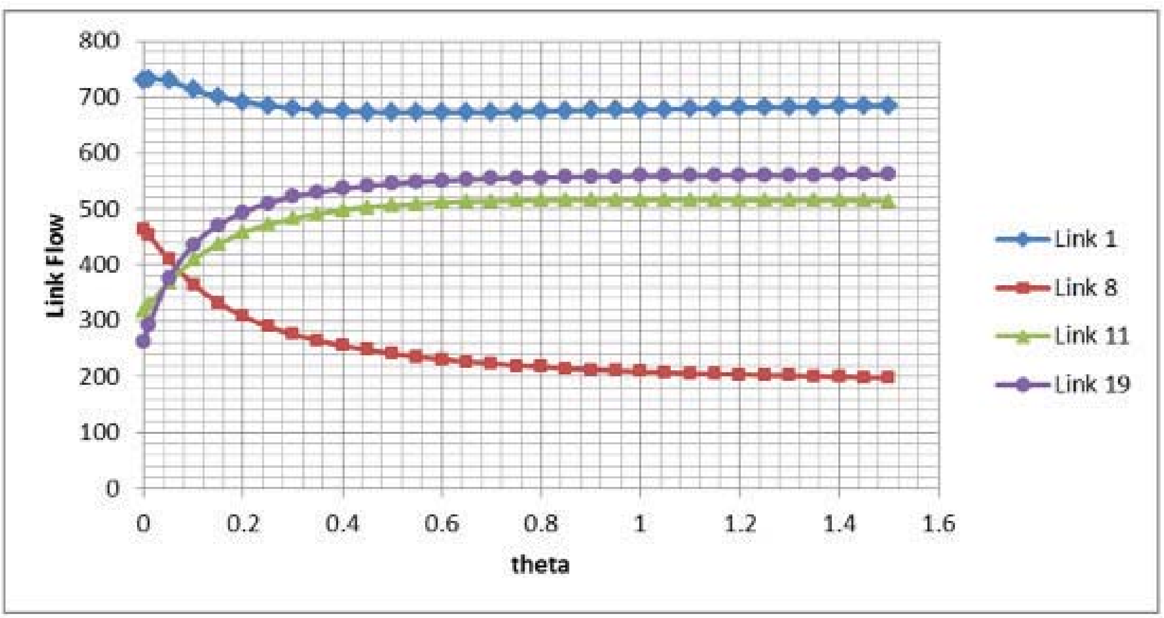


Figure 5. Effects of model specification and dispersion parameter calibration on predicted flow.

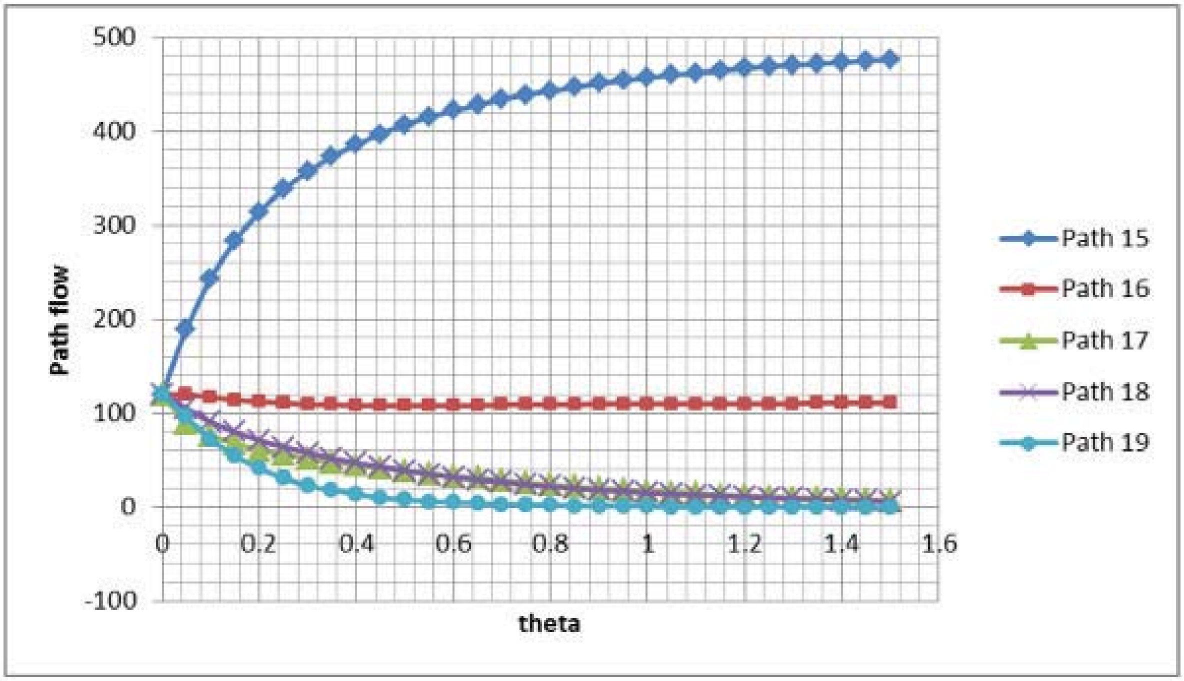


(a) MNL model.

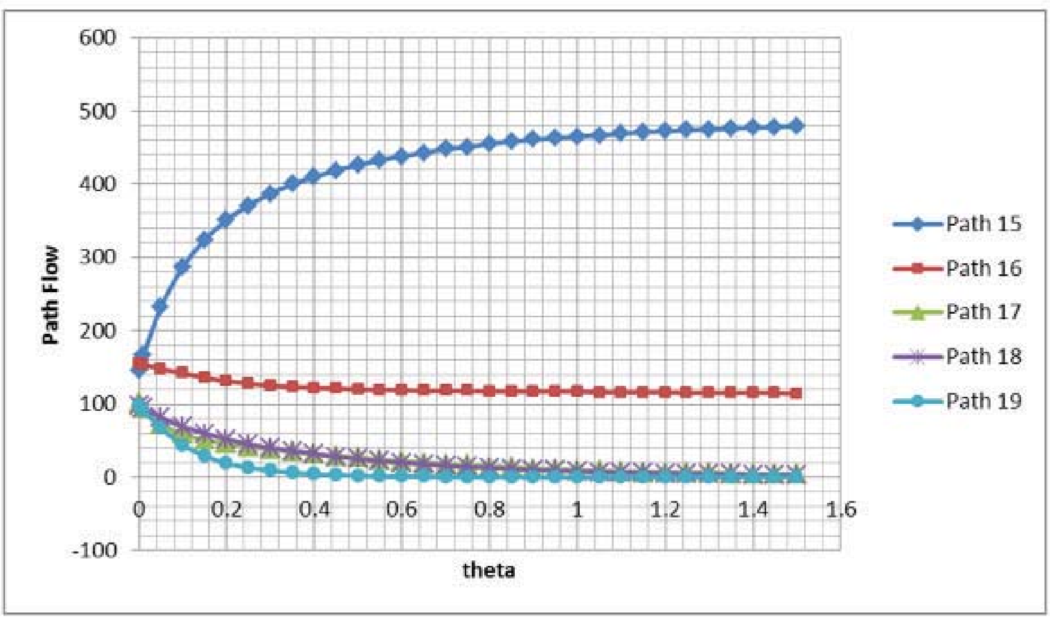


(b) PCL model

Figure 6. Sensitivity of link flows to the value of dispersion parameter (θ).



(a) MNL model



(b) PCL model.

Figure 7. Sensitivity of path flows to the value of dispersion parameter (θ).

This argument is made using typical values for input calibration parameters. Therefore, we made a sensitivity analysis to investigate the level of congestion at which SUE and DUE give rise to very similar results. The result is enlightening for determining a scope of SUE models. For this purpose, we designed some experiments in which the vector of travel demand was multiplied by a factor of ƛ to change the congestion level. As illustrated in Figure 8, DUE and SUE get similar in congested networks, but the congestion level at which the two methods become quite similar is different when the dispersion parameter is estimated. A high degree of similarity is shown having a typical value of one for θ, even for the base demand vector of our example. While, DUE and SUE are yet different setting $\theta=0.15$ and of λ=3. Even for λ=40, our experience revealed that marginal differences still exist between the two approaches. Furthermore, PCL and DUE outputs are compared in Figure 9 for different levels of congestion, and a dispersion parameter of 0.15. Link flows of DUE are regressed versus PCL-SUE model. The values of estimated intercept, slope, and their 95% confidence intervals along with the coefficient of determination (R^2)are illustrated in Figure 9. According to this figure, as λ increases, the intercept becomes more statistically insignificant and the slope is estimated more precisely. A general increase in R^2, on the other hand, indicates an increase in similarity between SUE and DUE results as the network becomes more congested. Our finding in this area, however, is somewhat different than what was stated by Sheffi and Powell [1981]. According to their statement, one can conclude that even in moderately congested networks, SUE and UE approaches both lead to quite similar results which in fact questions the need for resorting to SUE models and undertaking their more computational difficulties. Our finding, however, is rather different and shows a considerable difference between UE and SUE even for highly congested conditions. The source of this disagreement between the two conclusions is indeed in specification of the input parameter.

4.2 Multi-Criteria Model

The multi-criteria route choice model requires income of the travelers, in addition to travel time and cost, to perform the assignment. To obtain this information, travelers are divided into four income groups (<5, 5-10, 10-20, and >20 million Iranian Rials), each of which has an exponential distribution, assuming the average monthly income of 10 and 15million Iranian Rials in zone 1 and 4, respectively.

Equilibrium flows are obtained by a path-based stochastic equilibrium algorithm, the steps of which are as follows. This is a modification of MSA algorithm [Sheffi and Powell, 1982] for expanding utilities to multi-criteria case and differentiating classes of income. The adaptation of the algorithm has originally been made in this study.

Step 0: *Initialization:*

i. Compute monetary path costs (c_k^{rs}), having path lengths (l_k^{rs}), fuel price (FP), average rate of fuel consumption (FC), and path tolls,

$$toll_k^{rs} = \sum_a Toll_a . \delta_{ak}^{rs} \quad \forall\, rs, \forall k \in \mathbb{K}^{rs}$$
$$c_k^{rs} = (FP \times FC \times l_k^{rs}) + toll_k^{rs} \quad \forall\, rs, \forall k \in \mathbb{K}^{rs}$$

ii. Set each link's travel times equal to free-flow times $(T_a = T_a^0 \ \ \forall a)$, and compute initial path times $(t_k^{rs(0)} = \sum_a T_a^{(0)} \delta_{ak}^{rs} \ \ \forall\, rs, \forall k \in \mathbb{K}^{rs})$.

iii. Compute representative utility of each path for each income group (i):

$$V_{k,i}^{rs(0)} = \alpha t_k^{rs(0)} + \beta \frac{\ln(c_k^{rs})}{I_i} \quad \forall\, rs, \forall k \in \mathbb{K}^{rs}, \forall i$$

iv. Perform a stochastic network loading to obtain initial flows . $(f_{k,i}^{rs(0)} \ \ \forall\, rs, \forall k \in \mathbb{K}^{rs}, \forall i)$

v. Compute:

$$f_k^{rs(0)} = \sum_i f_{k,i}^{rs(0)} \quad \forall\, rs, \forall k \in \mathbb{K}^{rs}.$$

vi. Set counter $n = 1$.

Step 1: *Update*:

i. Compute link flows:

$$x_a^{(n)} = \sum_{rs} \sum_k f_k^{rs(n)} \delta_{ak}^{rs} \quad \forall a$$

ii. Update link travel times:

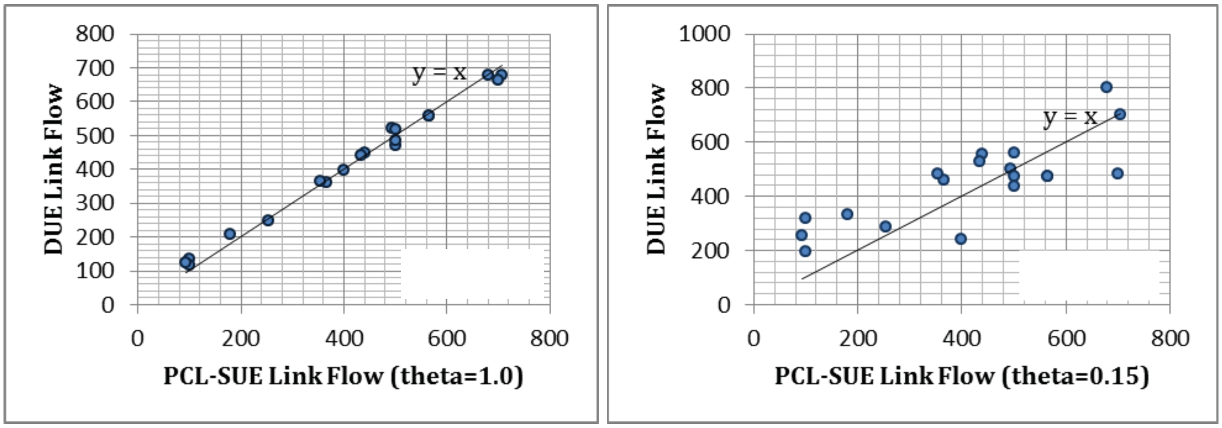
$$T_a^{(n)} = T_a\left(x_a^{(n)}\right) \quad \forall a$$

iii. Update path travel times:

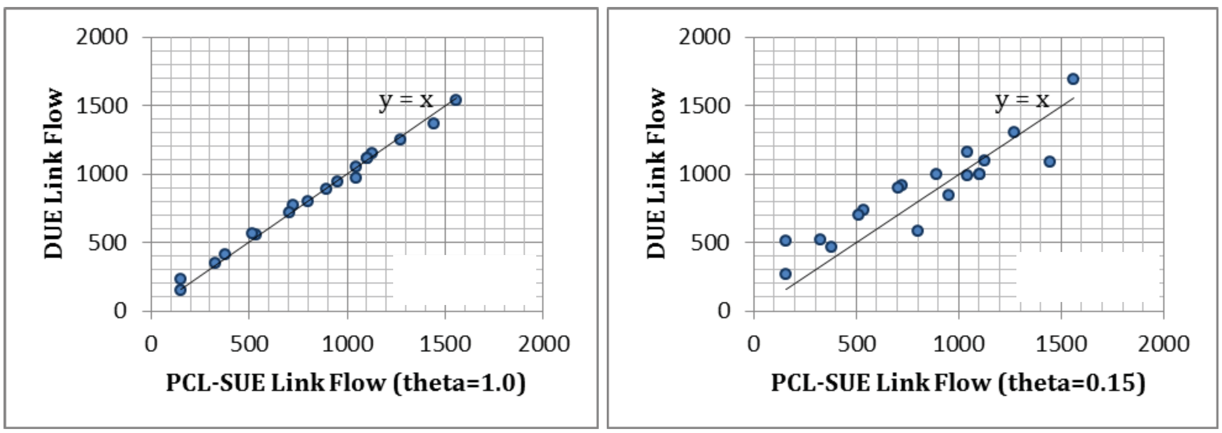
$$t_k^{rs(n)} = \sum_a T_a^{(n)} \delta_{ak}^{rs} \quad \forall\, rs, \ \forall k \in \mathbb{K}^{rs}$$

iv. Update path utilities:

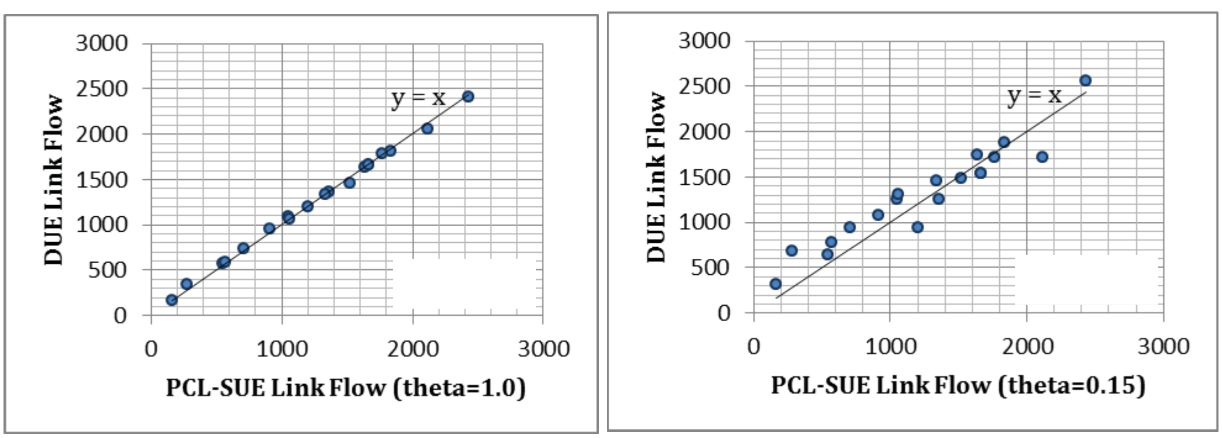
$$V_{k,i}^{rs(n)} = \alpha t_k^{rs(n)} + \beta \frac{\ln(c_k^{rs})}{I_i} \quad \forall\, rs, \ \forall k \in \mathbb{K}^{rs}, \ \forall i$$



(a) Original demand : $\lambda = 1$

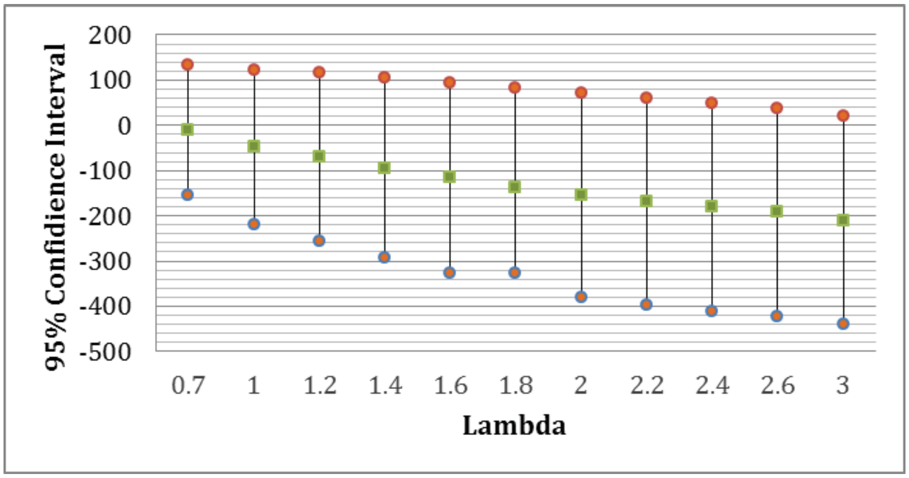


(b) Doubled demand : $\lambda = 2$

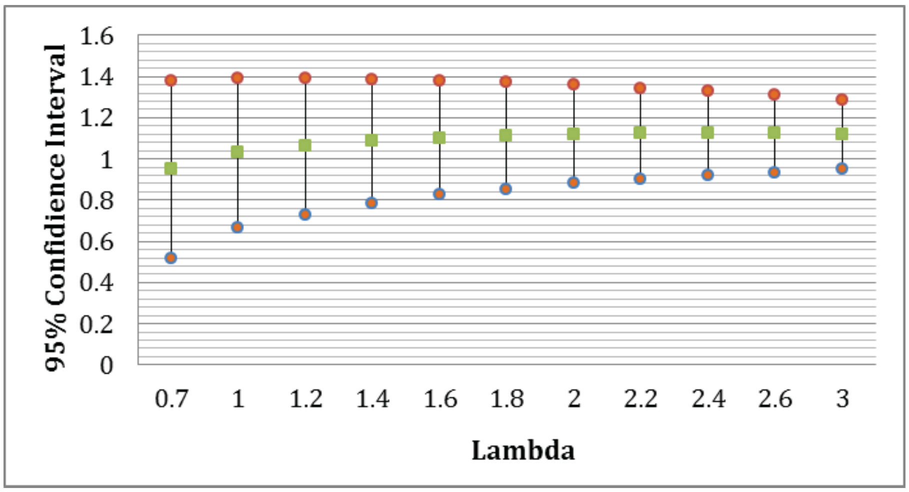


(c) Tripled demand : $\lambda = 3$

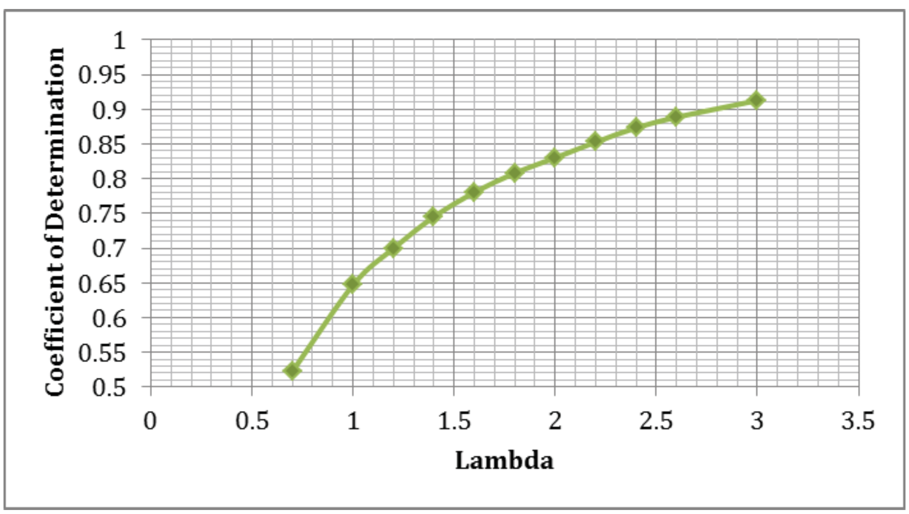
Figure 8. Comparison of UE and SUE for different congestion ocenarios



(a) Confidence interval for intercept.



(b) Confidence interval for slope



(c) Coefficient of determination (R^2)

Figure 9. Regression of UE versus SUE flows for different levels of congestion

Step 2: *Direction Finding:*

i. Compute path choice probabilities $(P_{k,i}^{rs(n)})$ and perform a stochastic network loading to compute auxiliary path flows $(h_{i}^{rs(n)})$:

$$h_{k}^{rs(n)} = \sum_{i} h_{k,i}^{rs(n)} = \sum_{i} (q_{i}^{rs} \times P_{k,i}^{rs(n)})$$

Step 3: *Move:*

i. Compute path flows as convex combinations of auxiliary flows and existing flows:

$$f_{k}^{rs(n+1)} = f_{k}^{rs(n)} + \frac{1}{n}\left(h_{k}^{rs(n)} - f_{k}^{rs(n)}\right)$$

Step 4: *Convergence Check.*

i. Compute RMSE as a measure of convergence:

$$RMSE = \sqrt{\left(\frac{1}{\sum_{rs} \mathcal{K}^{rs}}\right) \sum_{rs} \sum_{k} \left(h_{k}^{rs(n)} - f_{k}^{rs(n)}\right)^{2}}$$

ii. If $RMSE < \varepsilon_0$ or $n > N_0$, then set the existing flows as SUE, otherwise, set $n = n + 1$ and go to step 1. ε_0 and N_0 are predetermined constants.

Tables 4 and 5 compares the equilibrium traffic flow of stochastic logit-family traffic assignment models based on the estimated multi-criteria path utility. The multi-criteria assignment is sensitive to travel cost of the travelers, and therefore, can predict effects of a wide range of pricing policies. For illustration purposes, we have set a fixed toll of 10, 10, and 20 thousands Iranian Rials, respectively, for links 5, 7 and 8. This pricing scheme is denoted by Tolled and its equilibrium flows are compared to the Base condition, in Tables 4 and 5. Paired combinatorial logit model, for example, predicts a total travel time of 108,271 minutes, and a total vehicle-kilometers of 59,733 (setting an average occupancy rate of 1.2) for the base condition. According to the PCL forecasts, total travel time and total vehicle-kilometer is expected to improve by 1.63 and 0.2 percent respectively under the aforementioned pricing policy.

Table 4. Equilibrium link flows of multi-criteria models

Link	DUE	MNL		C-logit		PSL		CNL		PCL	
		B*	T**	B	T	B	T	B	T	B	T
1	706	737	721	736	723	741	728	741	719	732	715
2	494	463	479	464	477	459	472	459	481	468	485
3	100	430	390	400	359	422	380	374	331	379	337
4	700	370	410	400	441	378	420	426	469	421	463
5	440	686	551	616	491	642	514	658	505	636	493
6	366	480	560	520	591	521	595	457	545	475	560
7	354	586	524	556	490	556	491	557	493	569	504
8	180	392	302	317	239	335	254	344	245	340	245
9	100	221	188	234	198	219	185	220	183	238	201
10	254	365	336	321	292	337	306	338	311	331	303
11	500	393	391	442	437	430	426	435	430	434	429
12	500	474	549	442	507	463	534	457	539	460	540
13	566	377	421	478	524	435	481	427	475	436	483
14	680	866	851	759	747	798	788	800	784	800	785
15	500	607	609	558	563	570	574	565	570	566	571
16	434	623	579	522	476	565	519	573	525	564	517
17	94	292	275	257	239	249	232	243	233	272	257
18	400	171	204	207	239	210	240	215	248	195	228
19	566	377	421	478	524	435	481	427	475	436	483

* Base

** Tolled

Table 5. Equilibrium path flows of multi-criteria models

OD	Path	Link Sequence	DUE	MNL		C-logit		PSL		CNL		PCL	
				B*	T**	B	T	B	T	B	T	B	T
1-2	1	2-18-11	400	171	204	207	239	210	240	215	248	195	228
	2	1-5-7-9-11	0	52	38	45	32	42	30	46	31	48	35
	3	1-5-7-10-15	0	30	24	25	19	24	19	27	21	28	22
	4	1-5-8-14-15	0	32	22	27	18	26	18	29	19	32	21
	5	1-6-12-14-15	0	24	28	25	28	31	34	24	29	25	29
	6	2-17-7-9-11	0	40	39	31	30	29	27	25	24	33	32
	7	2-17-7-10-15	0	24	24	18	18	18	17	15	16	18	18
	8	2-17-8-14-15	0	27	21	21	16	20	15	18	14	22	16
1-3	9	1-6-13-19	366	252	296	345	390	314	358	296	344	306	352
	10	1-5-7-10-16	203	122	102	101	81	109	89	125	97	112	89
	11	1-5-8-14-16	136	127	91	95	65	101	70	119	76	109	70
	12	1-6-12-14-16	0	97	120	73	89	92	112	74	102	73	98
	13	2-17-7-10-16	50	96	103	95	100	94	98	92	107	100	109
	14	2-17-8-14-16	44	105	88	91	75	89	74	93	74	100	82
4-2	15	2-12-14-15	500	235	271	253	291	243	280	277	316	279	317
	16	3-5-7-9-11	100	130	111	159	136	149	128	149	128	157	134
	17	3-5-7-10-15	0	79	72	69	62	74	67	65	59	61	54
	18	3-5-8-14-15	0	87	69	71	55	81	64	70	53	64	46
	19	3-6-12-14-15	0	68	77	49	56	53	60	39	45	40	49
4-3	20	4-13-19	200	97	98	113	114	99	101	112	112	106	107
	21	4-12-14-16	0	38	41	34	36	35	38	38	42	37	40
	22	3-6-13-19	0	27	27	20	20	22	22	19	19	24	24
	23	3-5-7-10-16	0	13	11	14	12	18	16	14	12	13	12
	24	3-5-8-14-16	0	14	11	13	10	17	13	13	10	13	10
	25	3-6-12-14-16	0	11	12	7	8	8	9	5	5	7	8

* Base

** Tolled

5. Summary, Conclusions and Future Research Directions

5.1 Summary

A range of logit-type stochastic route choice models from the standard logit and its modifications to GEV-based models, in which theoretical deficiencies of early models are addressed to a possible extent, were discussed.

A univariate and a multi-criteria logit path utility function was estimated based on experimental data and were applied to a stochastic logit-based TA model. Significance of calibrating the dispersion parameter in a univariate route choice model was emphasized and a heuristic method was proposed to estimate this parameter. The proposed estimation method does not require choice data and hence, overcomes the problem of choice-set generation, the main challenge which makes estimation of route choice models highly challenging. The proposed estimation method, however, utilizes a simplifier underlying assumption which makes us regard the method as an approximate but practical approach of calibration.

5.2 Conclusion

This study was primarily set out to emphasize the importance of estimation in probabilistic route choice modeling, which has received much less attention than model specification in the literature. The chief findings of this study can be outlined as follows:

- Running different stochastic models of TA with the common value of input parameter θ=1 revealed that it would not lead to a considerably different result than DUE approach. Accordingly, application of this category of TA models in this way could not be practically justifiable.
- For a specific value of dispersion parameter, our investigation showed that there is not a highly significant difference between simple and advanced choice models in STA. This indicates that simple modifications of MNL model, such as C-logit and PSL—which simply addresses path correlation without bringing about

too much computational difficulty in analysis— could suffice for many planning purposes.

• Comparison also showed that estimation of the input parameter of SUE models on TA outcome is far more influential than that of utilization of theoretically appealing choice models. As a result, in real planning it seems that analyst has to give more priority to precise estimation of the parameters than to selection of a complicated model. In other words, we found that calibration of the route utility based on the experimental data is more crucial than adapting advanced specifications for the choice model. Hence, application of advanced models in this area seems not to be justifiable without precise calibration.

• A marginal investigation, as a sensitivity analysis, was also conducted showing that in spite of the former statements made in previous studies [Daganzo and Sheffi, 1977; Sheffi and Powell, 1981], even in high level of congestion there is considerable differences between the prediction of UE and SUE models for network flow pattern.

5.3. Future Research Directions

This study is one of the early efforts that estimates path utility based on experimental data, and has certain limitations that deserve more investigation in future studies:

• The deployed traffic assignment model is unimodal with fixed demand.

• Path utility function could encompass more explanatory variables, should a rich set of data and more complicated traffic assignment routines were available.

• Random taste variation and correlation over repeated choices can simultaneously be accounted for in a mixed discrete choice specification.

• Random utility theory is capable of evaluating the effects of supply management policies by introducing the concept of consumer surplus (CS). In traditional approaches, the performance of network and the response of travelers to changes in the system, such as road pricing and new road designs, were conducted by overall measures such as total travel time. While these overall measures more evaluates the consequences from the supplier point of view than users, the CS measure can not only consider the problem from travelers' viewpoint, but also can discriminate between the effects of policies on different groups of users which can be crucial for equity considerations. Evaluation of consumer surplus variation due to changes in the traffic system, however, is challenging for non-linear-in-variables models, and is open to further research.

• In addition, application of the studied SUE models in real-sized networks will introduce two further issues to the problem: path generation and optimization of the step-size in the convex combination procedure, which both were beyond the scope of this study. In spite of some previous researches in the literature, these two problems also require more investigations.

6. Acknowledgement

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7. References

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