A Vehicle Routing Problem for Modeling Home Healthcare: a Case Study

Mona Issabakhsh [1], Seyyed-Mahdi Hosseini-Motlagh [2], Mir-Saman Pishvaee [3], Mojtaba Saghafi Nia[4]

Received: 06. 09. 2016 Accepted: 08. 01. 2017

Abstract

Compared to center-based hemodialysis (HD), peritoneal dialysis (PD) has many advantages among which cost effectiveness and comfort of patients are the most important ones. On the other hand the number of PD patients is so small and even decreasing worldwide due to difficulties of this mode of dialysis. Therefore to encourage dialysis patients to choose PD, health system must provide a proper set of care services proportional to special needs of these patients. Applying operations research (OR) as an efficient mathematical tool and considering the realistic assumptions such as travel time uncertainty, first a Vehicle Routing Problem model is presented to serve PD patients at home with special logistic services. Thereafter, based on the criticality of timeliness in providing healthcare service, a conservative method called robust optimization, is applied to handle time uncertainty. The corresponding results show that the proposed method at the maximum uncertainty level has less than 30% variations in results and in comparison with the deterministic model increases the costs only by 1.2%. With small variations in results, this model can handle the travel time uncertainty properly and is highly appropriate and practical to be used in a sensitive application like healthcare where timeliness is crucial.

Keywords: Home healthcare, peritoneal dialysis, operations research, vehicle routing problem, uncertainty, robust optimization.

Corresponding author E-mail: motlagh@iust.ac.ir

[1] MSc Student, School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran
[2] Assistant Professor, School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran
[3] Assistant Professor, School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran
[4] Instructor, School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

1. Introduction

The process of removing waste fluid and products from body is called dialysis. It becomes necessary when kidneys fail to filter the blood properly. In hemodialysis (HD) an external machine is used as a filter while in peritoneal dialysis (PD) the Peritoneum is applied instead of filter.
Many researchers have compared PD and HD from different aspects such as economic efficiencies, mortality rates and convenience. According to economic evaluations, annual treatment costs of PD are 30% – 40% less than center-based HD costs [Klarenbach and Manns, 2009]. Patients using PD method, also would have better chance of overall and early survival [Perl et al., 2015]. Not only is PD cost effective for either patients or National Health Service (NHS), but also it has the potential to enhance the life quality of patients [Treharne et al., 2014]. Moreover, PD is flexible in the sense that based on patient condition it can be done at home without anyone's help and does not need regular commute to a dialysis center. Therefore, satisfaction and autonomy can lead patients with end-stage renal disease to prefer PD to HD. As the most important deciding actor, patients choose between PD and HD based on their preferences and special conditions [Perl et al. 2015, Liem et al. 2007, McDonald et al. 2009, Rubin et al. 2004, Cheraghi et al. 2017].
Although PD has many advantages, this method usage decreased in developed countries by 5.3% from 1997 to 2008. According to statistics revealed in 2008, only 11% of renal failure patient population used PD [Jain et al., 2012]. The main reason why patients prefer HD may be that it is done only three times a week compared to PD, which must be done 4 to 5 times every day or automatically each night by automated peritoneal dialysis (APD) machine. Another reason could be that while HD is done in a dialysis center where trained nurses monitor the whole procedure, PD have to be done at home without any professional aid [Rubin et al., 2004]. Reported Statistics from 2009 to 2014 in United States show that among every 128 patients 16 percent withdraw from PD because of lack of service [Workeneh et al., 2015]. A large percentage of renal failure patients are elderlies, who have worse mental and physical conditions and need special care [Stevens,Viswanathan and Weiner, 2010, Yu et al., 2016]. Based on a research done in Singapore, 50% of elderly PD patients were not capable of doing their daily treatment tasks independently [Griva et al. 2014, Riahi et al. 2013]. Taking all the aforementioned matters into account, it can be concluded that to encourage renal patients to choose PD, heath care systems should provide appropriate home care services for them.
Having provided a proper set of services for patients with end-stage renal disease at home, PD would seem as an ideal choice for them. Serving PD patients at home can be considered as a Home Health Care (HHC) issue. By constructing an appropriate model based on special needs of PD patients and optimizing the function of this model, PD can be replaced by HD and finally such a model can be well used in reality by HHC companies.
The contributions of this paper to the present literature are that:

1. The first practical operations research model to serve PD patients is presented.
2. The model is constructed to satisfy PD patients needs to both health commodities (PD packs, drugs etc.) and health staff such as nurses and technicians of APD machines. No other HHC research has considered the routing of health commodities and staff together.
3. A hard worst case robust optimization approach is presented to face the uncertainty in vehicle travel time, in order to guaranty the timeliness of serving PD patients. No HHC research has applied this robust optimization approach.

This model is both conservative and cost effective in the sense that it contains realistic assumptions about time uncertainty in service giving and has much less costs compared with center-based HD. The remainder of this paper is organized as follows: In following section the relevant literature is reviewed. In Section 1.1, the proposed mathematical model is demonstrated. Section 2 and 3 concern with computational results and finally, conclusions and a number of directions for future research are given in Section 4.

1.1 Literature Review

Nowadays it is more fruitful to serve patients at home. As it was stated in Bertels and Fahle [Bertels and Fahle, 2006], services such as visiting patients and treating them at home are called HHC. According to the growing trend of antimicrobial resistance and the risk of getting infection in hospitals due to the weakened immune system of patients, HHC would be an appropriate option for them. Besides, providing HHC service is much less expensive than patient hospitalization and related hoteling costs. These are not the only advantages of HHC and other benefits such as punctuality and desirability of service, time saving, reducing traffic load and patients' comfort can also be also noted [Cappanera, Scutellà and Visintin, 2014, Lanzarone and Carello, 2014, Jokar and Hosseini-Motlagh 2015].

Despite the importance of HHC just a limited number of papers have considered this issue. Ramus et al. [Torres-Ramos et al., 2014] presented a mathematical model for HHC scheduling and routing problem as an integration of nurse rostering problem and vehicle routing problem (VRP). In this study a mixed integer linear programming model based on time windows limitation and other related constraints is constructed and the problem is solved using exact mathematical methods. By extending the Barnhart et al. [Barnhart et al., 1998] and Bredström and Rönnqvist [Bredstrom and Rönnqvist, 2007] models, Done et al. [Dohn et al., 2008] solved the home care crew scheduling problem with branch-and-price method. In this mixed integer model, continuous variable is used to schedule visits while binary variable is applied to assign visits to healthcare workforce. Kergosien, Lenté and Billaut [Kergosien, Lenté and Billaut, 2009] solved the crew routing problem of HHC staff using branch-and-bound approach. The effect of pattern generation policies on scheduling and routing decisions in HHC was surveyed by Cappanera and Scutell [Cappanera and Scutellà, 2013]. In this investigation the pattern was equivalent to a set of scenarios which incorporate different decision levels.

Most of articles in the field of HHC applied heuristic and meta-heuristic methods to optimize large-scaled problems. Triki, Garaix and Xie [Triki, Garaix and Xie, 2014] used a two-stage method to solve the periodic HHC problem with time windows. They applied a MIP-based neighborhood search and tabu search (TS) algorithm to solve and improve the proposed model. Cheng and Rich [Cheng and Rich, 1998] surveyed the nurse scheduling problem and applied two heuristic methods to solve this problem. Trautsamwieser, Gronalt and Hirsch [Trautsamwieser, Gronalt and Hirsch, 2011] proposed the daily scheduling of HHC service in the time of natural disaster in Austria. Variable neighborhood search (VNS) was applied to solve the proposed model. Begur, Miller and Weaver [Begur, Miller and Weaver, 1997] designed a decision support system for the United States HHC staff planning problem. Clarke and wright savings algorithm and the nearest neighborhood heuristic method was applied to optimize routes. [Nikkhah Qamsari et al., 2017, Cheraghi and Hosseini-Motlagh, 2017, Majidi et al., 2017]

Some articles separated the allocation and routing decisions in HHC claiming that handling these two simultaneously is computationally harder than a two-phase manner. Yalçındag et al. [Yalçındağ et al., 2014] considered the routing and assignment problem in HHC by a two-phase approach. They estimated the travel time by means of Kernel regression. Bertels and Fahle [Bertels and Fahle, 2006] firstly found a feasible solution to HHC nurse rostering and routing problem applying constraint programming and then optimized the solution using simulated annealing (SA) and TS meta-heuristics. Nickel, Schröder and Steeg [Nickel, Schröder and Steeg, 2012] considered the allocation and routing problem of HHC in Germany. They applied a two-stage approach for weekly planning, in which a heuristic method was applied to construct an initial solution and thereafter the obtained solution was optimized. Braekers et al. [Braekers et al., 2016] studied a multi-objective routing and scheduling problem in HHC. They used a combination of multi-directional local search and large neighborhood search (LNS) to solve the problem. In contrast with previous works, Liu et al. [Liu et al., 2013, Liu, Xie and Garaix, 2014] proposed the ordinary and periodic routing problem of commodities in HHC. They considered two nodes other than patients' homes and depots including hospital and laboratory in vehicle tour which provided special pickup and delivery services for patients. They solved the proposed problem using genetic algorithm (GA) and TS meta-heuristics. They also handled the periodic problem with a combination of multi-directional local search heuristic and LNS. Kergosien, Ruiz and Soriano [Kergosien, Ruiz and Soriano, 2014] study was also different from HHC literature; they proposed the problem of gathering biological samples from patients' homes as a VRP in HHC. They constructed an integer programming model and solved it by TS and VNS meta-heuristics.

HHC companies perform many daily logistic activities such as providing repair and maintenance services for medical equipment (which are vital for patients and cannot be transferred to a repair shop), delivering medicines to patients' homes, gathering unused devices and medicines, picking up laboratory samples and bringing physicians to patients' bedside [Liu et al., 2013, Kergosien, Ruiz and Soriano, 2014]. Therefore, to serve patients at home, a given HHC company is faced with a VRP with some specific features and constraints such as pickups and deliveries and time windows. Patients are usually dispersed and far apart from each other. In order to reduce service costs along with enhancing service level, it is better to serve all the pickup and delivery needs of each patient simultaneously with one vehicle. The VRP with simultaneous delivery and pickup and time windows (VRPSDPTW) was introduced by Min [Min, 1989] for the first time. In this model along with the basic assumptions of vehicle routing problem, customers' delivery and pickup demands are responded simultaneously within a specific time windows.

Timeliness of the service is a critical issue in health care and violation from the due time is so costly that it is not allowed, on the other hand keeping patients waiting can result in dissatisfaction and customer loss. Another point is that in reality travel time of vehicles is uncertain. This uncertainty can be caused by traffic load, car accidents, bad weather conditions, etc. Therefore providing a timely service would be challenging and needs careful management. This means that an appropriate method must be chosen to face uncertainties; A method which is able to minimize the violations from due times and keeps costs at a desirable level [Castillo-Salazar, Landa-Silva and Qu, 2014, Holte and Mannino, 2013]. Table 1 shows the brief summary of HHC logistics articles introduced in this section. Literature gap is more apparent in this table.

In this investigation a special mathematical model called robust vehicle routing problem with simultaneous delivery, pickup and time windows (RVRPSDPTW) is proposed in order to plan and optimize giving home care service to PD patients. The developed model would be appropriate for any home care company which serves PD patients and wishes to optimize its activities and minimize routing costs as well as any tardiness in service giving. The basis of this model is elicited from Liu et al. [Liu et al., 2013] who have designed an HHC model for routing health commodities. However, in this paper we have considered the routing of both health commodities and staff. Additionally, we have added some realistic features to our model such as the uncertainty in the travel time of the vehicle and applied a hard worst case robust model to face this uncertainty. To the best of our knowledge, none of the researches in the field of HHC has applied such a robust model.

The contribution of this paper is threefold: (1) in the proposed model soft time windows (SFW) is taken into account, which is more realistic by allowing violation from time windows bounds and controlling this violation with penalties in objective function [Zare-Reisabadi and Mirmohammadi, 2015]. Also the limit on each tour time length as a constraint in the model is considered since home care companies have limited work shifts in reality; (2) in this study the uncertainty in the travel time of vehicles which is a realistic assumption and is critical to be considered and handled in providing healthcare services was perceived. Uncertainty is handled with a hard worst case (HWC) robust method with no constraint violation which would be the best model for health service; (3) the model is designed and moderated with respect to the special needs of PD patients.

Table 1. Literature review summary

No.	Paper	Year	Routing			Considering Uncertainty		application
			staffs	commodities	both	yes	no	
1	Done et al. [21]	2008	*				*	-
2	Kergosien et al. [24]	2009	*				*	-
3	Cappanera and Scutella [25]	2013	*				*	-
4	Ramus et al. [19]	2014	*				*	-
5	Begur et al. [28]	1997	*				*	-
6	Cheng and Rich [26]	1998	*				*	-
7	Bertels and Fahle [3]	2006	*				*	-
8	Trautsamwieser et al. [27]	2011	*				*	natural disaster
9	Nickel et al. [29]	2012	*				*	-
10	Yalçındag et al. [20]	2014	*				*	-
11	Triki et al. [33]	2014	*				*	-
12	Braekers et al. [30]	2016	*				*	-
13	Liu et al. [4]	2013		*			*	-
14	Liu et al. [32]	2014		*			*	-
15	Kergosien et al. [31]	2014		*			*	-

2. Methods

With respect to the advantages of PD compared to center-based HD, a RVRPSDPTW model is presented to serve special needs of PD patients. Applying this model, HHC companies can serve PD patients with the lowest costs. In this model two types of facilities called dialysis center (DC) and laboratory (Lab) are considered to provide special service for PD patients. Patients can ask for four types of demands:

1. Requesting medicines and PD packs from HHC company drug store or depot (patients belong to D_1 set).
2. Asking for physicians, nurses and APD machine technicians (D_2 set)
3. Requesting gathering unused or close-to-corruption medicines and damaged devices and taking them to company depot (P_1 set).
4. Asking for gathering biological samples like blood and urine samples and taking them to Lab (P_2 set).

Type 1 deliveries are picked from company depot at the start of each route and type 1 pickups are also carried to the depot at the end of the route. Type 2 deliveries and also physicians and nurses must be picked up from DC and type 2 pickups must be taken to Lab. As a result, routes have to be constructed with respect to the types of deliveries and pickups. For example if a patient has type 2 deliveries, it should be visited after the DC in the route and if it has type 2 pickups, it must be seen before Lab.

Nodes 0 and $n+1$ also represent the initial and final depots respectively. Each route stars from depot 0 and finally ends at depot $n+1$. Middle nodes include patients' homes, DC and Lab. Each patient is served by just one vehicle and each vehicle visits each node only once per tour.

Delivering commodities, physicians, nurses and technicians are done simultaneously. It is assumed that all k vehicles are homogeneous and have same capacities equal to Q and all commodities and personnel can be transferred by the same type of vehicle. For instance the tour of a given vehicle can be like what is shown in Figure 1. The vehicle leaves the depot 0 with D_1 commodities and after (or before) serving D_1 patients, goes to DC to load D_2 demands. After visiting D_2 patients and gathering P_2 patient's returning items, it heads toward Lab. Finally after serving P_2 patients it returns to depot $n+1$.

It should be mentioned that each patient can have all four types of demands. The basic assumptions are as follows:

1. Each tour starts from depot 0 and ends at depot $n+1$ and each node is visited only once.
2. All the demands of each patient are served by a single vehicle.
3. D_2 demands are not counted in calculating the capacity of vehicle.
4. If there is at least one D_2 demand in a tour, a physician / technician accompanies the driver of the vehicle during the tour.
5. Before serving D_2 patients each vehicle goes to DC.
6. Each vehicle goes to Lab after visiting P_2 patients.
7. The service time of the picking up and delivering commodities are equal to zero and is just considered in delivering D_2 demands.

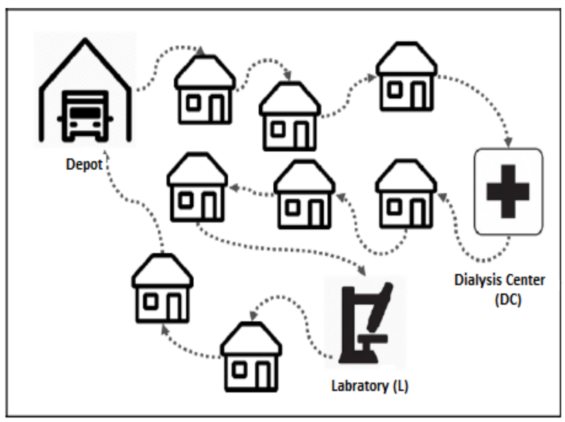

Figure 1. A potential route of a given vehicle

The following notations are used to model the aforementioned problem.

2.1 Sets and Indices

V: The set of all nodes $V = \{0,1,\ldots,n,n+1\} \cup \{DC, L\}$ (Nodes $0, n+1, DC$ and L represent the initial and final depots, Dialysis Center and Laboratory respectively.)

N: Set of patients' homes $N = \{1,\ldots,n\}$

i, j: Indices of patients $\quad i, j \in N$
D_1 : The set of D_1 patients.
D_2 : The set of D_2 patients.
P_1: The set of P_1 patients.
P_2: The set of P_2 patients.
K: The set of vehicles.

2.2 Parameters

Q: The capacity of vehicle.
C_{ij}: The cost of traveling from node i to node j.
t_{ij}: The travel time from node i to node j.
aa_i: The earliest allowed service time at node i.
bb_i: The latest allowed service time at node i.
st_i: The service time at node i.
d_{1i}: The amount of D_1 demand of i^{th} patient.
d_{2i}: The amount of D_2 demand of i^{th} patient.
p_{1i} : The amount of P_1 demand of i^{th} patient.
p_{2i} : The amount of P_2 demand of i^{th} patient.
T: The maximum time length of each tour.
M: A big number.
Δ: The penalty of constraint violation in realization models.
Ψ: The level of uncertainty

2.3 Decision Variables

x_{ijk}: A binary variable that if vehicle k goes directly from node i to node j will be equal to one; otherwise equal to zero.
B_{ik}: The time of beginning service at node i by vehicle k.
y_{ijk} : The amount of P_1 and P_2 pickups in (i, j) arc carried by vehicle k.
w_{ijk} : The amount of D_1 and D_2 pickups in (i, j) arc carried by vehicle k.
E_{ik}: The variable of earliness of vehicle in STW.
L_{ik}: The variable of lateness of vehicle in STW.

2.4 Deterministic Mathematical Model

$$\boldsymbol{Min} \sum_{i\epsilon V}\sum_{j\epsilon V}\sum_{k\epsilon K} c_{ij}x_{ijk} + \sum_{i\epsilon V}\sum_{k\epsilon K}(E_{ik} + L_{ik}) \tag{1}$$

s.t.

$$\sum_{j\epsilon V}\sum_{k\epsilon K} x_{jik} = 1 \qquad \forall i\epsilon N \tag{2}$$

$$\sum_{j\epsilon V} x_{ijk} \le \sum_{j\epsilon V} x_{jlk} \qquad \forall i\epsilon \mathrm{P}_2, k\epsilon K \tag{3}$$

$$\sum_{j\epsilon V} x_{jik} \le \sum_{j\epsilon V} x_{dcjk} \qquad \forall i\epsilon D_2, k\epsilon K \tag{4}$$

$$\sum_{i\epsilon V} x_{0,ik} \le 1 \qquad \forall k\epsilon K \tag{5}$$

$$\sum_{i\epsilon V} x_{i,n+1,k} \le 1 \qquad \forall k\epsilon K, \tag{6}$$

$$\sum_{j\epsilon V} x_{jik} = \sum_{j\epsilon V} x_{ijk} \qquad \forall i\epsilon N \cup \{l, dc\}, k\epsilon K \tag{7}$$

$$\sum_{j\epsilon V} x_{jik} \le 1 \qquad \forall i\epsilon\{l, dc\}, k\epsilon K \tag{8}$$

$$B_{jk} \ge B_{ik} + t_{ij} + st_i - M(1 - x_{ijk}) \qquad \forall i\epsilon V, j\epsilon V \setminus 0, k\epsilon K \tag{9}$$

$$\sum_{i\epsilon V}\sum_{k\epsilon K} y_{jik} - \sum_{i\epsilon V}\sum_{k\epsilon K} y_{ijk} = p_{1j} + p_{2j} \qquad \forall j\epsilon N \cup \{dc\} \tag{10}$$

$$\sum_{i\epsilon V}\sum_{k\epsilon K} w_{ijk} - \sum_{i\epsilon V}\sum_{k\epsilon K} w_{jik} = d_{1j} \qquad \forall j\epsilon N \cup \{dc, l\} \tag{11}$$

$$\sum_{i\epsilon V} w_{0ik} = \sum_{i\epsilon V}\sum_{j\epsilon D_1} x_{ijk}d_{1i} \qquad \forall k\epsilon K \tag{12}$$

$$\sum_{i\epsilon V} y_{ilk} - \sum_{i\epsilon V} y_{lik} = \sum_{i\epsilon V}\sum_{j\epsilon D_2} x_{ijk} p_{2i} \qquad \forall k\epsilon K \qquad (13)$$
$$B_{lk} \geq B_{ik} + t_{il} + st_i - M(1 - \sum_{j\epsilon V} x_{ijk}) \qquad \forall i\epsilon P_2, k\epsilon K \qquad (14)$$
$$B_{ik} \geq B_{dck} + t_{dci} - M(1 - \sum_{j\epsilon V} x_{ijk}) \qquad \forall i\epsilon D_2, k\epsilon K \qquad (15)$$
$$E_{ik} \geq aa_i \sum_{j\epsilon V\setminus\{0\}} (x_{ijk} - B_{ik}) \qquad \forall i\epsilon N, k\epsilon K, \qquad (16)$$
$$L_{ik} \geq B_{ik} - bb_i \qquad \forall i\epsilon N, k\epsilon K \qquad (17)$$
$$y_{ijk} + w_{ijk} \leq Q x_{ijk} \qquad \forall i,j\epsilon V, k\epsilon K \qquad (18)$$
$$\sum_{i\epsilon V}\sum_{j\epsilon V} x_{ijk}\ (t_{ij} + st_i) \leq T \qquad \forall k\epsilon K \qquad (19)$$
$$x_{ijk} \in \{0,1\} \qquad \forall i,j\epsilon V, k\epsilon K \qquad (20)$$
$$y_{ijk} \geq 0 \qquad \forall i,j\epsilon V, k\epsilon K \qquad (21)$$
$$B_{ik} \geq 0 \qquad \forall i\epsilon V, k\epsilon K \qquad (22)$$
$$w_{ijk} \geq 0 \qquad \forall i,j\epsilon V, k\epsilon K \qquad (23)$$
$$E_{ik} \geq 0 \qquad \forall i\epsilon N, k \in K \qquad (24)$$
$$L_{ik} \geq 0 \qquad \forall i\epsilon N, k \in K \qquad (25)$$

The first part of Equation (1) concerns with minimizing the sum of travel costs of vehicles while the second part minimizes deviations from the time windows bounds. Equation (2) guarantees the visit of each patient in each tour only once. According to Equations (3) and (4) vehicle visiting D_2 and P_2 patients should also visit DC and the Lab respectively. Constraints (5) and (6), state that each vehicle must start its tour from the depot and at last return to it. Equation (7) guarantees the equality of in-put and out-put flow in each node. According to Constraint (8) each vehicle in each tour can visit the Lab and DC at most once. Equation (9) is time continuity constraint of nodes; Equations (10) and (11) are pickup and delivery demands flow equations. Constraints (12) and (13) make sure that delivery demands of D_1 patients is gathered from the depot and P_2 pickup demands are delivered to the Lab respectively. Equations (14) and (15) ensure visiting DC before serving D_2 patients and giving service to P_2 patients before going to the Lab. Equations (16) and (17) are STW constraint. Equations (18) and (19) are capacity and tour time bound constraints respectively. At last, Constraints (20) to (25) concern with variable definition.

2.5 Robust Mathematical Model

Uncertainty can be defined as a situation in which available data and information is incomplete. Different procedures are introduced to cope with such situations among which robust is a risk-averse approach that is developed to be used in optimization models. Soyster [Soyster, 1973] introduced the first robust approach which was a HWC model with the highest conservativeness. In 1995, Mulvey et al. introduced a more flexible robust procedure for scenario-based optimization problems. Years later, Ben-Tal and Nemorovski [Ben-Tal and Nemirovski, 1998] and El Ghaoui, Oustry and Lebret [El Ghaoui, Oustry and Lebret, 1998] extended Soyster's [Soyster, 1973] procedure and developed new robust models based on convex programming. This was a significant step toward robust theory. Findings of this research later were gathered in Ben-Tal,El Ghaoui and Nemirovski [Ben-Tal,El Ghaoui and Nemirovski, 2009] book which now is known as one of the best resources of robust theory. Along the same research line, later Bertsimas and Sim [Bertsimas and Sim, 2004] and Fischetti and Monaci [Fischetti and Monaci, 2009] developed a worst case model with less conservativeness built on the basis of Soyster's [Soyster, 1973] work.

In general a robust decision is one that withstands the environmental uncertainty with minimum fluctuations. In optimization a robust solution must have feasibility robustness and optimality robustness simultaneously; the former has the meaning that it should be almost always feasible and the latter means it must have minimum difference from the optimal solution for all possible values of uncertain parameters. The higher the level of uncertainty in a model is, the more the conservativeness of the model and the higher the costs of the problem will be. Therefore, based on cost considerations and the sensitiveness of the application, different models should be applied [Pishvaee, Razmi and Torabi, 2012, Ghatreh Samani and Hosseini-Motlagh, 2017].

As it was mentioned in the previous parts, timeliness is a crucial issue in healthcare. Being late in giving healthcare service can be equal to serious damage to one's health or even death. The cost of delay in healthcare is substantial, therefore to address uncertainty in such situations, worst happenings or "worst case" must be considered. Thus, an appropriate method which handles the uncertainty with minimum or even no violations from constraints should be considered in health scope. Based on these considerations, a HWC convex model is used here to handle uncertainty in vehicle travel time.

Suppose ξ_j is a variable which constructs the uncertain part of each parameter by varying between $-\Psi$ and Ψ. Ψ shows the level of uncertainty.

$$-\Psi \leq \xi \leq \Psi \tag{26}$$

By considering U uncertainty set for uncertain parameters in a closed convex set, the aim is to find a set of solutions which remains feasible for any ξ. According to equation (27) in box uncertainty set, uncertain parameters are modeled based on infinity norm (U_∞) J_i shows the set of uncertain parameters.

$$U_\infty = \{\xi | \|\xi\| \leq \Psi\} = \{\xi_j | |\xi_j| \leq \Psi \ \forall j \epsilon J_i\} \tag{27}$$

Figure 2 demonstrates the geometric justification of box uncertainty set in two dimensions or in other words with two uncertain parameters in a given constraints. It can be seen that the uncertainty space between the two uncertain parameters in the left side of the Figure 1 is pictured to a square shown on the right side of the Figure 1.

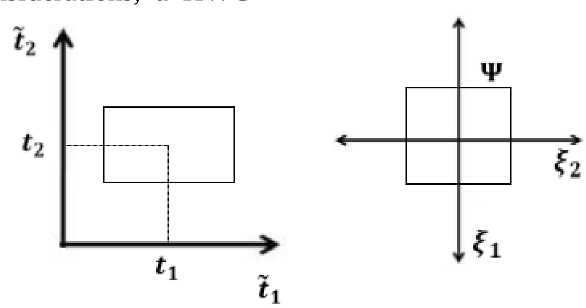

Figure 2. Geometric justification of box uncertainty set.

$$\tilde{t}_{ijd} = t_{ijd} + \hat{t}_{ijd}\xi_j \tag{28}$$

$$B_{jk} \geq B_{ik} + t_{ij} - M(1 - x_{ijk}) + \Psi\hat{t}_{ij} \qquad \forall i \epsilon V, j \epsilon V \setminus 0, k \epsilon K \tag{29}$$

$$B_{lk} \geq B_{ik} + t_{il} - M(1 - \sum_{j \epsilon V(S)} x_{ijk}) + \Psi\hat{t}_{il} \qquad \forall i \epsilon P_2, k \epsilon K \tag{30}$$

$$B_{ik} \geq B_{hk} + t_{hi} - M(1 - \sum_{j \epsilon V} x_{ijk}) + \Psi\hat{t}_{hi} \qquad \forall i \epsilon D_2, k \epsilon K \tag{31}$$

$$\sum_{i \epsilon V} \sum_{j \epsilon V} x_{ijk}\ t_{ij} + \Psi(\sum_{i \epsilon V} \sum_{j \epsilon V} x_{ijk}\ \hat{t}_{ij}) \leq T \qquad \forall k \epsilon K \tag{32}$$

The uncertain parameter (time of vehicles) can be separated into two parts of deterministic and uncertain constraint (28).

Constrains (9), (14), (15) and (19) are uncertain. According to Li, Ding and Floudas [Li, Ding and Floudas, 2011] and based on box uncertainty set, the robust counterpart of these constraints can be written as constraint (29)-(32).

Actually in box uncertainty set, Ψ percent of the uncertain part of the parameter is added to each uncertain constraint. This way, all the supplies won't be used in the process of solving the problem and a portion will be saved for potential fluctuations in uncertain parameters. For instance, a given HHC company must plan to serve its customers without any delay, despite the uncertainties in vehicles' travel time. To schedule such services and avoid lateness, some extra time must be considered.

3. Results

Next step is to solve the proposed model on a random instance to verify its functionality in real world for PD patients. Therefore the proposed model is encoded in the General Algebraic Modeling System (GAMS) software and solved on a 2-GHz dual core computer with 4-GB memory, based on Table 2 parameters.

The robust counterpart of the mentioned problem is solved considering uncertainty in travel time. The first phase in verifying a robust model is to set $\Psi = 0$. This way, the deterministic and robust models will be equivalent and are supposed to provide exactly the same answers. Besides, enhancing Ψ, the level of uncertainty in model increases and the objective function gets worse. Table 3 and Figure 3 show the details of this analysis.

Table 2. Parameters

Parameters	Quantity
c	Random coordinates- Euclidean distance
t	Likewise c
d_1, d_2, p_1, p_2	$\sim Uniform\{0, \dots, 40\}$
st	20 minutes
T	180
k	2
Q	100

Table 3. The comparison between the objective function of deterministic (D) and robust (R) models

Model type	Uncertainty Level (Ψ)	Objective Function value
D	-	293.4
R	0	293.4
	0.2	293.4
	0.4	293.8
	0.6	294.9
	0.8	296
	0.9	297.1

Comparing the objective function of deterministic model and the worst case robust model with $\Psi = 0.9$, we can see that the cost increases only by 1.2%. This increase of cost is relatively small in comparison to the conservativeness that the robust model provides against constraint violations. It becomes even more negligible in health scope where nothing is more important than serving patients without any delay.

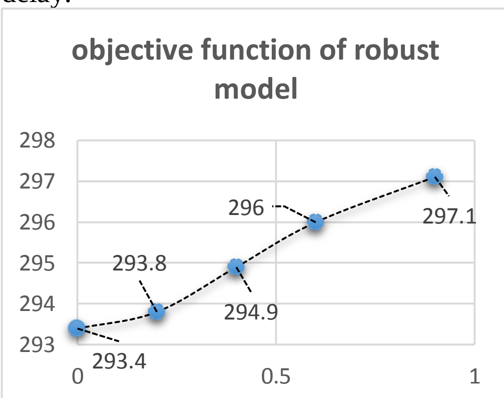

Figure 3. The objective function of R models with different uncertainty levels

4. Discussion

According to the robust logic, it is expected that the robust model results in lower constraint violations compared to deterministic model. Here in a health care case, constraint violation means tardiness in serving patients which could cause severe damage to one's health and is not allowed. The higher the uncertainty level goes, the more conservative the model gets and consequently the lower constraint violations will be. Realization models are used to validate the proposed robust procedure. A realization model is a model likewise the deterministic model in which a penalty variable is considered for each constraint that in case of constraint violation adds some extra cost to the objective function [Pishvaee, Rabbani and Torabi, 2011, Pishvaee, Razmi and Torabi, 2012].

First, five nominal models are constructed according to parameters shown in Table 2; these models are run for seven uncertainty levels ($\Psi = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.85$). Then, the solution of each model is put in a realization model as parameters and 35 realization models are run as it has been illustrated in and Table 4. The results show that as the uncertainty level enhances, the robust model gets more conservative and will have lower constraint violations. It can be seen from Figure 4 that the results of realizations for the maximum uncertainty level ($\Psi = 0.85$) have really low variations (77% less variations than $\Psi = 0.3$). It means this model is completely capable of handling any fluctuations in travel time; in other words it is quite reliable which counts a critical factor in healthcare services.

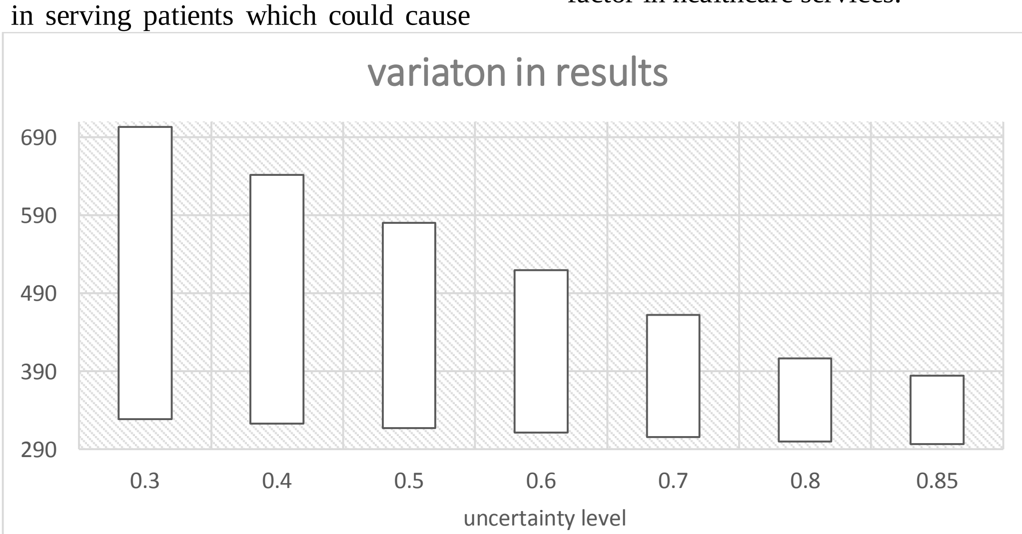

Figure 4. Variation in robust realization models with different uncertainty level

Table 4. Comparing R realization models with different uncertainty levels

Model parameters Q	T	N	Level of uncertainty (Ψ)	Constraint violation penalty (Δ)	Objective function of robust realization model	Mean of Objective function of robust realization models	Standard deviation (SD) of Objective function of robust realization models
80	180	7	0.3	15	328.8 594.82 492 703.39 390	501.802	151.5054
80	180	7	0.4	15	323 550.54 463.6 641.92 375.3	470.872	129.0905
80	180	7	0.5	15	317.17 505.46 434.7 580.45 360.8	439.716	106.5349
80	180	7	0.6	15	311.36 463.1 406.36 519.82 346.6	409.448	84.61852
80	180	7	0.7	15	305.54 420.54 378.9 461.92 332.47	379.874	63.55883
80	180	7	0.8	15	300 378 349.63 406.63 318.3	350.512	43.28589
80	180	7	0.85	15	296.82 356.7 335.45 384.52 311.31	336.96	35.0822

Table 5. Constraint violations of R realization models with different uncertainty levels

Model parameters Q	T	N	Δ	Level of uncertainty (Ψ)	Constraint violation
80	200	7	15	0.1	741.78
				0.2	708.3
				0.3	703.4
				0.4	641.92
				0.5	580.45
				0.6	519.82
				0.7	463.4
				0.8	407.83
				0.9	352.24
				1	297.42

To find the best level of uncertainty or the level in which constraint violations is decreases significantly, a realization model is run for different uncertainty levels as shown in Table 5.

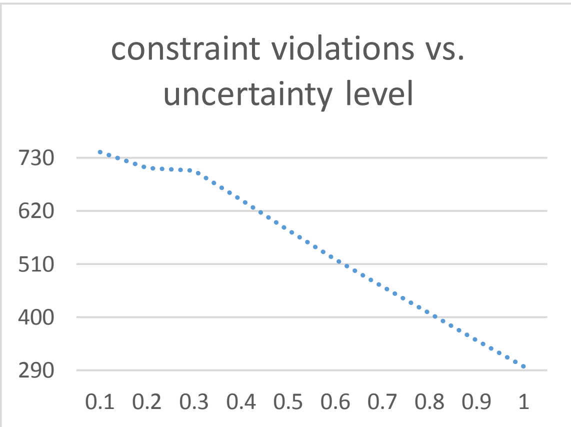

Figure 5. Constraint violation vs. uncertainty level

It can be seen from Figure 5 that by increasing the level of uncertainty, constraint violations decrease significantly and just between $\Psi = 0.2$ and $\Psi = 0.3$ it remains almost the same. It means to avoid constraint violations the level of uncertainty should be enhanced as much as possible. In the next step, for four problem sizes (7 to 10 patients) 20 realization models- each problem size five realizations- are made. The purpose of constructing these realization models is to compare constraint violations in deterministic models with their robust counterparts. The relative parameters of realization models and details of this analysis can be seen in Table 6.

Figure 6 to 9 show that robust realization models have both less deviations and variations compared with deterministic ones in terms of constraint violations. Again this analysis demonstrates the reliability of the proposed robust model to handle travel time fluctuations in comparison with deterministic models in different problem settings. In the first group of iterations the constraint violations of robust model is 14% less than the deterministic model in average, it means 14% less tardiness in serving patients which is considerable.

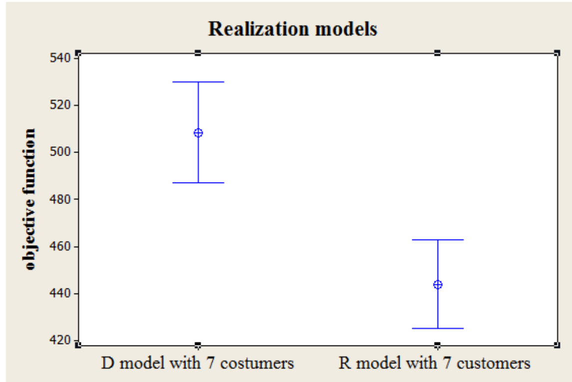

Figure 6. Comparing mean and SD of D and R realization models with 7 costumers

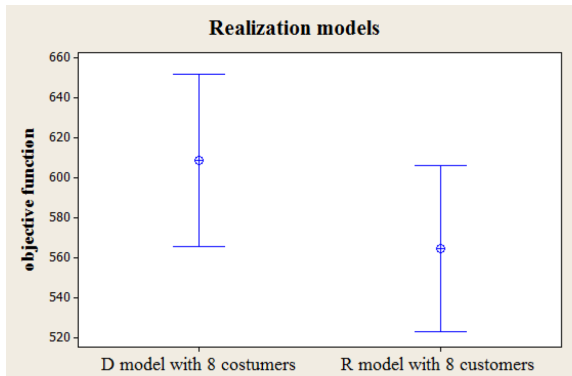

Figure 7. Comparing mean and SD of D and R realization models with 8 costumers

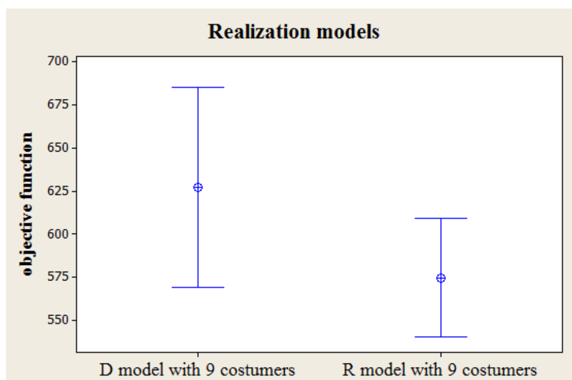

Figure 8. Comparing mean and SD of D and R realization models with 9 costumers

Table 6. Comparing R and D realization

Model parameters			% deviation from the nominal part	Level of uncer tainty (Ψ)	Constraint violation penalty (Δ)	Objective function of realization model		Mean of Objective function of realization models		Standard deviation (SD) of Objective function of realization models	
Q	T	N				D	R	D	R	D	R
						528	320.66				
						498	317.665				
80	180	7	20%	0.4	15	513	317.447	508.46	444	17.3	15.3
						518.9	317.447				
						484.4	318.347				
						643.41	568.855				
						628.71	569.178				
100	200	8	10%	0.25	15	628.71	597.94	608.673	546.78	34.7	33.4
						571.07	530.46				
						571.463	530.46				
						651.7	600.75				
						693.35	591.81				
100	200	9	10%	0.25	15	622.8	591.81	627.12	574.77	45.8	27.7
						581.85	544.61				
						585.9	544.86				
						672.92	626.31				
						645.32	626.58				
100	220	10	10%	0.25	15	681.32	665.8	680.832	650.92	23.7	23.6
						702.3	678.66				
						702.3	657.24				

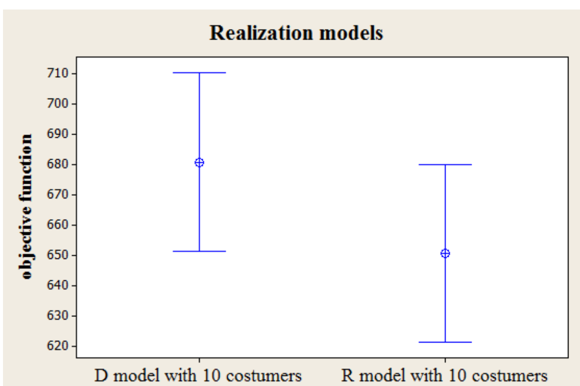

Figure 9. Comparing mean and SD of D and R realization models with 10 costumers

5. Conclusions

In this study a special RVRPSDPTW model proper to serve PD patients' needs is proposed. In this VRP model two places other than depots and patients' homes are also considered in vehicles' routes to satisfy special delivery and pickup demands of PD patients. This model is designed according to special needs of PD patients. However, it contains many realistic assumptions that makes it usable for different kinds of HHC services with slight modifications. It is suitable to provide patients at home with different health services. Therefore, HHC managers can apply it to plan for serving their customers. The model is built based on realistic assumptions such as STW for arrival time, limited tour time and travel time uncertainty of vehicles. Due to the importance of timeliness in health care, a conservative model is applied to handle uncertainty. The results of different analyses in numerical examples show that robust model has less constraint violations and therefore is more reliable; however its costs is more than the deterministic one. The cost increase of robust model is not considerable; for instance in a model with 7 patients the cost of the robust model with maximum uncertainty level is just 1.2% more than the deterministic model. This robust model has 14% less constraint violations compared with the deterministic one.

This study can be extended in many ways. First, the desirability of patients as "waiting time" can be considered as a second objective function of this problem and the model can be solved via a bi-objective procedure. Second, the periodic service giving to patients can be studied which is more realistic, for example weekly planning of serving PD patients. At last by moderating the assumption of this model it can be used for other HHC services like taking care of elderlies at home.

6. Acknowledgements

We thank the staffs of the Dialysis Center of Hasheminejad Kidney Center and Samen Pharmaceutical Corporation that provided insight and expertise who greatly assisted the research.

7. References

-Barnhart, C., Johnson, E. L., Nemhauser, G. L., Savelsbergh, M. W. and Vance, P. H. (1998) "Branch-and-price: Column generation for solving huge integer programs", Operations Research, Vol. 46, pp. 316-329.

-Begur, S. V., Miller, D. M. and Weaver, J. R. (1997) "An integrated spatial DSS for scheduling and routing home-health-care nurses", Interfaces, Vol. 27, pp. 35-48.

-Ben-Tal, A., El Ghaoui, L. and Nemirovski, A. (2009). "Robust optimization", Princeton University Press.

-Ben-Tal, A. and Nemirovski, A. (1998) "Robust convex optimization", Mathematics of Operations Research, Vol. 23, pp. 769-805.

-Bertels, S. and Fahle, T. (2006) "A hybrid setup for a hybrid scenario: combining heuristics for the home health care problem", Computers and Operations Research, Vol. 33, pp. 2866-2890.

-Bertsimas, D. and Sim, M. (2004) "The price of robustness", Operations Research, Vol. 52, PP. 35-53.

-Braekers, K., Hartl, R. F., Parragh, S. N. and Tricoire, F. (2016) "A bi-objective home care scheduling problem: Analyzing the trade-off between costs and client inconvenience", European Journal of Operational Research, Vol. 248, pp. 428-443.

-Bredstrom, D. and Rönnqvist, M. (2007) "A branch and price algorithm for the combined vehicle routing and scheduling problem with synchronization constraints" NHH Dept. of Finance and Management Science Discussion Paper.

-Cappanera, P. and Scutellà, M. G. (2013) " Home care optimization: Impact of pattern generation policies on scheduling and routing decisions", International Network Optimization Conference.

-Cappanera, P., Scutellà, M. G. and Visintin, F. (2014) "Home Care Services delivery: equity versus efficiency in optimization models", Proceedings of the International Conference on

Health Care Systems Engineering, Springer, pp. 1-13.

-Castillo-Salazar, J. A., Landa-Silva, D. and Qu, R. (2014) "Workforce scheduling and routing problems: literature survey and computational study", Annals of Operations Research, pp. 1-29.

-Cheng, E. and Rich, J. L. (1998) "A home health care routing and scheduling problem", URL http://citeseerx. ist. psu. edu/viewdoc/summary.

-Cheraghi, S., & Hosseini-Motlagh, S. M. (2017). "Optimal blood transportation in disaster relief considering facility disruption and route reliability under uncertainty". International Journal of Transportation Engineering,Vol.4, pp. 225-254.

-Cheraghi, S., Hosseini-Motlagh, S. M., & Ghatreh Samani, M. (2017). "Integrated planning for blood platelet production: a robust optimization approach". Journal of Industrial and Systems Engineering, vol.10,pp. 55-80.

-Dohn, A., Rasmussen, M. S., Justesen, T. and Larsen, J. (2008) " The home care crew scheduling problem", Proceedings of the 1st international conference on applied operational research (ICAOR'08), Yerevan, Armenia. Lecture Notes in Management Science, pp. 1-8.

-El Ghaoui, L., Oustry, F. and Lebret, H. (1998) "Robust solutions to uncertain semidefinite programs", SIAM Journal on Optimization, Vol. 9, pp. 33-52.

-Fischetti, M. and Monaci, M. (2009) "Light robustness", Robust and online large-scale optimization. Springer, pp. 61-84.

-Ghatreh Samani, M., & Hosseini-Motlagh, S. M. (2017)."A Hybrid algorithm for a two-echelon location-routing problem with simultaneous pickup and delivery under fuzzy demand", International Journal of Transportation Engineering, Vol 5,pp. 59-85.

-Griva, K., Yu, Z., Chan, S., Krisnasamy, T., Yamin, R. B. A., Zakaria, F. B., Wu, S. Y., Oei, E. and Foo, M. (2014) "Age is not a contraindication to home-based dialysis–Quality-of-Life outcomes favour older patients on peritoneal dialysis regimes relative to younger patients", Journal of Advanced Nursing, Vol. 70, pp. 1902-1914.

-Holte, M. and Mannino, C. (2013) "The implementor/adversary algorithm for the cyclic and robust scheduling problem in health-care", European Journal of Operational Research, Vol. 226, pp. 551-559.

-Jain, A. K., Blake, P., Cordy, P. and Garg, A. X. (2012) "Global trends in rates of peritoneal dialysis", Journal of the American Society of Nephrology, Vol. 23, pp. 533-544.

-Jokar, A., & Hosseini-Motlagh, S. M. (2015). "Impact of capacity of mobile units on blood supply chain performance: Results from a robust analysis",International Journal of Hospital Research,Vol 4,pp. 101-105.

-Kergosien, Y., Lenté, C. and Billaut, J.-C. (2009) "Home health care problem: An extended multiple traveling salesman problem", 4th Multidisciplinary International Conference on Scheduling: Theory and Applications (MISTA'09), Dublin (Irlande), pp. 10-12.

-Kergosien, Y., Ruiz, A. and Soriano, P. (2014) "Published. A routing problem for medical test sample collection in home health care services", Proceedings of the International Conference on Health Care Systems Engineering, Springer, pp. 29-46.

-Klarenbach, S. and Manns, B. (2009) "Economic evaluation of dialysis therapies", Seminars in Nephrology, Elsevier, pp. 524-532.

-Lanzarone, E. and Carello, G. (2014) "Applying the Cardinality–Constrained Approach in Health Care Systems: The Home Care Example". Proceedings of the International Conference on Health Care Systems Engineering, Springer, pp. 61-72.

-Li, Z., Ding, R. and Floudas, C. A. (2011) "A comparative theoretical and computational study on robust counterpart optimization: I. Robust linear optimization and robust mixed integer linear optimization", Industrial and Engineering Chemistry Research, Vol. 50, pp. 10567-10603.

-Liem, Y. S., Wong, J., Hunink, M., de Charro, F. T. and Winkelmayer, W. (2007) "Comparison of hemodialysis and peritoneal dialysis survival in

The Netherlands", Kidney International, Vol. 71, pp. 153-158.

-Liu, R., Xie, X., Augusto, V. and Rodriguez, C. (2013) "Heuristic algorithms for a vehicle routing problem with simultaneous delivery and pickup and time windows in home health care", European Journal of Operational Research, Vol. 230, pp. 475-486.

-Liu, R., Xie, X. and Garaix, T. (2014) "Hybridization of tabu search with feasible and infeasible local searches for periodic home health care logistics", Omega, Vol. 47, pp. 17-32.

-Majidi, S., Hosseini-Motlagh, S. M., & Ignatius, J. (2017). "Adaptive large neighborhood search heuristic for pollution-routing problem with simultaneous pickup and delivery". Soft Computing,pp. 1-15.

-Hosseini-Motlagh, S. M., Majidi, S., Yaghoubi, S., & Jokar, A. (2017). "Fuzzy green vehicle routing problem with simultaneous pickup-delivery and time windows". RAIRO-Operations Research.

-McDonald, S. P., Marshall, M. R., Johnson, D. W. and Polkinghorne, K. R. (2009) "Relationship between dialysis modality and mortality", Journal of the American Society of Nephrology, Vol. 20, pp.155-163.

-Min, H. (1989) "The multiple vehicle routing problem with simultaneous delivery and pick-up points", Transportation Research Part A: General, Vol. 23, pp. 377-386.

-Nickel, S., Schröder, M. and Steeg, J. (2012) "Mid-term and short-term planning support for home health care services", European Journal of Operational Research, Vol. 219, pp. 574-587.

-Nikkhah Qamsari, A., Hosseini Motlagh, S. M., & Jokar, A. (2017). "A two-phase hybrid heuristic method for a multi-depot inventory-routing problem". International Journal of Transportation Engineering,Vol 4,pp. 287-304.

-Perl, J., Davies, S. J., Lambie, M., Pisoni, R. L., McCullough, K., Johnson, D. W., Sloand, J. A., Prichard, S., Kawanishi, H. and Tentori, F. (2015) "The Peritoneal Dialysis Outcomes and Practice Patterns Study (PDOPPS): unifying efforts to inform practice and improve global outcomes in peritoneal dialysis", Peritoneal Dialysis International, pdi. 2014.00288.

-Pishvaee, M. S., Rabbani, M. and Torabi, S. A. (2011) "A robust optimization approach to closed-loop supply chain network design under uncertainty", Applied Mathematical Modelling, Vol. 35, pp. 637-649.

-Pishvaee, M. S., Razmi, J. and Torabi, S. A. (2012) "Robust possibilistic programming for socially responsible supply chain network design: A new approach", Fuzzy Sets and Systems, Vol. 206, pp. 1-20.

-Riahi, N., Hosseini-Motlagh, S. M., & Teimourpour, B. (2013). "A Three-phase Hybrid Times Series Modeling Framework for Improved Hospital Inventory Demand Forecast". International Journal of Hospital Research, Vol 2,pp. 133-142.

-Rubin, H. R., Fink, N. E., Plantinga, L. C., Sadler, J. H., Kliger, A. S. and Powe, N. R. (2004) "Patient ratings of dialysis care with peritoneal dialysis vs hemodialysis", Jama, Vol. 291, pp. 697-703.

-Soyster, A. L. (1973) "Technical note—convex programming with set-inclusive constraints and applications to inexact linear programming", Operations Research, Vol. 21, pp. 1154-1157.

-Stevens, L. A., Viswanathan, G. and Weiner, D. E. (2010) "CKD and ESRD in the elderly: Current prevalence, future projections, and clinical significance", Advances in Chronic Kidney Disease, Vol. 17, pp. 293.

-Torres-Ramos, A. F., Alfonso-Lizarazo, E. H., Reyes-Rubiano, L. S. and Quintero-Araújo, C. L. (2014) "Published. Mathematical Model for the Home Health Care Routing and Scheduling Problem with Multiple Treatments and Time Windows", Proceedings of the 1st International Conference on Mathematical Methods and Computational Techniques in Science and Engineering (MMCTSE 2014), pp. 140-145.

-Trautsamwieser, A., Gronalt, M. and Hirsch, P. (2011) "Securing home health care in times of natural disasters", OR Spectrum, Vol. 33, pp. 787-813.

-Treharne, C., Liu, F. X., Arici, M., Crowe, L. and Farooqui, U. (2014) "Peritoneal dialysis and in-centre haemodialysis: a cost-utility analysis from a UK payer perspective", Applied Health Economics and Health Policy, Vol. 12, pp. 409-420.

-Triki, N., Garaix, T. and Xie, X. (2014) "Published. A two-phase approach for periodic home health care planning", Automation Science and Engineering (CASE), 2014 IEEE International Conference, pp. 518-523.

-Workeneh, B., Guffey, D., Minard, C. G. and Mitch, W. E. (2015) "Causes for Withdrawal in an Urban Peritoneal Dialysis Program", *International journal of nephrology*.

-Yalçındağ, S., Matta, A., Şahin, E. and Shanthikumar, J. G. (2014) "Published. A two-stage approach for solving assignment and routing problems in home health care services", Proceedings of the International Conference on Health Care Systems Engineering, Springer, pp. 47-59.

-Yu, Z. L., Seow, Y.-Y., Seow, P. S. and Tan, B. L. K. (2016) "Effectiveness of a day care program in supporting patients on peritoneal dialysis and their caregivers", International Urology and Nephrology, pp. 1-7.

-Zare-Reisabadi, E. and Mirmohammadi, S. H. (2015) "Site dependent vehicle routing problem with soft time window: Modeling and solution approach", Computers and Industrial Engineering, Vol. 90, pp. 177-185.