A Hybrid Algorithm for a Two-Echelon Location-Routing Problem with Simultaneous Pickup and Delivery under Fuzzy Demand

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Abstract

Location-Routing Problem (LRP) emerges as one of the hybrid optimization problems in distribution networks in which, total cost of the system would be reduced significantly by simultaneous optimization of locating a set of facilities among candidate locations and routing vehicles. In this paper, a mixed integer linear programming model is presented for a two-echelon location-routing problem with simultaneous pickup and delivery. In the investigated problem, one echelon of facilities, which is called the middle depot echelon, is positioned between central distribution centers and customers echelons. The number and capacity of middle depots and vehicles are considered to be limited. Besides, each network customer demands for both receiving a type of commodities and delivering another type to vehicles to be returned to the depot. In the literature of location routing problem, the majority of researches have been conducted in the deterministic conditions. However, we present a model in which data uncertainty is also taken into account and customers' demand is assumed to be a fuzzy parameter. We utilize a fuzzy programming approach to cope with uncertain demands. Moreover, a combined heuristic method based on simulated annealing (SA) algorithm and genetic algorithm (GA) is devised for solving the presented model. The results achieved from solving the problem in different sizes of numerical examples imply that the proposed hybrid algorithm outperforms other algorithms within reasonable length of time. The effectiveness of the proposed solution method is examined through a comprehensive numerical experiments. Finally, valuable insights are provided via conducting a number of sensitivity analyses.

Keywords: Location-routing problem; Two echelons; Fuzzy numbers; Credibility theory; Hybrid algorithm.

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1. Introduction

Economic consideration is one of the most significant issues in business environment so enterprises have always been seeking the ways to reduce costs in different parts of organizations. The fact is that a great portion of these expenses belongs to logistics costs. Being a substantial part of any supply chains, distribution networks need to be appropriately designed to reduce costs and improve responsiveness of the chain. Increasing the efficiency of distribution systems can be considered as one of the primary goals of integrated logistics systems. Thus, optimization of logistics systems has become a critical problem in supply chain management in recent years. Integrated problems in distribution networks can be categorized into location-routing problems (LRP), location-inventory problems (LIP), inventory-routing problems (IRP), vehicle-routing problems (VRP), and so on [Majidi, Hosseini-Motlagh and Ignatius, 2017]. The location-routing problem (LRP) will convert to the vehicle-routing problem (VRP) if the location of facilities is predetermined. Location-routing problems are in the set of NP-hard problems, and solving each problem separately (i.e. once, the location-allocation problem and then routing problem) would result in sub-optimal solutions [Cheraghi, Hosseini-Motlagh and Ghatreh Samani, 2016].

Location and routing decisions are the two interdependent elements of a distribution network. Deciding on locating facilities without accounting for routing considerations may increase total network cost. [Laporte, 1987].

LRP is applicable in many fields such as food and beverage distribution, postal parcels and pharmaceuticals deliveries and military applications.

In some cases, customers may have pickup and delivery demands at the same time. In such situations, the problem is called location-routing problem with simultaneous pickup and delivery (LRPSPD), which copes with determining the location of facilities and vehicles routes in such a way that both delivery and pickup demands for each customer are supposed to be simultaneously satisfied by vehicles to minimize total cost [Karaoglan et al. 2011]. Delivering car spare parts and collecting defective parts and so on can be considered as the applications of this problem.

Noteworthy, in large-scale optimization problems, enough knowledge about the exact value of some parameters including demands, costs, travelling time and so on is not accessible [Riahi, Hosseini-Motlagh and Teimourpour, 2013; Majidi, Hosseini-Motlagh, Yaghoubi and Jokar, 2017; [Jokar and Hosseini-Motlagh, 2015]]. In such cases, we encounter fuzzy impreciseness (i.e., the lack of knowledge about the precise value of a parameter). Thus, we need to refer to the professional experts' subjective knowledge to have an estimation of the value of fuzzy parameters [Cheraghi, Hosseini-Motlagh and Ghatreh samani, 2016]. In such situations, the parameter fuzziness is handled by applying fuzzy programming approaches whose one of the effective methods among those, which have been frequently addressed in the literature, is based on the fuzzy credibility approach which has been devised in this research. The remainder of the paper is organized as follows. In Section 2, the recent literature of location-routing problems is reviewed. The problem description and mathematical formulation of the proposed model is provided in Section 3. In Section 4, a fuzzy approach is developed to deal with the demand fuzziness. In Section 5, our heuristic method, which is the combination of meta-heuristic algorithms (i.e., simulated annealing and genetic algorithms) is presented. Section 6 provides several numerical examples to evaluate the effectiveness of the proposed method. Finally, concluding remarks and future works recommendations are given in Section 7.

2. Literature Review

In this section, we first briefly review the related literature on the location-routing problem (LRP) and its derivatives (i.e. the location-routing problem with simultaneous pickup and delivery (LRPSPD) and the two-echelon location routing problem (2E-LRP)), then investigate the papers which have taken into account the data uncertainties.

The first study of location- routing problems refers to Webb [Webb, 1968]. The study was expanded by Watson-Gandy and Dohrn [Watson-Gandy and Dohrn, 1973], Nambiar, Gelders and Van Wassenhove [Nambiar, Gelders and Van Wassenhove, 1981] and Madsen [Madsen, 1983]. The location-routing problem could be classified based on different criteria. The first classification was provided by Min, Jayaraman and Srivastava [Min, Jayaraman and Srivastava, 1998]. Nagy and Salhi [Nagy and Salhi, 2007] classified this problem based on standard and non-standard structures and the type of objective functions. Recently, Prodhon and Prins [Prodhon and Prins, 2014] presented a classification of location-routing problems according to solution methods and modeling approaches.

LRPSPD, a branch of LRP, was firstly introduced by karaoglan et al. [karaoglan et al. 2009]. They proposed two mixed integer programming (MIP) formulations, which are flow-based and node-based formulations, respectively. They presented several polynomial-size valid inequalities to strengthen the models. In another effort, Karaoglan et al. [Karaoglan et al. 2011] addressed a mathematical model for LRPSPD and applied an exact algorithm based on branch-and-cut (BandC) algorithm to solve the problem. They also developed simulated annealing (SA) to improve the initial solution during the search process of the branch-and-cut algorithm. In the investigated problem, vehicles are considered to be homogeneous and their numbers are limited. In another work, Karaoglan et al. [Karaoglan et al., 2012] presented a two-phase heuristic method based on simulated annealing, called tp-SA. The quality of the two proposed formulations (i.e. flow-based and node-based formulations) is compared with each other with respect to their ability to obtain optimal solutions. Later on, the many-to-many location-routing problem with simultaneous pickup and delivery appealed to Rieck, Ehrenberg and Zimmerman [Rieck, Ehrenberg and Zimmerman, 2014]. In their proposed model, the location of hub facilities was determined. The model was solved in both small size by using exact methods and in large size via genetic algorithm (GA).

Jacobsen and Madsen [Jacobsen and Madsen, 1980] presented a two-echelon location-routing problem (2E-LRP) for the first time. A central facility was predetermined in the first echelon and several local facilities were established in customers' side. They employed three heuristic methods to solve the problem. A two-echelon model for the location-routing problem was addressed by Wasner and Zäpfel [Wasner and Zäpfel, 2004] and the limitation on the capacity of vehicles as well as delivery constraints are taken into account. However, the capacity of facilities is considered to be unlimited. They tailored a two-phase algorithm in which the location of each facility is determined in the first phase, afterwards, vehicles routing is dealt with in the second phase. Ambrosino and Scutella [Ambrosino and Scutella, 2005] developed a two-echelon location-routing problem in which customers were visited in several clusters. A two-phase method was devised to solve their proposed model. In the first phase, customers' clustering, vehicles allocation to each cluster and the location of local facilities are determined by employing an integer programming model. Then, in the second phase, a travelling salesman problem (TSP) is solved for each cluster by using a branch-and-cut algorithm. Eventually, the solutions are improved through replacing local facilities with each other. Nikbakhsh and Zegordi [Nikbakhsh and Zegordi, 2010] worked on a model for the two-echelon capacitated location-routing problem, in which the capacity of both vehicles and facilities was assumed to be limited. They regarded constraints on maximum tour length for vehicles and applied a heuristic method along with a meta-heuristic algorithm based on simulated annealing (SA) to solve the problem. Moreover, the corresponding results were evaluated by solving the problem in different sizes.

Although the related papers on the subject of LRP have been mostly considered in deterministic conditions, inadequate knowledge of uncertain parameters such as demand, travel time and so on has made the researchers consider uncertainty conditions in their works to have a better perception of reality. In this regard, a location-routing problem with time windows (LRPTW) was presented by Zarandi et al. [Zarandi et al., 2013]. They considered travel time as a fuzzy parameter and employed simulated annealing (SA) to solve the model, then compared the respective results with those existing in the literature. Golozari, Jafari, and Amiri [Golozari, Jafari, and Amiri, 2013] worked on a routing problem while considering the constraint on maximum route length. In the concerned problem, customers' demand and travel time as well as service time were regarded to be fuzzy parameters. They used simulated annealing (SA) algorithm to solve the presented model. Nadizadeh and Hosseini Nasab [Nadizadeh and Hosseini Nasab, 2014] proposed a model for the location-routing problem under fuzzy demand over a multi-period planning horizon, and devised a clustering approach to solve the proposed model.

Recently, Riquelme-Rodríguez, Gamache and Langevin [Riquelme-Rodríguez, Gamache, and Langevin, 2016] addressed the first method for a periodic capacitated location arc routing problem for suppressing dust in hauling roads. They proposed two methods for finding the initial location of water depots in the road and then compared their performance with the aim of minimizing penalty costs arising from the lack of humidity in roads as well as routing costs. Afterwards, the initial location of water depots and the initial vehicle routing were modified by applying an exchange algorithm and an adaptive large neighborhood search algorithm, respectively.

A novel bi-objective multi-product capacitated vehicle routing problem was presented by Tavakkoli-Moghaddam, Raziei, and Tabrizian [Tavakkoli-Moghaddam, Raziei, and Tabrizian, 2016] in which the fleet of vehicles was heterogeneous, and demand amounts as well as volume of products were considered to be tainted with uncertainty. Minimizing the cost of used vehicles, fuel consumption along with the shortage of products are the two objectives of the problem. They applied the ε-constraint method to solve the proposed bi-objective model and devised a fuzzy programming approach to deal with the uncertainty. Hiassat, Diabat and Rahwan [Hiassat, Diabat and Rahwan, 2017] proposed a mixed integer programming model for a location-inventory-routing problem for perishable products. Their research aimed to determine the location and required number of depots, the level of inventory at each retailer, and the travelling routes. They developed a Genetic Algorithm approach to solve the under-investigated problem and obtained near-optimal solutions in reasonable length of time. A novel approach for location routing problem was presented by Schiffer and Walther [Schiffer and Walther, 2017] while considering strategic planning for electric logistics fleets. The approach considers the decisions of charging station siting and vehicle routing simultaneously to illustrate the significance of jointly consideration of siting and routing decisions. Applying the proposed approach, they minimized total costs, distance, and the number of vehicles and charging stations concurrently. In an effort, Nikkhah Qamsari, Hosseini-Motlagh and Jokar [Nikkhah Qamsari, Hosseini-Motlagh and Jokar, 2017] developed a two phase hybrid heuristic approach to solve the multi-depot multi-vehicle routing problem while accounting for inventory constraints. Their concerned model aimed to minimize total cost including inventory holding cost at distribution centers and the customers' side as well as transportation costs. They applied a variable neighborhood search algorithm to modify the initial solution obtained in construction phase. They illustrated the capability of their proposed algorithm to find near-optimal solutions within reasonable computing time.

To the best of our knowledge, the majority of studies have addressed the two-echelon location-routing problem in deterministic condition and a study on this subject while considering uncertainty conditions is non-existent. To fill this gap, our research is differentiated from the ones existing in the literature of LRP by considering the following contributions:

- A mixed integer linear programming model for a two-echelon location-routing problem with simultaneous pickup and delivery under uncertainty is proposed.
- The uncertainty in customers' demand is accounted for in the form of fuzzy numbers which is handled by applying a fuzzy credibility programming approach.
- The proposed model is solved by means of a hybrid solution approach which is the combination of genetic and simulated annealing algorithms.

3. Problem description

This paper puts forward a 2E-LRPSPD under fuzzy demands by considering two types of facilities, i.e., the central and secondary facilities where products get transferred from central depots to secondary ones and then are distributed among the customers or picked up from customers to be returned to the facilities. The secondary facilities have an intermediate role in the distribution network and are regarded as temporary places for storing, loading and unloading goods. The concerned 2E - LRPSPD can be schematically depicted in Figure 1. In this research, we seek to optimize the number of open facilities along with routing between established secondary and central facilities as well as routing between customers and secondary facilities with the aim of customers' demand satisfaction and minimizing the network total cost including establishment cost, travelling cost and vehicles fixed cost. In the concerned network, travelling route starts from a main depot and ends at the same depot. Each vehicle belongs to one route and is in charge of delivering goods from main depot to the secondary depot and from secondary depot to customers such that each customer is visited exactly once and the customers' demands do not exceed the capacity of the vehicle, and picking up goods from customers to return to the same depot.

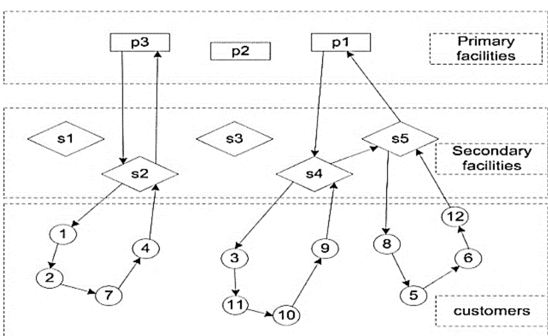

Figure 1.The considered two-echelon location-routing problem

3.1 Mathematical Formulation in Deterministic Mood

Consider graph $G = (V, E)$ in which V represents the set of network nodes which includes V_O , the set of central depots, V_R, the set of middle depots and V_C, the set of customers. In this graph, V_1 and V_2 indicates the nodes of the first and second echelons, respectively, such that $(V_1 = V_O \cup V_R)$ and $(V_2 = V_R \cup V_C)$. The set of total existing arcs of the graph *(E)* includes undirected arcs connecting central distribution centers to middle ones, middle distribution centers to customers and customers to each other. The connecting arcs must satisfy the following triangular inequality $(d_{ij} \leq d_{ik} + d_{kj})$. Set K includes vehicles which are used between middle distribution centers and customers in the second echelon. The aforementioned problem is studied under the following constraints:

- Each vehicle travels a route while starting from a specific depot and finishing in the same depot.
- Each route can provide services for only one vehicle.
- Each customer is allowed to be served by only one vehicle.
- Customers' pickup and delivery demands are satisfied simultaneously and do not exceed the capacity of vehicles.
- Total demands from customers allocated to a depot do not exceed the depot capacity.
- Total demands in a route do not exceed the capacity of vehicle assigned to that route.

We use the following components to formulate the proposed model.

Model sets, parameters and decision variables

3.1.1. Sets

I	The set of customers
O	The set of central depots
R	The set of candidate middle depots
K	The set of vehicles

3.1.2. Technical parameters

CD_O	The capacity of each central depot
RD_R	The capacity of each middle depot
d_i	The amount of each customer's delivery demand
p_i	The amount of each customer's pickup demand
CV	The capacity of each vehicle

3.1.3. Cost parameters

F_R	Establishment fixed cost of each middle depot
G_{OR}	Travel cost between a pair of central and middle depots
H_{ij}	Travel cost from customer i to j
FC_k	Fixed cost of each vehicle

3.1.4. Decision variables

$\mathbf{x_{OR}}$	The amount of commodity transported from central depot O to middle depot R.	$(\forall R \in V_R, \forall O \in V_O)$
$\mathbf{y_{ij}}$	Binary variable, equal to 1 if a vehicle moves to node j from node i; 0, otherwise.	$(\forall i, j \in V_2)$
$\mathbf{O_R}$	Binary variable, equal to 1 if middle depot R is established; 0, otherwise.	$(\forall R \in V_R)$
$\mathbf{p_{iR}}$	Binary variable, equal to 1 if customer i is allocated to middle depot R; 0, otherwise.	$(\forall i \in V_C, \forall R \in V_R)$
$\mathbf{Z_i}$	Total demands delivered to customers before meeting customer i	$(\forall i \in V_2)$
$\mathbf{W_i}$	Total demands picked up from customers after meeting customer i	$(\forall i \in V_2)$

3.1.4 Objective Function

$$Min \sum_{i \in V_2} \sum_{j \in V_2} H_{ij} y_{ij} + \sum_{O \in V_O} \sum_{R \in N_R} G_{OR} x_{OR} + \sum_{R \in V_R} F_R w_R + \sum_{i \in V_2} \sum_{R \in V_R} FC_k p_{iR} \quad (1)$$

3.1.5 Model Constraints

$$\sum_{R \in V_R} x_{OR} \leq CD_O \qquad \forall O \in V_O \quad (2)$$

$$\sum_{O \in V_O} x_{OR} \leq RD_R * O_R \qquad \forall R \in V_R \quad (3)$$

$$\sum_{O \in V_O} x_{OR} \geq \sum_{i \in V_2} d_i * p_{iR} \qquad \forall R \in V_R \quad (4)$$

$$\sum_{j \in V_2} y_{ij} = 1 \qquad \forall i \in V_C \quad (5)$$

$$\sum_{j\in V_2} y_{ji} = \sum_{j\in N_2} y_{ij} \qquad \forall i \in V_2 \qquad (6)$$

$$\sum_{R\in V_R} p_{iR} = 1 \qquad \forall i \in V_C \qquad (7)$$

$$\sum_{i\in V_2} d_i * p_{iR} \le RD_R * O_R \qquad \forall R \in V_R \qquad (8)$$

$$\sum_{i\in V_2} p_i * p_{iR} \le RD_R * O_R \qquad \forall R \in V_R \qquad (9)$$

$$Z_j - Z_i + CV * y_{ij} + \left(CV - d_i - d_j\right)y_{ji} + d_i \le CV \qquad \forall i, j \in V_c, i \neq j \qquad (10)$$

$$W_j - W_i + CV * y_{ij} + \left(CV - p_i - p_j\right)y_{ji} + p_i \le CV \qquad \forall i, j \in V_c, i \neq j \qquad (11)$$

$$Z_i + W_i - d_i \le CV \qquad \forall i \in V_C \qquad (12)$$

$$d_i + \sum_{j\in V_C, j\neq i} d_j y_{ij} \le Z_i \qquad \forall i \in V_C \qquad (13)$$

$$p_i + \sum_{j\in V_C, j\neq i} p_j y_{ij} \le W_i \qquad \forall i \in V_C \qquad (14)$$

$$Z_i + (\ CV - d_i)(\sum_{R\in V_R} y_{iR}) \le CV \qquad \forall i \in V_C \qquad (15)$$

$$W_i + (\ CV - p_i)(\sum_{R\in V_R} y_{Ri}) \le CV \qquad \forall i \in V_C \qquad (16)$$

$$y_{iR} \le p_{iR} \qquad \forall i \in V_C, \forall R \in V_R \qquad (17)$$

$$y_{Ri} \le p_{iR} \qquad \forall i \in V_C, \forall R \in V_R \qquad (18)$$

$$y_{ij} + p_{iR} + \sum_{m\in V_R, m\neq R} p_{jm} \le 2 \qquad \forall i, j \in V_c, i \neq j\ , \forall R \in V_R \qquad (19)$$

$$y_{ij} \in \{0,1\} \qquad \forall i, j \in V_2 \qquad (20)$$

$$x_{OR} \ge 0 \qquad \forall O \in V_O, \forall R \in V_R \qquad (21)$$

$$p_{iR} \in \{0,1\} \qquad \forall i \in V_C \qquad (22)$$

$$O_R \in \{0,1\} \qquad \forall R \in V_R \qquad (23)$$

$$Z_i \ge 0 \qquad \forall i \in V_2 \qquad (24)$$

$$W_i \ge 0 \qquad \forall i \in V_2 \qquad (25)$$

The objective function (1) aims to minimize the total cost of the network consisting of travel costs in the first and second distribution levels, establishment fixed cost of middle depots, and vehicles fixed costs. Constraint (2) shows the capacity limitation of central depots. In other words, the amount of goods stored in a central depot and distributed to a middle depot does not exceed the capacity of the central depot. The limited capacity of middle depots is represented by constraint (3). Better to say, the amount of goods received by a middle depot does not exceed the capacity of the depot. Constraint (4) is the inflow and outflow conservation constraint for middle depots. It guarantees that the amount of goods received by each middle depot is equal to the amount of goods dispatched from the depot. In other words, middle depots act as a bridge between central depots and customers. Each customer is allowed to be visited exactly once by any vehicles. This is guaranteed by constraint (5). Constraint (6) denotes the inflow and outflow conservation constraint for each customer. Indeed, this constraint states that each customer is once visited and delivered the goods and is left while picking up the required goods. Constraint (7) ensures that each customer can be assigned to only one middle depot. Constraints (8) indicates that total demand which is delivered from each middle depot to customer does not exceed the capacity of the middle depot. Constraint (9) ensures that total pickup demand from customers to each middle depot does not exceed the capacity of the depot. Constraints (10) and (11) determine delivery and pickup flows in each arc, considering the capacity of vehicles, respectively. In other words, total delivery demand to each customer and total pickup demand from each customer do not exceed the capacity of the vehicle. Constraint (12) shows the maximum capacity of each vehicle. Constraints (13) and (15) define lower bound and upper bound of total delivery demand variable. Similarly, constraints (14) and (16) denote lower bound and upper bound of total pickup demand variable. Constraints (17)-(19) denote sub-tour elimination constraints. Better to say, these constraints prevent undesirable tours in which some customers are neglected to be visited or vehicles, starting from a specific depot, do not return to the same depot at the end of the service. Constraints (20)-(25) specify the type of decision variables.

4. Fuzzy Programming Approach

In this section, we devise a fuzzy programming approach based on the credibility theory to cope with the customers' fuzzy demands. The problem is handled by applying a credibility-based chance constrained programming method as an efficient fuzzy approach because it enables the decision maker to satisfy the chance constraints at least at a minimum confidence level α, and can be applied for both triangular and trapezoidal fuzzy numbers [Cheraghi, Hosseini-Motlagh, 2016]. Let $\tilde{\vartheta}$ be a fuzzy variable with membership function $\mu(x)$ and r be a real number. The credibility measure can be formulated as follows. (Equation 26) [Liu and Liu, 2002] Since the $Pos\{\tilde{\vartheta} \leq \mathrm{r}\} = Sup_{x \leq r}\mu(\mathrm{x})$ and $Nec\{\tilde{\vartheta} \leq \mathrm{r}\} = 1 - Sup_{x>r}\mu(\mathrm{x})$ the relationship (26) can be substitute by the following equation. Thus, the expected value of $\tilde{\vartheta}$ based on credibility measure is represented by equation (28). If $\tilde{\vartheta}$ be a trapezoidal fuzzy number, as shown in Figure 2, such that $\tilde{\vartheta} = (\vartheta_{(1)}, \vartheta_{(2)}, \vartheta_{(3)}, \vartheta_{(4)})$, the expected value of $\tilde{\vartheta}$ will be equal to $(\vartheta_{(1)} + \vartheta_{(2)} + \vartheta_{(3)} + \vartheta_{(4)})/4$, and the credibility measure will be determined by equations (29) and (30). It is shown that if $\alpha \geq 0.5$, the credibility measure will be equivalent by equations (30). [Zhu and Zhang, 2009]

4.1 The Equivalent Auxiliary Crisp Model

Assuming that the chance constraints are satisfied with the minimum confidence level 0.5, or better to say, $\alpha > 0.5$, the proposed model can be converted to the equivalent crisp one using the relationships (31) and (32). However, the rest of the elements will remain unchanged. According to the above descriptions, the equivalent crisp model can be presented as equations (33)-(42).

$$Cr\left\{\widetilde{\vartheta} \le r\right\} = \frac{1}{2}\left(Pos\left\{\widetilde{\vartheta} \le r\right\} + Nec\left\{\widetilde{\vartheta} \le r\right\}\right) \quad (26)$$

$$Cr\left\{\widetilde{\vartheta} \le \mathrm{r}\right\} = \frac{1}{2}\left(Sup_{x\le r}\mu(\mathrm{x}) + 1 - Sup_{x>r}\mu(\mathrm{x})\right) \quad (27)$$

$$E\left[\widetilde{\vartheta}\right] = \int_0^{\infty} Cr\left\{\widetilde{\vartheta} \ge \mathrm{r}\right\} dr - \int_{-\infty}^{0} Cr\left\{\widetilde{\vartheta} \le \mathrm{r}\right\} dr \quad (28)$$

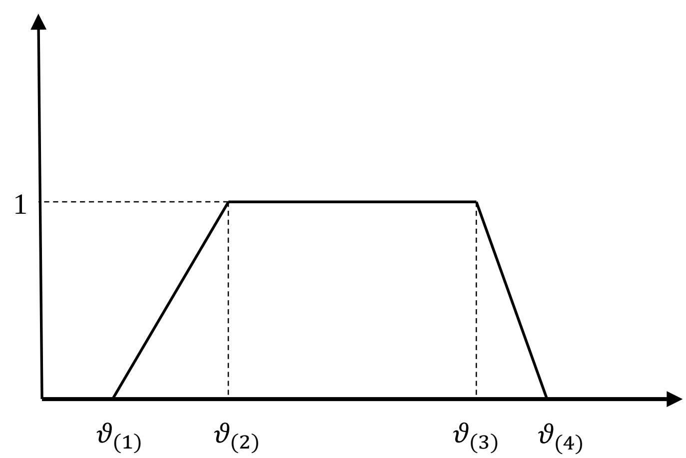

Figure 2. A trapezoidal fuzzy variable

$$Cr\left\{\widetilde{\vartheta} \le \mathrm{r}\right\} = \begin{cases} 0 & r \in (-\infty, \vartheta_{(1)}] \\ \dfrac{r - \vartheta_{(1)}}{2\left(\vartheta_{(2)} - \vartheta_{(1)}\right)} & and\ r \in (\vartheta_{(1)}, \vartheta_{(2)}] \\ \dfrac{1}{2} & r \in (\vartheta_{(2)}, \vartheta_{(3)}] \\ \dfrac{r - 2\vartheta_{(3)} + \vartheta_{(4)}}{2(\vartheta_{(4)} - \vartheta_{(3)})} & andr \in (\vartheta_{(3)}, \vartheta_{(4)}] \\ 1 & andr \in (\vartheta_{(4)}, +\infty] \end{cases} \quad (29)$$

$$Cr\left\{\widetilde{\vartheta} \geq \mathrm{r}\right\} = \begin{cases} 1 & r \in (-\infty, \vartheta_{(1)}] \\ \frac{2\vartheta_{(2)} - \vartheta_{(1)} - r}{2(\vartheta_{(2)} - \vartheta_{(1)})} & and r \in (\vartheta_{(1)}, \vartheta_{(2)}] \\ \frac{1}{2} & r \in (\vartheta_{(2)}, \vartheta_{(3)}] \\ \frac{\vartheta_{(4)} - r}{2(\vartheta_{(4)} - \vartheta_{(3)})} & and\ r \in (\vartheta_{(3)}, \vartheta_{(4)}] \\ 0 & and r \in (\vartheta_{(4)}, +\infty] \end{cases} \tag{30}$$

$$Cr\left\{\widetilde{\vartheta} \leq \mathrm{r}\right\} \geq \alpha \Leftrightarrow r \geq (2-2\alpha)\vartheta_{(3)} + (2\alpha - 1)\vartheta_{(4)} \tag{31}$$

$$Cr\left\{\widetilde{\vartheta} \geq \mathrm{r}\right\} \geq \alpha \Leftrightarrow r \geq (2\alpha - 1)\vartheta_{(1)} + (2-2\alpha)\vartheta_{(2)} \tag{32}$$

$\boldsymbol{d_{i,f(n)}^{s}}$ The amount of each customer's delivery demand under each scenario

$\boldsymbol{p_{i,f(n)}^{s}}$ The amount of each customer's pickup demand under each scenario

$$\sum_{O \in V_O} x_{OR}^{S} \geq \sum_{i \in V_2} \left((2-2\alpha) * \mathrm{d}_{\mathrm{i},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{d}_{\mathrm{i},f(4)}^{\mathrm{S}}\right) * p_{iR}^{S} \qquad \forall \mathrm{R} \in V_R, \forall s \tag{33}$$

$$\sum_{i \in V_2} \left((2-2\alpha) * \mathrm{d}_{\mathrm{i},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{d}_{\mathrm{i},f(4)}^{\mathrm{S}}\right) * p_{iR}^{S} \leq RD_R * O_R \qquad \forall \mathrm{R} \in V_R, \forall s \tag{34}$$

$$\sum_{i \in V_2} \left((2-2\alpha) * \mathrm{p}_{\mathrm{i},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{p}_{\mathrm{i},f(4)}^{\mathrm{S}}\right) * p_{iR}^{S} \leq RD_R * O_R \qquad \forall \mathrm{R} \in V_R, \forall s \tag{35}$$

$$\begin{aligned} Z_j^S - Z_i^S + CV * y_{ij}^S & \\ + \Big(CV - \Big((2-2\alpha) * \mathrm{d}_{\mathrm{i},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{d}_{\mathrm{i},f(4)}^{\mathrm{S}}\Big) & \\ - \Big((2-2\alpha) * \mathrm{d}_{\mathrm{j},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{d}_{\mathrm{j},f(4)}^{\mathrm{S}}\Big)\Big) y_{ji}^S & \\ + \Big((2-2\alpha) * \mathrm{d}_{\mathrm{i},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{d}_{\mathrm{i},f(4)}^{\mathrm{S}}\Big) \leq CV & \end{aligned} \qquad \forall \mathrm{i}, \mathrm{j} \in V_c, i \neq j, \forall s \tag{36}$$

$$W_j^S - W_i^S + CV * y_{ij}^S + \left(CV - \left((2-2\alpha) * \mathrm{p}_{\mathrm{i},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{p}_{\mathrm{i},f(4)}^{\mathrm{S}}\right) - \left((2-2\alpha) * \mathrm{p}_{\mathrm{j},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{p}_{\mathrm{j},f(4)}^{\mathrm{S}}\right)\right) y_{ji}^S + \left((2-2\alpha) * \mathrm{p}_{\mathrm{i},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{p}_{\mathrm{i},f(4)}^{\mathrm{S}}\right) \le CV \quad \forall \mathrm{i},\mathrm{j} \in V_c, i \ne j, \forall s \qquad (37)$$

$$Z_i^S + W_i^S - \left((2-2\alpha) * \mathrm{d}_{\mathrm{i},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{d}_{\mathrm{i},f(4)}^{\mathrm{S}}\right) \le CV \quad \forall \mathrm{i} \in V_C, \forall s \qquad (38)$$

$$\left((2-2\alpha) * \mathrm{d}_{\mathrm{i},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{d}_{\mathrm{i},f(4)}^{\mathrm{S}}\right) + \sum_{j \in V_C, j \ne i} \left((2-2\alpha) * \mathrm{d}_{\mathrm{j},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{d}_{\mathrm{j},f(4)}^{\mathrm{S}}\right) y_{ij}^S \le Z_i^S \quad \forall \mathrm{i} \in V_C, \forall s \qquad (39)$$

$$\left((2-2\alpha) * \mathrm{p}_{\mathrm{i},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{p}_{\mathrm{i},f(4)}^{\mathrm{S}}\right) + \sum_{j \in V_C, j \ne i} \left((2-2\alpha) * \mathrm{p}_{\mathrm{j},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{p}_{\mathrm{j},f(4)}^{\mathrm{S}}\right) * y_{ij}^S \le W_i^S \quad \forall \mathrm{i} \in V_C, \forall s \qquad (40)$$

$$Z_i^S + \left(CV - \left((2-2\alpha) * \mathrm{d}_{\mathrm{i},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{d}_{\mathrm{i},f(4)}^{\mathrm{S}}\right)\right)\left(\sum_{R \in V_R} y_{iR}^S\right) \le CV \quad \forall \mathrm{i} \in V_C, \forall s \qquad (41)$$

$$W_i^S + \left(CV - \left((2-2\alpha) * \mathrm{p}_{\mathrm{i},f(3)}^{\mathrm{S}} + (2\alpha - 1) * \mathrm{p}_{\mathrm{i},f(4)}^{\mathrm{S}}\right)\right)\left(\sum_{R \in V_R} y_{Ri}^S\right) \le CV \quad \forall \mathrm{i} \in V_C, \forall s \qquad (42)$$

5. Solution Methods

5.1 Genetic Algorithm (GA)

Genetic algorithm, presented by Holland [Holland, 1975], is a random research technique based on natural mechanism, combination and mutation genetic rules. Genetic algorithm starts with an initial set of random solutions which are named initial populations. Each population member is called a chromosome, which shows a solution for the problem and evolves during iterative periods. The population changes in each period and creates a new generation which is nearer to optimal solution than the previous generation.

5.2 Simulated Annealing (SA) Algorithm

In some combined optimization problems which require high computing time and have wide solution space, using SA is more effective. The main concept of this algorithm is derived from physical and thermodynamic melting principles. In this way, the temperature of a solid body *(T)* increases till it melts, then body temperature reduces gradually. In metallurgical engineering perspective, this process seeks to put atoms together in such a way that the physical state of the body forms in the best possible shape. The relationship between physical concepts and combined optimization is that different solutions in combined optimization are equivalent to different physical states of a solid body and solution costs are equivalent to different energy levels *(E)*.

5.3 The Proposed Algorithm

In this research, a hybrid algorithm which is the combination of dynamic programming, genetic and simulated annealing algorithms is presented. The visual representation of the proposed hybrid algorithm can be shown in Figure 3.

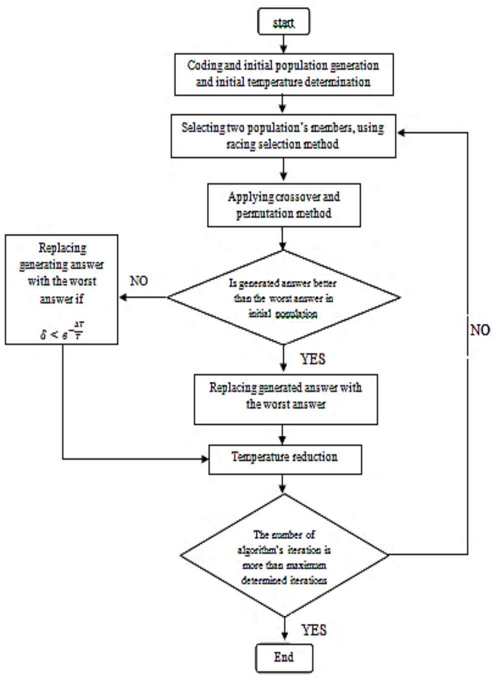

Figure 3. Flow chart of the GA-SA for the concerned problem

5.4 Chromosome Displays Mode

In this part, we intend to display a feasible solution mode for the problem. In the following, a chromosome is shown for 3 intermediate depots under specific scenarios. As can be seen, the designed chromosome for this problem is represented in 3 parts.

5.4.1 Part 1

The first part indicates a state in which whether an intermediate depot is activated or not. It is equal to 1 if the intermediate depot is activated and 0, otherwise. The matrix dimension is *1*R*, in which *R* represents the number of intermediate depots.

intermediate depot	1	2	3
Be activated or not	1	1	0

5.4.2 Part 2

The dimension of the matrix belonging to the second part is *1*l*, in which *l* represents the number of customers. Digit *1* indicates intermediate depot activation and digit 0 indicates intermediate depot inactivation. A depot is chosen for each customer among the activated depots (in this example intermediate depots *1* and *2* are activated).

Customer	1	2	3	4	5	6	7
Customer allocation	1	1	1	2	1	1	1
Scenario	1	1	1	1	1	1	1

5.4.3 Part 3

Chromosome's third part shows how to meet the demand of customers allocated to each intermediate depot. In other words, it expresses the route and the pattern of customers' serving. Considering the first row of the previous part of chromosome, customers 1, 2,3,5,6 and 7 are allocated to intermediate depot 1 based on the dynamic algorithm. So the first row of chromosome’s third part, which is relevant to the first intermediate depot, shows that the sequence of customers' serving is that a vehicle moves from intermediate depot 1 to customer 3 and subsequently to customers 5,6,2,7 and 1, then returns to the same depot.

On the other hand, as can be seen in chromosome’s second part, customer 4 is allocated to intermediate depot 2. Therefore, a vehicle moves from intermediate depot 2 to customer 4, and returns to the same depot after serving the customer.

The third intermediate depot is not activated, thus no customer is allocated to this depot.

Intermediate depot 1	3	5	6	2	7	1
Intermediate depot 2	4	-	-	-	-	-
Intermediate depot 3	-	-	-	-	-	-

To determine the optimal vehicles routes, we have employed dynamic programming, which improves the objective function value of initial generated solutions in comparison with random mode.

5.5 Finding the initial solution

Held and Krap [Held and Krap, 1962] proposed a dynamic programming approach to solve the sequential problems. Their solutions can be applied for scheduling problems, the traveling sales man (TSP) problem and assembly line balancing problems. Their proposed solution method is computationally efficient in specifying limited range. In addition, the approximate solutions might be achieved by solving the sequences of small sub-problems which have the same structures.

5.6 Fitness Function

Fitness function is a criterion to measure the quality of solution obtained by the chromosome. Each chromosome's fitness is computed based on objective function value in the mathematical model, the activation cost of each intermediate depot and customers' serving costs.

5.7 Choosing parents

In this phase, two members of the generated population are selected as parents, then a crossover operator is applied. The considered selection method in the presented algorithm is called "racing method" in which *P* members of the population are randomly selected. Then, the members, which have the best objective function values among these *P* members, are considered as parents.

5.8 Genetic operators

After selecting parents, the offspring must be generated by applying appropriate operators.

5.9 Crossover operators

In this paper, a crossover operator is used, in which we generate random numbers for each chromosome's first and second parts, and then, the crossover operator is applied to generate new offspring. Note that the first row of chromosome's second part (i.e. customers' allocation to intermediate depots) is reliant on chromosome's first part, this row is randomly generated based on initial solutions. On the other hand, chromosome's third part or routing part, completely depends on the first and the second parts. Accordingly, after applying crossover operator on the first and the second parts, we generate the third part based on initial generated solution. The following Example describes the crossover operator.

Assume that the first and the second parts of parents' chromosomes for a specific scenario is as follows:

First parent

Intermediate depot	1	2	3
Activated or not	1	1	0

Second parent

Intermediate depot	1	2	3
Activated or not	1	1	1

Assume that the second part is selected by random for crossover operator application, so the first part of offspring's chromosome is generated as below:

First offspring

Intermediate depot	1	2	3
Activated or not	1	1	1

Second offspring

Intermediate depot	1	2	3
Activated or not	1	1	0

Crossover operator is applied for parents under each scenario, separately.

5.10 Permutation Operator

In this phase, Firstly one population member is randomly selected, then a new solution is generated by creating a small change in the selected member. Permutation operator may increase the dispersion of solutions thus the searching process. In the presented algorithm, permutation operator is applied on chromosome's second part. The first part is the first row of chromosome's second part or customer's allocation to intermediate depots. The second part of permutation is applied on chromosome's third part or routing part.
Two types of permutation operators are used in the presented algorithm as follows:

5.10.1 Swap

In this method, two genes of the chromosome are selected and their places are changed with each other. It is assumed that represented chromosome in chromosome display part for a specific scenario is selected randomly for permutation operator.

Let the second and the fourth parts be selected.

Customer	1	2	3	4	5
Customer allocation	2	2	1	2	2

After changing the digits of places two and three, customers' allocation part is changed as follows:

Customer	1	2	3	4	5
Customer allocation	1	2	2	2	2

The same must be done for routing part as well.

5.10.2 Reversion

In this permutation operator, two genes are selected from the chromosome, then digits between two genes are re-arranged. In that example, points 1 and 3 of intermediate depot 2 of the chromosome's third part are selected for the reversion permutation operator.

Depot 2	4	1	5	2

The above part changes as follows, while applying permutation operator:

Depot 2	5	1	4	2

In this way, it can be seen that permutation and crossover are the two complementary operators. In other words, crossover operator impacts on two parts of a chromosome and permutation operator has influence on the other parts. It must be noted that in each permutation, selecting the permutation method occurs randomly with equal probability. Permutation operator is implemented for each scenario separately.

5.10.2.1 Comparing Generated Solution with the Worst Member of Population

After applying permutation and crossover operators and generating new solutions, each solution is compared with the worst member of initial population and would be replaced with the worst solution if it is better; otherwise, new solution is replaced with the worst existing solution in the population with the following probability:

$$\delta \leq e^{-\left(\frac{\Delta E}{T}\right)} \quad (58)$$

$$\Delta E = \frac{\text{newsol. Cost} - \text{sol. Cost}}{\text{sol. Cost}} \quad (59)$$

T=alpha*T (60)

in which new sol. cost represents objective function value of the new generated solution and T is the temperature in that iteration which reduces as equation (60), and finally δ is a uniformly generated number between 0 and 1.

5.11 Stop Criterion

As shown in Table 1, the appropriate values of the algorithm parameters are obtained by several runes of the algorithm using trial and error method. It is worth nothing that the considered criterion to stop algorithm is maximum number of generations.

Table 1.The algorithm parameters

Initial population	Maximum iteration (stop criterion)	Crossover rate	Permutation rate	Initial temperature	Temperature reduction rate
50	100	0.7	0.3	10	.99

6. Numerical experiments

To validate the proposed model and its solution approach, several numerical examples are investigated. The problem is solved under different α values. An analysis could be performed to examine the impact of changing service level on the network total cost. As can be observed in Figure.4, the increased value of α leads to the increased number of vehicles, which in turn increases the network total cost. However, we cannot see any changes in the number of vehicles when α grows from 0.6 to 0.65 and from 0.95 to 1. Therefore, the increase of objective function value could be possible due to the increase of transportation cost imposed by the increased value of service level to satisfy the fuzzy demands. Accordingly, decision makers (DM) need to determine conditions in which the demands are satisfied at higher confidence level α, however, it imposes higher costs on the network. Indeed, the DM has to make a tradeoff between cost and demand satisfaction to decide on an appropriate confidence level.

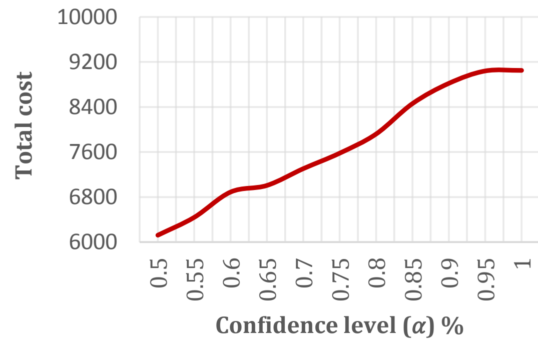

Figure 4. The impact of different α values on network total cost

In the following, we solve the problem in three different sizes (i.e. small size with 20 customers and 5 middle depots, medium size with 50 customers and 5 middle depots and large size with 100 customers and 5 and 10 middle depots) and their results are reported in Tables 2-4 using different methods including genetic algorithm, hybrid genetic algorithm, simulated annealing and also the combination of genetic and simulated annealing algorithms and dynamic programming approach.

The results imply that, on average, the combination of genetic and simulated annealing algorithms often has better performance in comparison with genetic algorithm in terms of network costs and CPU time. The proposed method which is the combination of genetic and simulated annealing algorithms and using dynamic programming to find initial solution, outperforms the three other methods in terms of network total cost, however, it increases CPU time a bit more. Therefore, a general observation confirms high efficiency of the proposed algorithm for the problem in different sizes.

The analyses of diagrams in Figure 5 indicate that computational time of the proposed hybrid algorithm increases as the service level $\boldsymbol{\alpha}$ is enhanced from 0.5 to 1, while genetic algorithm has a better performance in this situation since it reduces the computational time on average. On the other hand, the results, which is shown in Figure 6, represent the domination of the proposed hybrid algorithm performance over the three other methods in terms of network costs. As can be observed, genetic algorithm has the weakest performance among the algorithms. Moreover, although the combination of genetic algorithm and dynamic programming approach increases the computational time, it can result in the decreased network costs. As can be seen in Figure 7, in the problem of medium size, the computational time for solving the problem by genetic algorithm increases from 16 to 20 minutes as the value of α grows from 0.5 to 1, while the CPU time for the proposed hybrid algorithm decreases from 21 minutes at confidence level 0.5 and reaches 20 minutes at $\alpha = 1$. Other two methods do not show specific patterns in computational length of time. Figure 8 depicts the efficiency of the proposed hybrid algorithm among other three methods in which the lowest level of cost is achieved by applying this algorithm. Furthermore, genetic algorithm has the worst performance and other two algorithms perform quite similarly in terms of network costs.

Table 2. Detailed results for Prodhon's instances in small size

					Genetic algorithm		Genetic + Dp		Hybrid algorithm		Hybrid + DP	
	Central depot	Middle depot	Customer	α	Cost	CPU Time	Cost	CPU Time	Cost	CPU Time	Cost	CPU Time
20-5-1-2e	1	5	20	0.5	22575	15	22372	17	22446	14	22155	15
20-5-1-2e	1	5	20	0.75	24547	15	24413	18	24179	14	23687	17
20-5-1-2e	1	5	20	1	25487	14	25179	18	25145	12	24927	16
Avg					**24203**	**15**	**23988**	**18**	**23923**	**13**	**23590**	**16**
20-5-1b-2e	1	5	20	0.5	22059	13	21796	15	21881	13	21391	16
20-5-1b-2e	1	5	20	0.75	24296	15	24091	15	24096	14	23696	15
20-5-1b-2e	1	5	20	1	25436	13	25244	15	25163	15	24843	19
Avg					**23930**	**14**	**23710**	**15**	**23713**	**14**	**23310**	**17**
20-5-2-2e	2	5	20	0.5	22235	14	21897	16	22035	12	21685	15
20-5-2-2e	2	5	20	0.75	24745	14	24324	18	24435	13	23884	16
20-5-2-2e	2	5	20	1	25447	15	25151	17	25292	12	24793	16
Avg					**24142**	**14**	**23790**	**17**	**23920**	**12**	**23454**	**16**
20-5-2b-2e	2	5	20	0.5	22353	13	22148	17	22049	13	21828	15
20-5-2b-2e	2	5	20	0.75	24189	13	24078	15	24030	12	23694	15
20-5-2b-2e	2	5	20	1	25306	12	24956	17	25134	15	24631	15
Avg					**23949**	**13**	**23727**	**16**	**23737**	**13**	**23369**	**15**

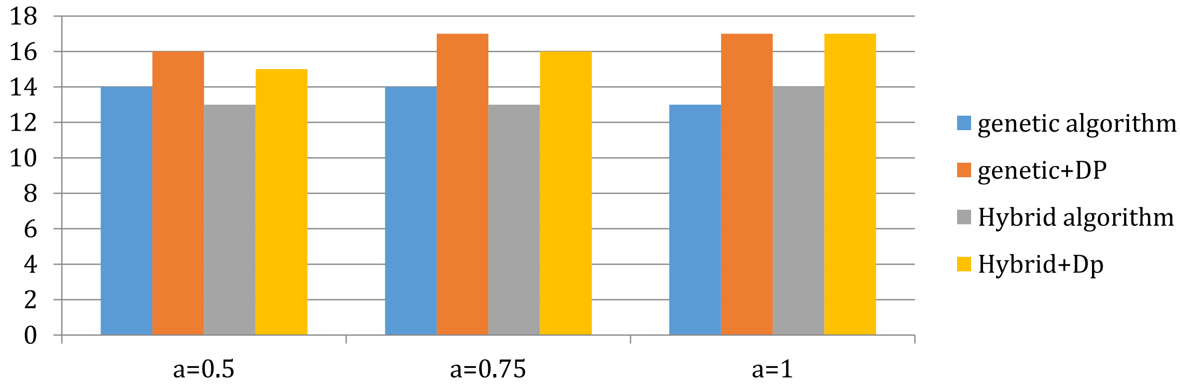

Figure 5. The comparison of CPU time performance in a small-size problem

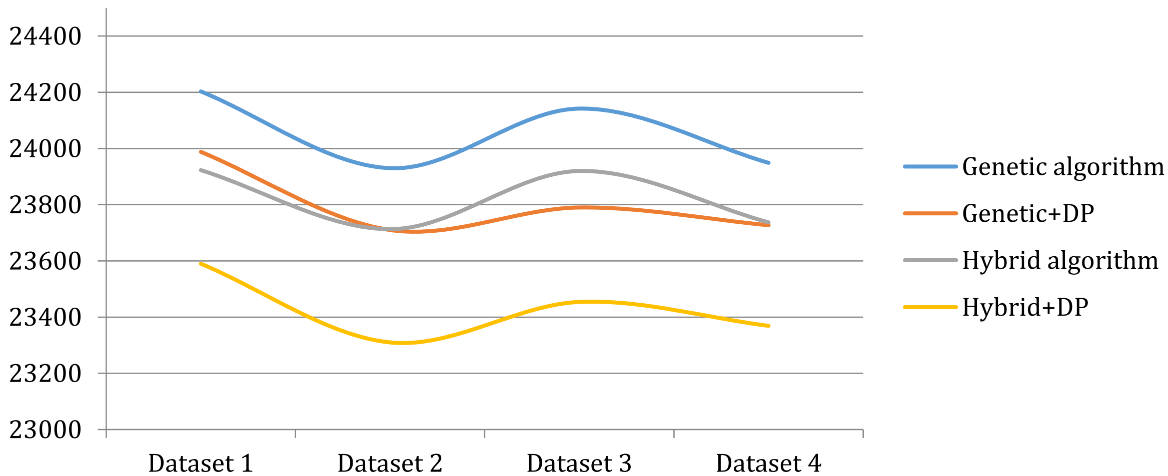

Figure 6. The comparison of cost performance in a small-size problem

Table 3. Detailed results for Prodhon's instances in Medium size

					Genetic algorithm		Genetic with Dp		Ga-SA		GA-SA with DP	
	Central depot	Middle depot	Customers	α	Cost	CPU Time	Cost	CPU Time	Cost	CPU Time	Cost	CPU Time
50-5-1-2e	1	5	50	0.5	42316	19	41368	22	41598	15	39737	18
50-5-1-2e	1	5	50	0.75	44872	17	43967	17	44203	20	42942	22
50-5-1-1e	1	5	50	1	47045	22	46455	19	46329	18	45152	20
Avg					**44744**	**19**	**43930**	**19**	**44043**	**18**	**42610**	**20**
50-5-1b-2e	1	5	50	0.5	40617	15	39859	19	39968	15	38504	22
50-5-1b-2e	1	5	50	0.75	43916	18	43041	19	43230	19	41620	20
50-5-1b-1e	1	5	50	1	46390	19	45763	21	45649	16	43691	21
Avg					**43641**	**17**	**42888**	**20**	**42949**	**17**	**41271**	**21**
50-5-2-2e	2	5	50	0.5	41165	16	40313	18	40589	16	38315	20
50-5-2-2e	2	5	50	0.75	43481	15	42921	22	42824	18	41640	20
50-5-2-2e	2	5	50	1	45636	18	45126	22	45034	15	43769	18

Avg					**43427**	**16**	**42877**	**21**	**42816**	**16**	**42241**	**19**
50-5-2b-2e	2	5	50	0.5	41655	16	41079	17	41152	16	39855	24
50-5-2b-2e	2	5	50	0.75	44528	20	43765	21	43960	16	42039	21
50-5-2b-2e	2	5	50	1	45295	17	44333	20	44641	18	42408	18
Avg					**43826**	**18**	**43059**	**19**	**43251**	**17**	**41434**	**21**
50-5-2bBIS-2e	2	5	50	0.5	40687	18	39710	21	39986	16	38057	20
50-5-2bBIS-2e	2	5	50	0.75	44519	16	43856	17	43963	16	42486	20
50-5-2bBIS-2e	2	5	50	1	47441	21	46644	17	46690	20	45421	20
Avg					**44215**	**18**	**43403**	**18**	**43546**	**17**	**41988**	**20**
50-5-2BIS-2e	2	5	50	0.5	41926	15	41156	20	41435	16	39551	22
50-5-2BIS-2e	2	5	50	0.75	44482	16	43709	17	43755	15	41867	20
50-5-2BIS-2e	2	5	50	1	45525	20	44588	21	44947	16	43185	20
Avg					**43977**	**17**	**43151**	**19**	**43379**	**16**	**41534**	**20**
50-5-3-2e	3	5	50	0.5	41453	17	40613	20	41028	16	39226	19
50-5-3-2e	3	5	50	0.75	44488	19	43789	17	43961	20	42055	21
50-5-3-2e	3	5	50	1	45490	20	44909	17	45066	16	43822	22
Avg					**43810**	**19**	**43103**	**18**	**43352**	**17**	**41701**	**21**
50-5-3b-2e	3	5	50	0.5	40455	15	39937	21	39748	16	38606	20
50-5-3b-2e	3	5	50	0.75	43211	17	42504	17	42485	19	40914	23
50-5-3b-2e	3	5	50	1	46988	21	46037	17	46251	17	44397	24
Avg					**43551**	**18**	**42826**	**18**	**42828**	**17**	**41306**	**22**

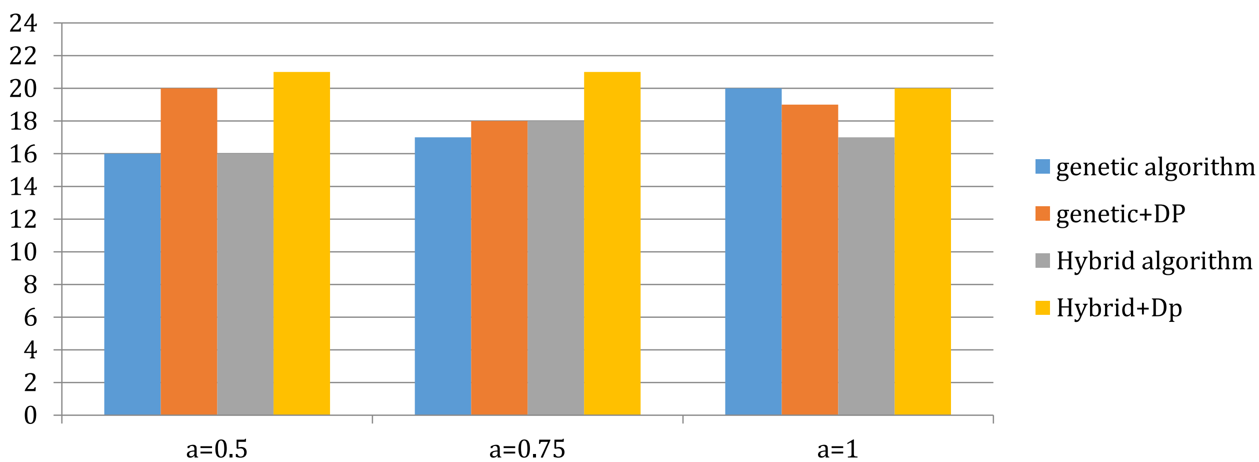

Figure 7. The comparison of CPU time performance in a medium-size problem

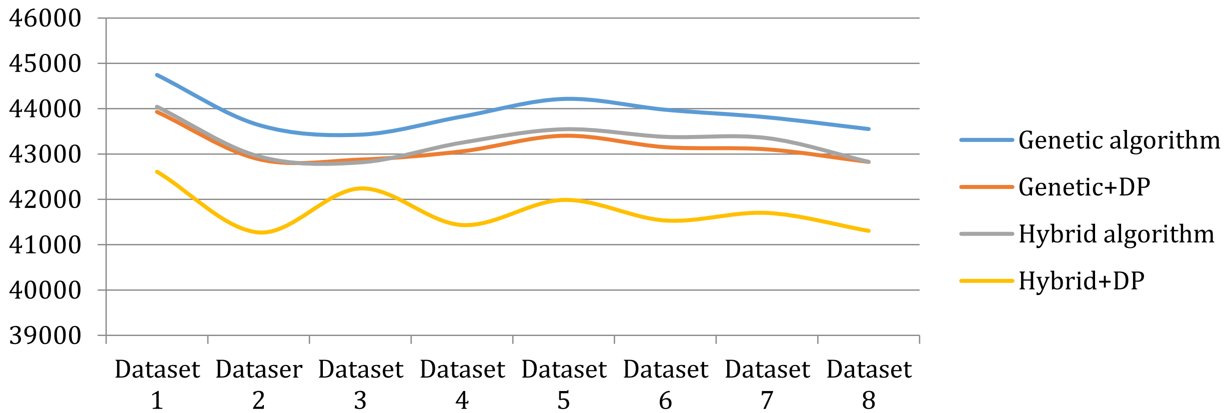

Figure 8. The comparison of cost performance in a medium-size problem

Table 4. Detailed results for Prodhon's instances in large size

	Central depot	Middle depot	Customers	α	Genetic algorithm		Genetic with Dp		Ga-SA		GA-SA with DP	
					Cost	CPU Time	Cost	CPU Time	Cost	CPU Time	Cost	CPU Time
100-5-1-2e	1	5	100	0.5	90528	83	86731	102	87645	93	82346	108
100-5-1-2e	1	5	100	0.75	96969	85	94613	102	94373	88	89592	96
100-5-1-2e	1	5	100	1	97934	90	94843	102	95258	81	91208	96
100-5-1b-2e	1	5	100	0.5	90710	82	88474	96	88684	88	84361	102
100-5-1b-2e	1	5	100	0.75	96718	82	93999	92	94626	87	89401	98
100-5-1b-2e	1	5	100	1	98297	91	94824	95	95709	90	90542	102
100-5-2-2e	2	5	100	0.5	90832	91	86855	102	87037	94	83062	108
100-5-2-2e	2	5	100	0.75	96571	89	94458	103	93560	92	89091	97
100-5-2-2e	2	5	100	1	97050	87	93926	98	93557	90	89900	108
100-5-2b-2e	2	5	100	0.5	93105	81	90025	102	90712	91	86902	98
100-5-2b-2e	2	5	100	0.75	96027	84	92983	105	92504	83	89402	98
100-5-2b-2e	2	5	100	1	98685	80	96317	91	96665	91	92009	108
100-5-3-2e	3	5	100	0.5	92869	88	89455	96	90683	89	84894	103
100-5-3-2e	3	5	100	0.75	95355	80	92683	98	92093	87	88948	95
100-5-3-2e	3	5	100	1	98118	85	94923	91	96022	94	90424	99
100-5-3b-2e	3	5	100	0.5	90260	83	86261	93	87645	93	82586	105
100-5-3b-2e	3	5	100	0.75	95797	92	93446	104	93618	94	90269	105
100-5-3b-2e	3	5	100	1	98709	86	96109	92	95373	95	01207	98
100-10-1-2e	1	5	100	0.5	123233	115	116719	113	119670	112	105960	118
100-10-1-2e	1	5	100	0.75	126105	112	121261	112	122373	106	112010	119
100-10-1-2e	1	5	100	1	128513	111	124371	110	124614	116	114566	120
100-10-1b-2e	1	5	100	0.5	124166	113	117949	120	119704	113	106923	117
100-10-1b-2e	1	5	100	0.75	126959	110	122268	111	122345	107	110343	115
100-10-1b-2e	1	5	100	1	127828	111	122782	119	122900	115	113009	124
100-10-2-2e	2	5	100	0.5	121991	108	115128	113	118716	107	105045	120
100-10-2-2e	2	5	100	0.75	126099	109	119965	113	122091	106	109011	120
100-10-2-2e	2	5	100	1	127985	110	122631	120	124901	121	112636	116
100-10-2b-2e	2	5	100	0.5	123749	112	119654	119	119075	107	108214	124
100-10-2b-2e	2	5	100	0.75	125661	111	119787	117	121682	110	110094	119
100-10-2b-2e	2	5	100	1	128390	105	123668	118	123444	123	113988	119
100-10-3-2e	3	5	100	0.5	124176	107	119106	115	120899	112	107672	121
100-10-3-2e	3	5	100	0.75	126239	113	120467	118	121485	107	110257	125
100-10-3-2e	3	5	100	1	128946	112	122801	110	125568	118	112757	125
100-10-3b-2e	3	5	100	0.5	121612	108	115624	114	117435	110	106567	116
100-10-3b-2e	3	5	100	0.75	125721	111	119740	117	122015	114	110374	123
100-10-3b-2e	3	5	100	1	127655	115	121086	114	123321	116	11721	125
200-10-1-2e	1	10	200	0.5	174067	154	168323	154	168075	147	155676	161

200-10-1-2e	1	10	200	0.75	182085	151	172672	160	176259	140	159064	155
200-10-1-2e	1	10	200	1	187467	144	181091	149	181713	152	170151	163
200-10-1b-2e	1	10	200	0.5	176669	141	167170	146	169890	145	154540	159
200-10-1b-2e	1	10	200	0.75	184859	152	178434	156	178004	148	163401	164
200-10-1b-2e	1	10	200	1	192845	140	184261	156	187094	154	173838	164
200-10-2-2e	2	10	200	0.5	179338	154	172086	159	172000	152	158174	150
200-10-2-2e	2	10	200	0.75	184940	148	176574	158	177402	144	159917	163
200-10-2-2e	2	10	200	1	193829	143	187412	153	189361	149	176823	160
200-10-2b-2e	2	10	200	0.5	178110	146	172082	157	171671	152	157737	157
200-10-2b-2e	2	10	200	0.75	184321	150	175999	151	177437	155	160942	163
200-10-2b-2e	2	10	200	1	194138	151	184657	155	186879	154	173510	153
200-10-3-2e	3	10	200	0.5	174845	140	165346	149	168546	150	152575	160
200-10-3-2e	3	10	200	0.75	181944	154	176330	152	177870	155	163530	155
200-10-3-2e	3	10	200	1	190538	151	181450	149	185284	155	169181	160
200-10-3b	3	10	200	0.5	177568	145	168755	153	172264	142	155968	158
200-10-3b	3	10	200	0.75	182274	140	175238	153	175575	144	159415	159
200-10-3b	3	10	200	1	190989	154	184039	147	186004	148	171927	151
Avg					**135007**	**115**	**129590**	**122**	**130728**	**117**	**116919**	**127**

6.1 The Validation of Proposed Solution Approach

To evaluate the performance of the proposed hybrid solution approach several small-size instances have been investigated in this section and the respective results are reported in Table 5. As can be observed, for instances 1 and 2, the results obtained from solving the concerned problem by the proposed hybrid solution method are the same as the ones obtained by the exact solver GAMS while having remarkably less computing time. However, for instances 3-5, the proposed solution approach achieves the solutions with small gap in comparison to the ones obtained by GAMS. For the problem in larger size the exact solver could not achieve a solution in reasonable length of time while the proposed algorithm performs well as the problem gets larger.

6.2 Sensitivity Analysis on The Confidence Level

This section aims to investigate the changes in the number of depots by varying service level α. To this aim the problem is solved for instances with 20 customers (small size) and 50 customers (medium size), as shown in Figures 9 and 10. A general observation is that the number of required depots increases as the service level enhances. Better to say, satisfying higher percentage of customers' total demand generally utilizes the current capacities of existing depots. As long as the capacities of current depots are adequate to respond customer's demands, no more depots will be required. However, the pattern will change if the existing depots cannot be responsible for satisfying customers' demand at the desired level. In such situation, more depots will be added to the current ones so that the desired satisfaction level is met**.**

As can be seen in Figure 11, the required number of middle depots will enhance from 2 to 4 as the confidence level increases from zero to 1 in small size. Similarly, the same pattern will be noticed in Figure 12 and the required number of middle depots will reach 5 under confidence level 1 in medium-size problem.

Another analysis could be performed to observe the changes in the number of required vehicles as a result of increasing the confidence level. To do so, as can be seen in Figures 11 and 12, the problem is solved in both small and medium sizes, respectively, under a number of confidence levels. The results imply that the number of vehicles which are required to satisfy customers' demands will increase when the confidence level enhances. This finding can be observed in the following figures as well. For example, in small-size problem, the number of required vehicles changes from 4 to 7 as the confidence level increases from 0 to 1. With similar pattern, the number of required vehicles increases from 9 to 12 as a result of increasing the confidence level from 0 to 1 in medium size.

6.3 Sensitivity Analysis on The Capacity of Vehicles

In this section, we analyze the impact of changes in the capacity of vehicles on the cost performance of the network. Figures 13 and 14 depict network cost reduction over different capacity levels for medium-size problem at confidence levels $\alpha = 0.7$ and $\alpha = 0.9$, respectively. As can be observed in the following diagrams, the increased capacity of vehicles results in decreased network costs. In some parts, however, the curve is steeper which shows reduced network costs and thus increased cost savings caused by the reduced transportation cost in total since the frequency of transportation decreases when the capacity of vehicles is increased. For instance, at confidence level 0.7, as the increase in capacity level reaches 5%, the amount of cost savings will become about 0.4% while this amount of savings will be 1.2% when the capacity increases to 30%. Consequently, these levels can be noticed as appropriate points for making a considerable reduction in network costs, however, choosing the level of capacity increase based on the decision makers' policies. In some parts, however, the slope of the curve will decrease little by little until it comes to zero and no changes would be seen as the capacity is increased, which means that no capacity shortage has occurred. In other words, the current capacity of vehicles can satisfy the desired amount of demands. Moreover, by increasing the service level from 0.7 to 0.9 the amount of cost savings will decrease resulting from the increase in network costs.

Table 5. summary of results (exact solver versus the proposed hybrid approach)

Instance no.	Central depot	Middle depot	customer	Network cost		CPU time (s)	
				GAMS	Hybrid algorithm	GAMS	Hybrid algorithm
1	1	2	5	11516	11516	328	12
2	1	3	8	12154	12154	391	14
3	1	4	10	12721	12793	407	15
4	1	5	12	15307	15391	364	15
5	1	5	15	17914	17973	312	13

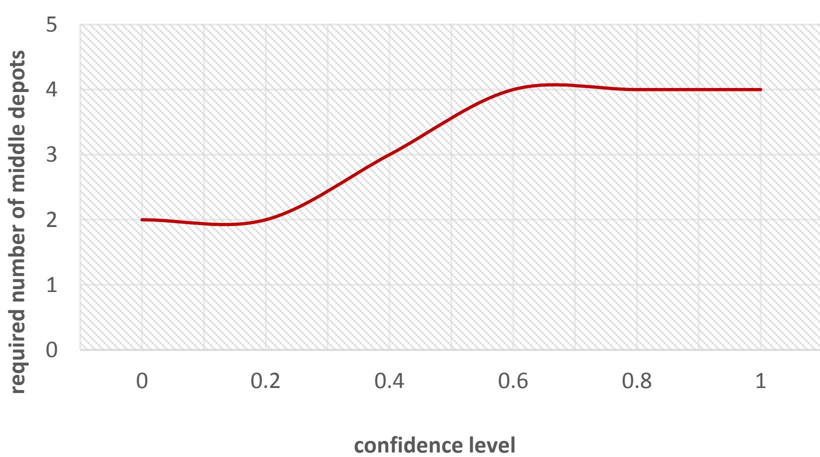

Figure. 9. The required number of middle depots under different confidence levels in small size (20-5-1-2e)

Figure 10. The required number of middle depots under different confidence levels in medium size (50-5-1-2e)

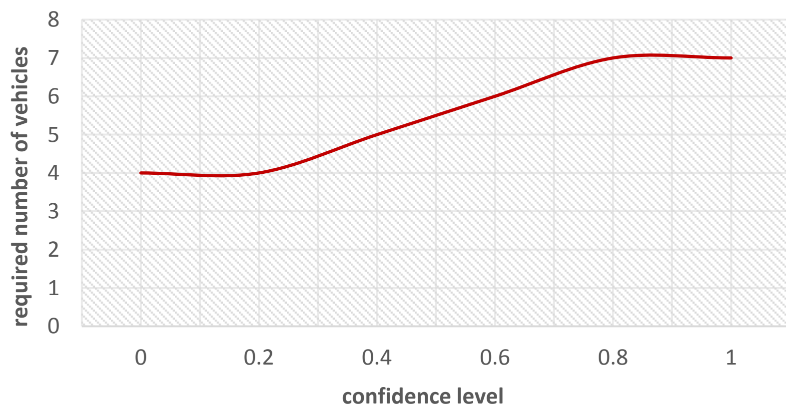

Figure 11. The required number of vehicles under different confidence levels in small size (20-5-1-2e)

Figure 12. The required number of vehicles under different confidence levels in medium size (50-5-1-2e)

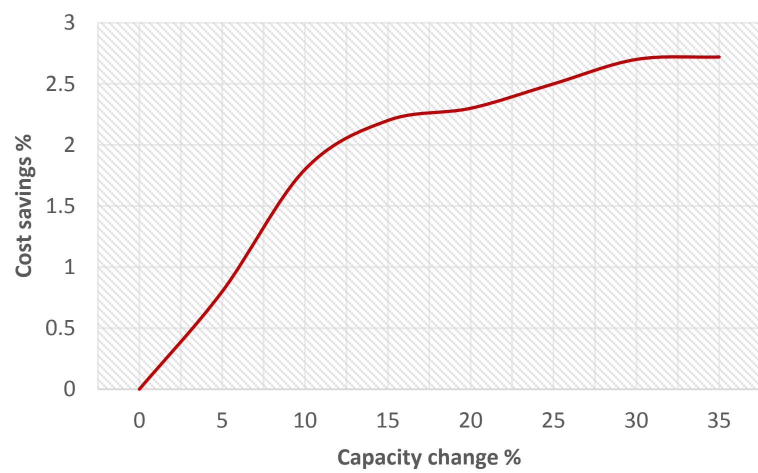

Figure 13. The impact of vehicle capacity changes on cost performance; $\alpha = 0.7$

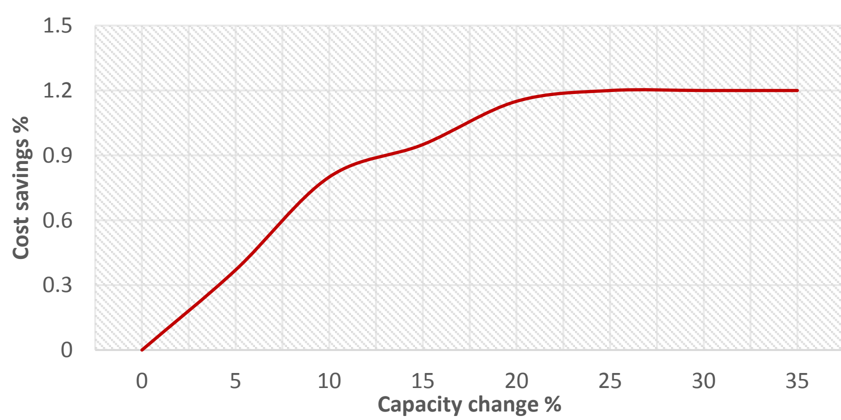

Figure 14. The impact of vehicle capacity changes on cost performance; $\alpha = 0.9$

7. Conclusions and Future Research Direction

This paper presents a mixed integer programming model for a two-echelon location-routing problem with simultaneous pickup and delivery. To come closer to reality, the amount of demands are considered to be a fuzzy parameter. To handle the demand uncertainty, a fuzzy programming approach based on the credibility theory is devised. A hybrid algorithm based on genetic algorithm (GA) and simulated annealing (SA) algorithm is tailored to solve the proposed model. The results achieved from solving the problem in different sizes imply that the proposed hybrid algorithm outperforms other algorithms within reasonable length of time. The domination of proposed hybrid algorithm over the other investigated algorithms even strengthens when the size of problem becomes larger. Noteworthy, a comprehensive sensitivity analysis has been performed and several valuable insights have been extracted as follows. (1) our findings from the sensitivity analysis of changes in the capacity of vehicles on the network costs imply that an appropriate increase in the capacity of vehicles can be devised as a strategy to decrease the total cost of network. For example, if the capacity of vehicles is increased up to %10, the system's cost savings will reach nearly %2. (2) fuzzy chance-constrained programming approach provides a confidence level for satisfying the demands such that the number of required vehicles and middle depots to satisfy customer's demands enhances as the confidence level increases. (3) an increase in credibility level would result in increasing total cost, thus, decreasing the cost savings of the network. (4) in comparison to genetic algorithm, the proposed hybrid solution approach generally improves the computing time in small, medium and large sizes of the problem. (5) using a mixed approach of dynamic programming and GA or hybrid approaches will increase the total computing time. A series of future research can be extended in this subject of investigation. For instance, scenario reduction methods could be used to reduce the problem size in large dimensions. Furthermore, using clustering methods could decrease the computing time and the problem complexity. Other heuristic algorithms can be applied to solve the problem and compare the corresponding results with the ones obtained by the proposed algorithm. Researchers could also investigate the problem over a multi period planning horizon considering inventory problem for intermediate depots and customers. A number of approaches such as robust optimization could be devised to handle the data uncertainties.

8. References

-Ambrosino, D. and Scutella, M. (2005) "Distribution network design: new problems and related models", European journal of operational research, Vol. 165, No. 3, pp. 610-624.

-Cheraghi, S., Hosseini-Motlagh, S. M. and Ghatreh Samani, M. R. (2016) "Integrated planning for blood platelet production: a robust optimization approach", Journal of Industrial and System Engineering, Vol. 10 (SI: Healthcare)

-Cheraghi, S. and Hosseini-Motlagh, S. M. (2016) "Optimal blood transportation in disaster relief considering facility disruption and route reliability under uncertainty", International Journal of Transportation Engineereing Vol. 4, No. 3, pp. 225-254.

-Golozari, F., Jafari, A. and Amiri, M. (2013) "Application of a hybrid simulated annealing-mutation operator to solve fuzzy capacitated location-routing problem", The International Journal of Advanced Manufacturing Technology, Vol. 67, No. 5-8, pp. 1791-1807.

-Holland, J. H. (1975) "Adaptation in natural and artificial systems", Ann Arbor, MI: University of Michigan Press.

-Held, M. and Karp, R. M. (1962) "A dynamic programming approach to sequencing problems", Journal of the Society for Industrial and Applied Mathematics, Vol. 10, No. 1, pp. 196-210.

-Hiassat, A., Diabat, A. and Rahwan, I. (2017) "A genetic algorithm approach for location-inventory-routing problem with perishable products", Journal of Manufacturing Systems, Vol. 42, pp. 93-103.

-Hosseini-Motlagh S, Cheraghi, S. and Ghatreh Samani, M. (2016) "A robust optimization model for blood supply chain network design. IJIEPR. Vol. 27, No. 4, pp. 425-444.

-Jacobsen, S. K. and Madsen, O. B. (1980) "A comparative study of heuristics for a two-level routing-location problem", European Journal of Operational Research, Vol. 5, No. 6, pp. 378-387.

-Jokar, A. and Hosseini-Motlagh, S.M. (2015) "Impact of capacity of mobile units on blood supply chain performance: results from a robust analysis", International Journal of Hospital Research, Vol. 4, No. 3, pp.101-105.

- Karaoglan, I., Altiparmak, F., Kara, I. and Dengiz, B. (2009) "Formulations for a location-routing problem with simultaneous pickup and delivery". Research paper,/http://w3. gazi. edu. tr/_fulyaal/Papers/LRPSPD_MIP Formulations.

- Karaoglan, I., Altiparmak, F., Kara, I. and Dengiz, B. (2011) “A branch and cut algorithm for the location-routing problem with simultaneous pickup and delivery”, European Journal of Operational Research, Vol. 211, No. 2, pp. 318-332.

- Karaoglan, I., Altiparmak, F., Kara, I. and Dengiz, B. (2011) “A branch and cut algorithm for the location-routing problem with simultaneous pickup and delivery”, European Journal of Operational Research, Vol. 211, No. 2, pp. 318-332.

-Laporte G (1988) Location Routing Problems. In: Golden B, Assad A (eds) Vehicle Routing: Methods and Studies, North-Holland, Amsterdam, pp. 293–318

-Liu, B. and Liu, Y. K. (2002) "Expected value of fuzzy variable and fuzzy expected value models", IEEE Transactions on fuzzy systems, Vol. 10, No. 4, pp. 445-450.

-Madsen, O. (1983) "Methods for solving combined two level location-routing problems of realistic dimensions", European Journal of Operational Research, Vol.12, No. 3, pp. 295-301.

-Majidi, S., Hosseini-Motlagh, S. M., Yaghoubi, S. and Jokar, A. (2017) " Fuzzy green vehicle routing problem with simultaneous pickup delivery and time windows", RAIRO-Operations Research. DOI:https://doi.org/10.1051/ro/2017007.

-Majidi, S., Hosseini-Motlagh,S. M. and Ignatius, J. (2017) "Adaptive large neighborhood search heuristic for pollution routing problem with simultaneous pickup and delivery", Soft Computing. DOI:https://doi.org/10.1007/s00500-017-2535-5

-Min, H., Jayaraman, V. and Srivastava, R. (1998) "Combined location-routing problems: A synthesis and future research directions", European Journal of Operational Research, Vol.108, No. 1, pp. 1-15.

-Nadizadeh, A. and Hosseini Nasab, H. (2014) "Solving the dynamic capacitated location-routing problem with fuzzy demands by hybrid heuristic algorithm", European Journal of Operational Research, Vol. 238, No. 2, pp. 458-470.

-Nagy, G. and Salhi, S. (2007) "Location-routing: Issues, models and methods", European Journal of Operational Research, Vol. 177, No. 2, pp. 649-672.

-Nambiar, J. M., Gelders, L. F. and Van Wassenhove, L. N. (1981) "A large scale location-allocation problem in the natural rubber industry", European Journal of Operational Research, Vol. 6, No. 2, pp. 183-189.

-Nikbakhsh, E. and Zegordi, S. H. (2010) "A heuristic algorithm and a lower bound for the two-echelon location-routing problem with soft time window constraints", Scientia Iranica Transaction E: Industrial Engineering, Vol. 17, No. 1, pp. 36-47.

-Nikkhah Qamsari, A. S., Hosseini-Motlagh, S. M., and Jokar, A. (2017) "A two-phase hybrid heuristic method for a multi-depot inventory-routing problem", International Journal of Transportation Engineering, Vol. 4, No. 4, pp. 284-304.

-Prodhon, C. and Prins, C. (2014) "A survey of recent research on location-routing problems", European Journal of Operational Research,Vol. 238No. 1, pp. 1-17.

-Riahi, N., Hosseini-Motlagh, S. M. and Teimourpour, B. (2013) " Three-phase Hybrid times series modeling framework for improved hospital inventory demand forecast" International Journal of Hospital Research Vol. 2, No. 3, pp. 130-138.

-Rieck, J., Ehrengerg, C. and Zimmerman, J. (2014) "Many-to-many location-routing with inter-hub transport and multi-commodity pickup-and-delivery", European Journal of Operational Research, Vol. 236, No. 3, pp. 863-878.

-Riquelme-Rodríguez, J., Gamache, M. and Langevin, A. (2016) " Location arc routing problem with inventory constraints", Computers and Operation Research, Vol. 76, pp. 84-94.

-Schiffer, M. and Walther, G. (2017) "The electric location routing problem with time windows and partial recharging", European Journal of Operational Research. DOI:https://doi.org/10.1016/j.ejor.2017.01.011.

-Tavakkoli-Moghaddam, R., Raziei, Z. and Tabrizian, S. (2016) "Solving a bi-objective multi-product vehicle routing problem with heterogeneous fleets under an uncertainty condition", International Journal of Transportation Engineering, Vol. 3, No. 3, pp. 207-225.

-Wasner, M. and Zapfel, G. (2004) "An integrated multi-depot hub-location vehicle routing model for network planning of parcel service", International Journal of Production Economics, Vol. 90, No. 3, pp. 403-419.

-Watson-Gandy, C. D. T. and Dohrn, P. J. (1973) "Depot location with van salesmen a practical approach", Omega, Vol. 1, No. 3, pp. 321-329.

-Webb, M. H. J. (1968) "Cost functions in the location of depots for multiple-delivery journeys", Journal of the Operational Research Society, Vol.19, No. 3, pp.311-320.

-Zarandi, M. H. F., Hemmati, A., Davari, S., and Turksen, I. B. (2013). “Capacitated location-routing problem with time windows under uncertainty”, Knowledge-Based Systems, Vol.37, pp. 480-489.

-Zhu, H. and Zahng, J. (2009) "A credibility-based fuzzy mathematical programming model for APP problem" Artificial Intelligence and Computational Intelligence, AICI'09. International Conference on. IEEE, pp. 455-459.