

# Part A- Experimental: Experimental Analysis of Crack Propagation in Pre-stressed Concrete Sleepers by Fracture Mechanics

S.Mohammad Farnam <sup>1</sup>, Fereydoon Rezaie <sup>2</sup>

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## Abstract

This study investigates propagation of mode I crack in B70 pre-stressed concrete sleepers by fracture mechanics approach. A new experimental analysis is done for notched B70 pre-stressed concrete sleepers with Replica test and image analysis. A scanning electron microscope test (SEM) and an image analysis are applied for the Replica test in order to determine crack length and crack mouth opening displacement (CMOD). The experimental data extracted from the three-point bending load test of B70 sleepers are analyzed with fracture mechanics method. In this study, the fracture mechanics parameters of a sleeper are investigated based on nonlinear fracture mechanics (NLFM) principles for concrete material. Sleepers with initial crack width of 8 mm and different initial crack lengths of 5 mm to 45 mm, with a 10 mm increasing step, are tested. Five specimens' of each group are loaded under three-point bending load test, in order to determine the propagated crack length, crack mouth opening displacement (CMOD), final load and the specimens' energy. The results showed that by increasing the crack-to-depth ratio, both final load and specimens' energy values are decreased linearly. Also, these analyses confirm that the structural behavior of the pre-stressed concrete sleepers can be predicted by a simple fracture mechanics test, such as beams in bending, provided that the related structural conditions like initial crack length and CMOD, are known.

**Keywords:** Fracture mechanics, crack propagation, pre-stressed concrete sleeper B70, crack length, CMOD.

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Corresponding author E-mail: frrezaie94@gmail.com

1. PhD Candidate, Department of Civil Engineering, Bu-Ali Sina University, Hamedan, Iran
2. Associate Professor, Department of Civil Engineering, Bu-Ali Sina University, Hamedan, Iran

## **1. Introduction**

Railway transportation is one of the most well-known kinds of transportation in many countries and maintaining its safety and operation is very important. For this reason, many items like different components of a railway line should be examined and studied. Sleepers are key components of every railway line which they receive the wheels load from the rails and transfer it to the sub layers. The force is then transferred to the ballast layer, while a constant spacing between the lines is maintained [Aikawa, 2013]. Sleepers are constructed from different materials, such as timber, steel, and concrete [Taherinezhad, 2014]. Timber sleepers were mostly common in the past, but nowadays, their usage has been limited due to their high price and wood shortage. On the other hand, while timber sleepers can be eaten by insects, concrete sleepers have a good resistance to environmental conditions. Thus, the using of concrete sleepers has been increased quickly in many railway lines across the world. Also, concrete sleepers are cheaper than timber or steel ones. With the development of railway lines and using of heavy wagons and high-speed trains, nonconcrete sleepers have proven to be inefficient. Therefore, sleepers with high-strength concrete and rebar are used to achieve higher capacities. Mono-block B70 pre-stressed concrete sleepers are more common in I.R. Iran railways due to their higher strength, more durability, and lighter weight.

Chen et al investigated parametric study on damage and load demand of pre-stressed concrete cross-tie and fastening systems with a numerical method for focused on developing an analytical framework of the element [Chen et al. 2014]. They also modeled and validated the fastening systems and concrete sleepers for improve the

design and performance of systems in 3D finite element model [Chen et al. 2014]. Rezaie, Shiri and Farnam investigated longitudinal crack propagation along pre-stressed concrete sleepers. They modeled the effects of extra pressure in rawl plugs by applying a uniform cylindrical pressure inside the holes. Also, an experimental test was performed to observe fracture and damage mechanism of B70 pre-stressed concrete [Rezaie, Shiri and Farnam, 2012]. They also studied on sensitivity analysis of the numerical model of B70 pre-stressed concrete with the changes of pre-stressing force and tensile strength in 2016 [Rezaie, Bayat and Farnam, 2016]. González-Nicieza et al developed a failure analysis of a railway track used for transporting heavy haul industrial freight. The aim of the study is to describe the method with which this type of failure should be analyzed. They developed a specific case, establishing the causes of failure and offering guidelines for improving the design and upkeep the sleepers and ballast on which the tracks are laid [González-Nicieza et al, 2008]. In 2009, experimental investigation was constructed at the University of Wollongong to achieve the accumulative impact damage and crack propagation for pre-stressed concrete sleepers by Kaewunruen and Remennikov [Kaewunruen and Remennikov, 2009]. They also described the detail of the high-capacity impact testing machine, as well as the instrumentation, the calibration, and the analysis of failure mode, crack propagation, flexural toughness, and energy absorption mechanisms with respect to railway pre-stressed concrete sleepers [Kaewunruen and Remennikov, 2011].

Concrete behavior is quasi-brittle in fracture mechanics point of view. As a result, linear elastic fracture mechanics (LEFM) theory can be probably used to describe crack growth, fracture toughness, and other parameters of concrete

fracture mechanics. The use of LEFM for concrete was documented by Kaplan [Kaplan, 1961]; however, it has been shown by a number of researchers, including Shah and Mac-Garry [Shah and Mac-Garry, 1971] that it is not an appropriate choice. Jokūbaitisa, Valivonisa and Marčiukaitisa discussed possible causes of damage and the deterioration of pre-stressed concrete sleepers with numerical method in 2016 [Jokūbaitisa, Valivonisa and Marčiukaitisa, 2016]. In order to determine fracture mechanics model of concrete, a large number of experimental and numerical studies have been done on the properties of the fracture of some concrete elements, given different conditions of the elements or by taking various sizes [Mechtcherine, 2009; Kim et al. 2009; Zhang et al. 2009]. Grassi and Davies are used lattice approach to describe the mechanical interaction of a corroding reinforcement bar, the surrounding concrete and the interface between steel reinforcement and concrete [Grassi and Davies, 2011]. Ohno and Ohtsu classified cracks in concrete with the simplified Green's functions for moment tensor analysis (SiGMA) and compared and discussed from results of three types of concrete failure tests [Ohno and Ohtsu, 2010]. In recent years, studies have been conducted analyzing fracture mechanics of reinforced concrete; however, very few of these studies have addressed the pre-stressed concrete, especially, using experimental and numerical methods [Rezaie, Bayat and Farnam, 2016; Rezaie and Farnam, 2015]. Rezaie and Farnam investigate on fracture mechanics of pre-stressed concrete sleepers with numerical method in 2015 [Rezaie and Farnam, 2015]. Therefore, in this paper, the

emphasis is placed on the experimental analysis of fracture and damage mechanics of B70 pre-stressed concrete sleepers. The main fracture parameters of a notched pre-stressed concrete sleeper, such as the crack length, crack mouth opening displacement (CMOD), final load and the specimens' energy are calculated with Replica test and NLFM theory image analysis.

## 2. Methodology of Research

### 2.1. Specimens

In this study, a negative three-point bending test is applied to the center of notched B70 pre-stressed concrete sleepers. Three-point bending test is recommended by many international railway codes to estimate the sleepers bearing capacity. As a matter of fact, in order to investigate the beam behavior of pre-stressed concrete sleepers, three-point bending test should be applied. The use of this method is also applicable for other structures. In fracture mechanics analysis, this test is also used for other structures with a notch [Wang et al. 2016; Dong et al. 2016]. The test method is based on negative center load test in "AREMA Chapter 30", "Australian Standard, AS 1085-14". In Figure 1, the B70 pre-stressed concrete sleepers are shown in three dimension and section. The concrete strength which is determined by Iranian code 301 must be at least equal to 59 MPa on 150×150 mm cubic samples after 28 days. Further information about concrete mix design is shown in Table 1 for 1 m<sup>3</sup> concrete.

Table 1. Mix design of concrete.

<i>Sand</i>	<i>Gravel</i>	<i>Cement</i>	<i>Water</i>	<i>D<sub>Max</sub></i>	<i>W/C</i>
300 Kg	400 Kg	170 Kg	60 Kg	25 mm	0.35

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**Figure 1. 3D and section of B70 pre-stressed concrete sleepers.**



**a. The center of sleepers is painted before cutting.**



**b. Center notched sleepers.**

**Figure 2. The center of sleeper specimens.**

Initial lengths of notches' are 5, 15, 25, 35, and 45 mm, respectively. In Figure 2a, the center of sleepers is painted before cutting and in Figure 2b, center notched sleepers are shown. Five specimens' are tested for each notch length. The width of notches which are created by a portal crane is 8 mm for all specimens. Longitudinal ST-160 strands are used for manufacturing of the B70 pre-stressed concrete sleepers. The other mechanical properties of strands are shown in Table 2. In production line of B70 sleepers, the prepared strand is pulled by a hydraulic jack up to 280 bars (equal to 5.4 ton for each strand). After strands pulling, concrete is placed in sleeper molds and is completely compacted by vibrating machines. The sleepers are then cured within 24

hours and ready to applying pre-stressing force by releasing the strands.

## 2.2 Experimental Content

Full scaled sleeper samples are used for the experiments after being manufactured and cured to the required strength and specifications. In this study, all sleeper specimens are tested by a three-point bending apparatus which is equipped by a digital load cell (ton unit) and a displacement gage as shown in Figure 3. The applied load is static and increases at a rate of 20 kN/min based on the recommendation of "Australian Standard, AS 1085-14". The bending load is applied in 8 equal steps.

**Table 2. Mechanical properties of strands.**

Strand type	Density	$\nu$	$f_y$	$E_t$	$E_s$	Diameter
ST-160	7850 kg/m <sup>3</sup>	0.3	1400 MPa	2000 MPa	200000 MPa	7 mm



**a. Three-pointed bending test with the load cell and the displacement gage.**

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**b. Digital load cell.**

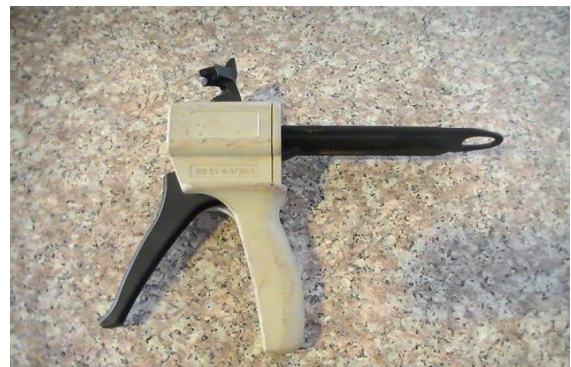


**c. Displacement gage.**

**Figure 3. Installed equipment on jack.**



**a. The package of Replica compound.**



**b. The injection tool.**



**c. The Replica compound injection with Replica set.**

**Figure 4. The Replica set and injection method.**

The main collected data includes crack length, crack mouth opening displacement (CMOD), final load and the specimens' energy of the sleepers pre-stressed concrete. In order to analyze the whole load development process and to test load initiation for each specimen, Replica test and image processing is used.

### 2.3 Analysis Methods

The main objective of testing notched B70 pre-stressed concrete sleepers is to determine the related parameters of crack growth, i.e. crack length, CMOD, failure mode and other fracture mechanics parameters. In this research, measuring of these parameters is done by a new method called Replica test. The Replica material is a carbon base compound of two hard and soft components, which is available as a package (Figure 4a). Replica set is an assembly of an injection tool (Figure 4b), the compound package and a nozzle. As shown in Figure 4c, the Replica compound can be injected in crack location by the device set.

The soft component gives high permeability to the Replica in order to penetrate through the pores of sleeper samples. The hard component makes the

Replica hard like solid materials and also the Replica easily separates from the sample.

Replica is a black flowing liquid that can penetrate in micro cracks and take crack form, as shown in Figure 4c. In this study, before the replica hardening, a plastic sheet is used in order to prevent wasting of the Replica liquid and also making its penetration possible in vertical surface cracks. After the replica hardening, it becomes an elastic material that can be separated from the cracks surface, as shown in Figure 5.

In this research, the Replica test is used for three-point negative bending test of B70 pre-stressed concrete sleepers. The Replica compound takes the three dimensional shape of cracks and therefore crack length, CMOD and failure form can be measured exactly. The analysis of Replica test results is performed by two different methods. The first method is done by scanning electronic microscope (SEM) test on small Replica samples. To perform SEM method, Replica samples are covered with a thin layer of gold at first (Figure 6). Then the sample is located under SEM apparatus lens (Figure 7) and output results like Figure 8 are derived. The second method is done by image analysis on large Replica samples (Figure 9)



Figure 5. Separation of elastic hardening Replica from the cracks surface.

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**Figure 6. Covering the Replica sample with the thin layer of gold.**



**Figure 7. SEM apparatus.**



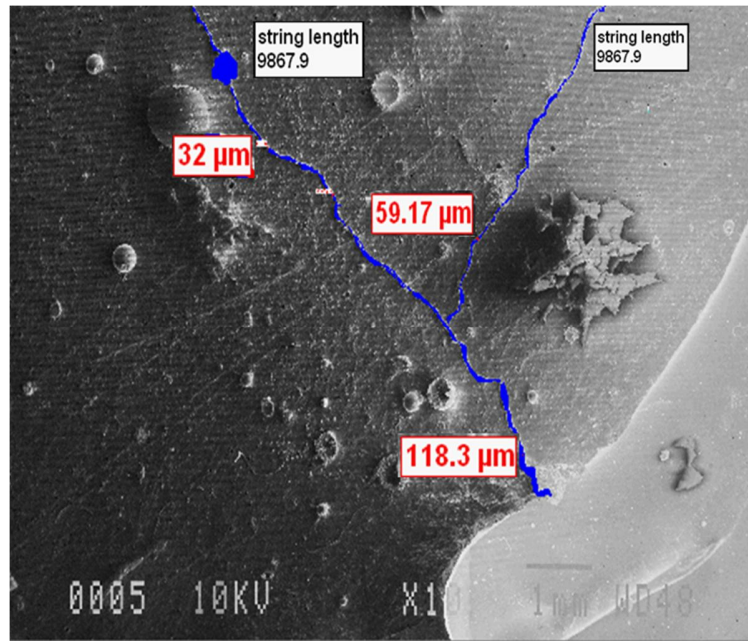


Figure 8. The SEM analysis of small Replica sample.



Figure 9. Image analysis on the large Replica samples.

### 3. Results

In this paper, load deformation diagrams such as load–crack length and load–CMOD curves, which might be of interesting to the design engineer, can be obtained using the abovementioned experimental procedure. Crack propagation of the 25 samples of pre-stressed concrete sleeper under 3-point-bending load is tested with the Replica test and image analysis. Sleepers are with initial crack lengths of 5 mm increasing to 45 mm (10 mm steps) and initial crack widths of 8 mm. Sleeper cracks are created in the notch and both sides of it, as shown in Figure 8. In this figure, crack propagation is almost symmetric in both sides of the notch [Rezaie and Farnam, 2015]. Therefore, both crack length and CMOD are only calculated in the notch itself and one side of it.

#### 3.1 Load-crack Length Diagram

Crack propagation is the most important parameter of a fracture mechanics based design [Zhang, Shi and Tu, 1995; Shahani and Moayeri Kashani, 2014]. As the crack grows larger, residual strength decreases in the element resulting in the load carrying capacity of the element being decreased. In this Section, crack propagation is calculated in terms of load –crack length curves in the notch. Crack growth of pre-stressed concrete sleepers is studied in the notch

and both sides of it using the Replica test and image analysis.

##### 3.1.1 Load-crack Length Diagram in Notch Position

Figure 10 shows the effect of notch length on the average of 5 sleeper specimens' in terms of load–crack length diagrams. Sleepers initial crack lengths of 5 mm and the corresponding results are compared together in average of five specimens' are made in each group.

It can be seen that for a given  $d_0$ , the larger the  $a_0$ , the lower the first crack strength and the 3-point-bending load test. As Figure 10 shows, resistance to crack growth is high in all of the curves. This is because of the initially sharp slopes of the curves (e.g. 612% for  $a_0=5$  mm). In the next step, resistance to crack growth considerably reduces. This is because of the relatively mild slopes of the curves in this step (e.g. 25% for  $a_0=5$  mm). In the final step, the sleepers' resistance to crack growth decreases and the cracks grow unstably without more loads being applied.

Although the mentioned conditions were the same for all curves, the curves are different. In Figure 10 resistance to crack growth increases as the notch length decreases. The load bearing percentage increase from 612% to 823% respectively in  $a_0= 5$  and 45 mm. In the next step, slope curves increase from 25% to 41% respectively in  $a_0= 5$  and 45 mm.

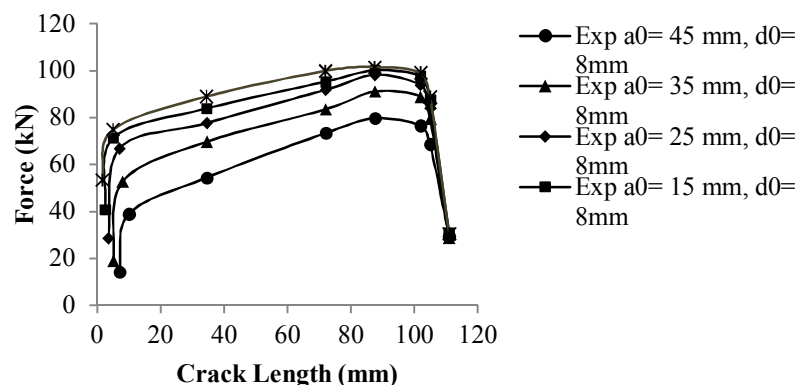


Figure 10. Effect of notch length  $a_0$  in the terms of load–crack length curves in the notch.

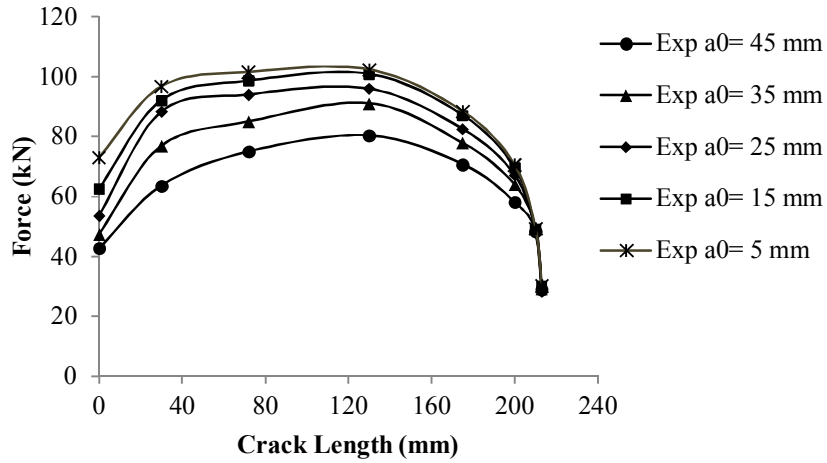


Figure 11. Effect of notch length  $a_0$  on the terms of load–crack length curves on the sides of the notch.

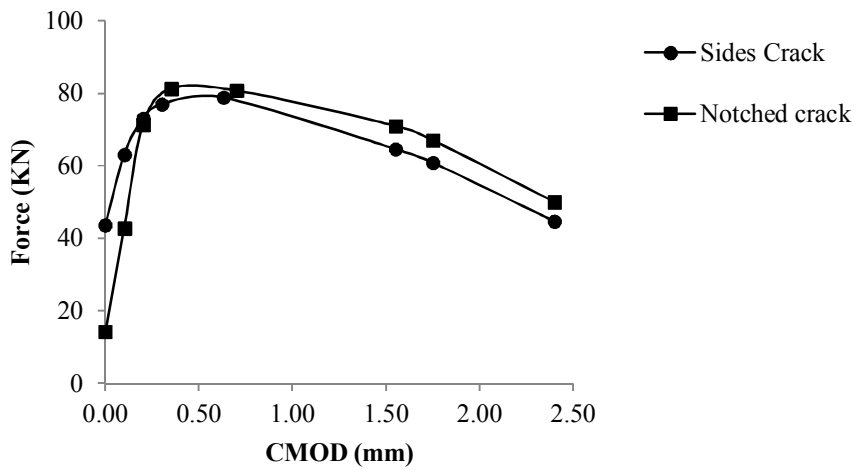


Figure 12. Load–CMOD curves in the notch and sides of the notch.

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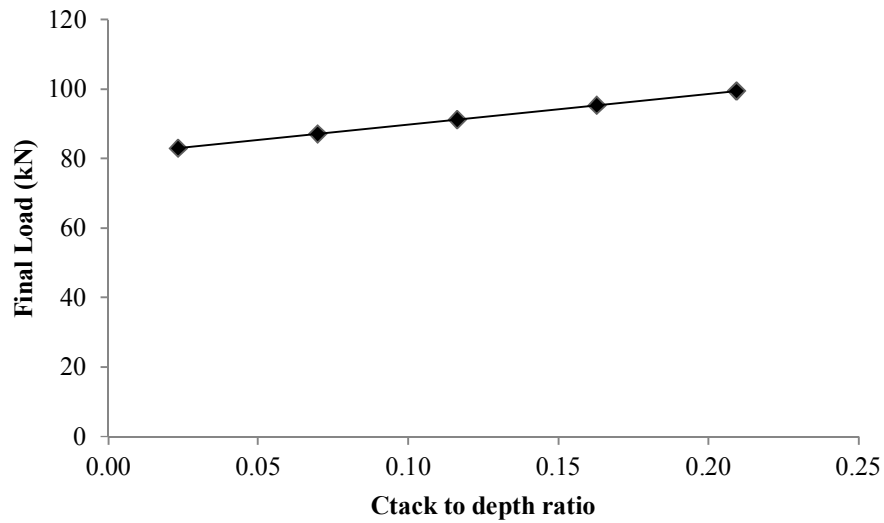


Figure 13. The diagram of final load-crack to depth ratio.

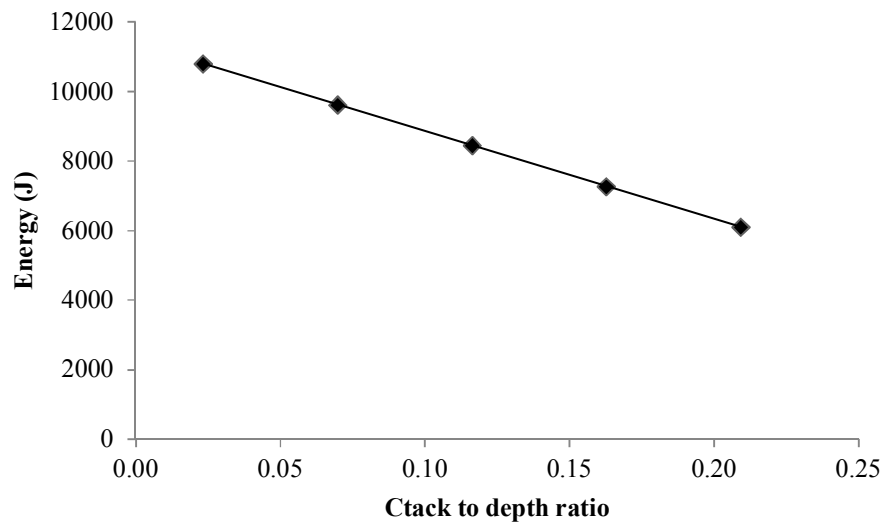


Figure 14. The diagram of specimens' energy-crack to depth ratio.

### 3.1.2 Load-crack Length Diagram on the Sides of the Notch

In addition to creating a crack in the notch, sleeper cracks are created on both sides of it. First crack is created at the center of the sleeper in the notch position, but as the loading process continues, cracks are created on both sides of the notch.

Figure 11 shows the effect of notch length on the average of 5 sleeper specimens' in terms of load-crack length diagrams on the sides of notch. Three specimens' of sleepers with the lengths with initial crack lengths of 5 mm are compared together. Sleepers with initial crack widths of 8 mm are tested and compared.

The difference between Figure 10 and Figure 11 is the notch resistance. In fact, the crack grows almost simultaneously with crack creation.

### 3.2 Load-CMOD Diagram

Crack mouth opening displacement (CMOD) is another important parameter in the fracture mechanics analysis. CMOD is used to calculate crack length and  $K_{IC}$  indirectly. The load-CMOD diagram is used to design structures and materials considering fracture mechanics. The Load-CMOD diagrams of pre-stressed concrete sleepers are shown in Figure 12. The Load-CMOD diagram of the notch and the sides of it are shown on the average of 5 sleeper specimens'. This figure is drawn to compare behavior of the notch and the sides of it. In Figure 12  $a_0=45$  mm and  $d_0=8$  mm.

The crack in the notch is created with 14.25 kN while the side cracks are created with 43.65 kN, as shown as in Figure 8. Therefore, the first crack is created at center of the sleeper and the side cracks are the created in the form of shear cracks. As it can be seen in Figure 12, the length and CMOD of the cracks grow with different rates. This important feature is more visible in the notch compared to its sides. Both cracks lose resistance to cracking in the nearly same load value i.e. 81 kN. In the greater loads, cracks of the sleeper grow unstably to final failure. Instability of the cracks loses resistance with difference rate. The crack of the notch loses resistance at a lower rate compared to side cracks, as shown as in Figure 12.

### 3.3 Final Load- Crack to Depth Ratio Diagram

The final load is another important parameter in the analysis of load bearing of structures and material. The fracture load that a structure completely loses its load bearing resistance is called final load. This parameter is used to determine the safety factor and for designing of structures. The final load changes vs. crack to depth ratio are shown on the average of 5 sleeper specimens' in Figure 13.

By increasing notch length and as a result increasing of crack to depth ratio, the final load decreases, as shown in Figure 13. The results

show that final load decreases almost linearly with low rate. In fact, the final load changes slightly depending on the crack to depth ratio. By increasing the crack to depth ratio equal to 21% at sleeper center, the final load decreases 17%.

### 3.4 Energy- Crack to Depth Ratio Diagram of the Specimens

The Specimens' energy changes vs. crack to depth ratio are shown in Figure 14. Specimens' fracture energy is another important parameter in materials and structures that has many applications in fracture mechanics analysis. In fact, Specimens' energy represents load bearing, structural damping and resistance against external loads. By increasing the specimens' fracture energy, the structure behaves better for the same load.

In this study, determining a relation between specimens' energy and crack to depth ratio changes is investigated. It is obvious that by increasing crack to depth ratio, the specimens' energy or the area under load-displacement diagram reduces. The results show that the specimens' energy decreases almost linearly vs. crack to depth ratio increasing, as shown as in Figure 14. By increasing the crack to depth ratio equal to 21% at sleeper center, the final load decreases 44%.

## 4. Discussion and Conclusion

B70 Pre-stressed concrete sleeper is widely used in I.R. Iran railway transition lines. In this research, the main fractural mechanics parameters are determined based on available foundations in fractural mechanics, such as crack length, CMOD, final load and specimens' energy. In the test procedure, full scaled specimens' of B70 pre-stressed concrete sleepers are used that are manufactured by an Iranian factory. Crack propagation parameters such as crack length, CMOD, final load and specimens' energy are important parameters in fractural behavior of structures and materials. By determining these parameters, fractural behaviors of pre-stressed concrete sleepers will be predictable. Sleeper cracks tend to appear in the notch and on both

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sides of it. Replica test and image analysis are used in this paper. The main structural parameters for sleeper behavior specimens' such as bending toughness and bending deformation response can be easily calculated through the results. These parameters can be obtained with load-crack length and load-CMOD curve of present sleepers. In this paper, sleepers with initial crack width of 8 mm and different initial crack lengths of 5 mm to 45 mm, with a 10 mm increasing step, are tested. Five specimens' of each group are loaded under three-point bending load test. The results show that the flexural behavior is very dependent on the initiation crack length,  $a_0$ . It is shown that cracks are first created at the center of the sleeper. Consequently, shear cracks appear at the sides of the sleeper. When considering resistance, crack growth length rate values in the notch are more significant compared to the sides. But, when considering instability, the situation is vice versa. Also results are shown the final loads and specimens' energy decrease almost linearly vs. crack to depth ratio. The final load changes slightly depending on the crack to depth ratio against specimens' energy. In fact, the specimens' energy changes at a faster rate than the final load depend on the crack to depth ratio. The results of paper are shown that crack parameters of pre-stressed concrete sleepers can be used to fracture mechanics analysis and design.

### 5. Acknowledgements

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