Estimating the Capacity of the Basic Freeway Section Using Headway Method in Developing Countries (Case Study: Iran)

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Abstract:

Although there are remarkable differences in drivers' behavior as well as vehicles characteristics, road capacity estimation methods in developing countries are same as developed ones. So, the discrepancy between the theoretical and practical values of capacity is inevitable. Although, capacity estimation methods can be classified based on numerous parameters, in developing countries, the availability of data is a remarkable factor to choose the appropriate method. Headway method is one of the most popular methods which is used to estimate capacity of basic freeways section regarding to the lack of available data in developing countries such as Iran. According to headway method theory, the distribution function of headways needs to be estimated in order to find the capacity of a road facility. However, in previous studies the mean of observed headway is used for capacity estimation.

The data applied in this study have been collected from Iran. Modelling had accomplished in 2 steps: 1) Filtering the input data that are related to following vehicles and just before queue. 2) Statistical modelling using the Minitab 16 software. Evaluating models by three parameters (applicability, validity and reasonability), it was proven that the lognormal distribution functions, is the best fit to observed data. According to results, the capacity of the basic freeway section is 12% less than the values proposed by HCM, which are the current bases for traffic studies in developing countries such as Iran.

Keywords: Capacity, headway, following vehicle, follower vehicle

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1. Introduction

Roadway Capacity estimation is one of the major aspects in traffic engineering and many studies have been conducted around this subject. Typically capacity estimation is based on analysis of significant data bases or is in accordance with theoretical algorithms and Physics laws while some methods are based on simulation results [Roess, Prassas and McShane, 2004]. The capacity and other flow characteristics depend heavily on driver behavior. Although, there are significant differences in drivers' behavior and vehicles characteristics in developing countries comparing with developed ones, HCM which is developed in United States is used directly for capacity studies around the world [Ashutosh, Senathipathi and Madhu, 2013]. Consequently, the results are almost unreal in practice and may cause unprofitable constructions. On the other hand, lack of required infrastructures in developing countries leads to insufficient data availability in traffic studies.

According to the aforementioned explanations, in this study, the capacity of freeway for a developing country, Iran, will be estimated in order to investigate possible differences with the HCM proposed values. Different methods have been proposed for capacity estimation like headway methods, maximum observed flow method, probabilistic methods and etc. Considering all aspects such as data requirement, location choice and observation period that are more meaningful in developing countries, time headway method is selected. Estimating the capacity, on the basis of the models related to headway, is carried out by assuming constant vehicle attitudes. These models are able to estimate the capacity of only one lane. Indeed, the assumption of models related to headway, are the separate functions of lane in multi-lane roads. The headway models can be used for one lane in both stable and unstable flow. [Minderhoud, Botma and Bovy, 1996]

According to inverse relation of headway and flow rate, the maximum flow rate, in other words "capacity", can be estimated using accepted headway of drivers' in a traffic flow. In this study, drivers' headway of follower vehicles and before queue is applied. In this case, the abnormal behavior of Iranian drivers in a congested flow will be eliminated. Followers' vehicles are found by the well-known General Motors' following model.

In order to estimate the capacity, the headway should be introduced as a deterministic value with the capability of describing the drivers' behavior. Although in previous studies in Iran the mean headway have been utilized, in this study the estimated distribution function of headways is used for finding the exclusive value of headway [Abtahi, Tamanaiee and Kermanshahi, 2011]. Finding the distribution function of headway is the vital part of this study, where eight well-known distribution functions in traffic studies are fitted to drivers' headway data of Iran freeways and the best model is introduced. Distribution functions are validated using three parameters_ applicability, validity and reasonability.

2. Literature Review

The headway attitudes were discussed in 1960s. Greenberg has found a connection between the microscopic traffic flow theory and the lognormal follower headway distribution [Greenberg, 1966]. Branston 1976 and Buckley 1968 presented the models based on headway. These models are based on following drivers' data at the time of perceiving the capacity level, and the compatibility of the hypothesis and the observed data had been proven [Branston, 1976; Buckley, 1968].

Mei and Bullen presented different distribution functions for time headways measured on a four-lane highway. According to their results, the lognormal distribution with a shift of 0.3 or 0.4 seconds was the best fit for the time headways in high traffic volumes [Mei and Bullen, 1993]. Sadeghhosseini in 2002 investigated the time headways at flow rates varying from 140 to 1704 vehicles per hour per lane on a highway and suggested the lognormal distribution [Sadeghhosseini, 2002]. Arasan and Koshy in 2003 determined the negative exponential distribution for headways distribution of urban arterial in India [Arasan and Koshy, 2003]. Bham and Ancha in 2006 analyzed the time headway of drivers in a basic freeway section and recommended the lognormal distribution for vehicles' headway [Bham and Ancha, 2006].

Some studies have been accomplished about headway distribution functions at different lanes of freeway, for instance Zwahlen, Oner and Suravaram's study 2007. They investigated one of the Ohio freeways and demonstrated that headway distribution functions at differ-

ent traffic lanes are the same [Zwahlen, Oner and Suravaram, 2007].

In addition to what has been mentioned, the empirical models are other forms of headway models. These models are introduced by cumulative distribution functions. Some studies, which were focused on estimating mode, the median and the coefficient of variation have been conducted, that Buckley research in 1968 has been the most reputed one. Also, Breiman et al. in 1977 and Griffiths & Hunt in 1991 have carried out some research in this field [Breiman et al., 1977; Griffiths & Hunt, 1991].

It should be considered that the discrepancy between distribution functions and results in different countries and also variety of the proposed models are problems of the presented models. The reason of the mentioned problem is that the drivers' behavior is completely different and their sensitivity impacts on time headway. Also, statistical characteristics of the models have not been presented comprehensively. In empirical models, determining the statistical characteristics of models is complicated and they cannot be generalized. In addition to what is expressed, Luttinen in 1996 investigated these models and demonstrated that the disadvantages of the headway models are interpreting of small data [Luttinen, 1996].

Although, the distribution function is used to model the drivers' behavior for choosing the time headway in traffic flow, in Iran, capacity is estimated by the mean value of observed headways. [Abtahi, Tamanaiee and Kermanshahi, 2011]

3. Data Collection

In this study 2 types of data, macroscopic and microscopic data, are utilized. Macroscopic parameters are mean speed, mean headway and flow, and in the form of microscopic parameters, vehicles speed, vehicle acceleration rate have been collected. The data related to macroscopic parameters are collected from the detectors which are available in Iran's freeways. Collected data are available in form of mean headway of 5-minute intervals. According to current limitations in available data in Iran, the microscopic parameters have been collected by video-recording and image processing.

Selected sections are base sections according to high-

way capacity manual (HCM) definition [TRB 2010] with 3-lanes and 120 km/hr speed limits. According to Base section definition of freeways, there is no access to studying sections. In Iran heavy vehicles are not allowed to use freeway facilities.

The total time needed for surveying the macroscopic parameters is considered one year in working days, and the time of surveying the microscopic parameters is 2-hour intervals in morning and evening in working days as well. The users of freeway in working days are commuters which are familiar to the road. All the data were provided in ordinary weather condition and dry pavement conditions.

According to the presented explanations, 3 sections have been selected, and related data will be used in this study.

4. Methodology

The following equations are bases for capacity estimation using headway models.

$$h_m = \sum \frac{h_p}{n} \tag{1}$$

$$q = \frac{1}{h_m}$$
 (Vehicle per second) (2)

In these equations:

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h_p = The time headway between vehicle P and its next vehicle (sec)

 $h_m =$ The mean headway of all the vehicles

q =The vehicles' flow rate

n = The number of vehicles passing the section during time t

As it is obvious from the above equation, capacity i.e. the maximum flows that can reasonably be estimated by reciprocal of the minimum headway. It would be possible to estimate the minimum mean headway and the capacity, if the headway distribution of vehicles is determined as a mathematics model.

In addition, the vehicles' headway depends on flow characteristics. According to Branston 1976 and Buckley 1968 hypotheses, vehicles at the capacity level cannot maneuver, so they are following [Branston, 1976; Buckley, 1968]. In other words, headway distribution function at the capacity level, are equal to the following vehicles' headway. This hypothesis has been considered

in most of headway models and this study. At the same time, according to investigations in Iran as a developing country, the drivers' behavior is remarkably different in period of time before queuing. In other word, the headway is significantly small at the time of queue. Consequently, in this study, the data related to queue, will be eliminated as a new hypothesis.

4.1 Following Vehicles

The car-following theories represent how vehicles follow others in an uninterrupted flow and are known as the most common parameters for driver's behavior definition. According to these models, vehicles are categorized in two groups which are following and free. The following vehicles cannot maneuver or change their direction, and their behavior depend on head vehicles named "leader". However, free vehicles can maneuver and move freely. Different models have been introduced in this field up to now. According to limited available data in General Motors Models have been chosen. [Tom and Krishna Rao, 2004]

General Motors presents the car-following definition regarding two assumptions: 1) The higher speed the more distance, 2) Heeding to safe [Tom and Krishna Rao, 2004]. The maximum following time can be estimated by using the equations which have been presented by General Motors. The equation 3 can be presented as: [Tom and Krishna Rao, 2004]

$$\Delta x_{n+1}^t = \Delta x_{safe} + \tau v_{n+1}^t \tag{3}$$

Differentiating the above equation with respect to time,

$$v_n^t - v_{n+1}^t = m_{n+1}^t \tag{4}$$

In this equation:

 Δx_{n+1}^t = The distance between the (n+1) vehicle and previous vehicle in tth second (meters)

 Δx_{safe} = Safe space distance in tth second (meters)

 $v_{n+1}^t = (n+1)$ vehicle's speed in tth second (meters per second)

 τ = Driver's sensitivity coefficient (seconds)

 $a_{n+1}^t = (n+1)$ vehicle's acceleration rate in tth second (meters per second squared)

The decision distance of the following vehicle will be considered as the desired distance if the speed of the leader vehicle is reduced. Therefore, the following time distance can be estimated by equation 5.

$$T_f = T_{safe} + \tau \tag{5}$$

In this equation:

 T_f = The desired time distance (seconds)

 T_{safe} =The safe time distance (seconds)

The safe time distance in equation 5 is the reaction time. Thus, the desired time distance can be estimated by calculating the sensitivity coefficient of driver. The sensitivity coefficient of driver will be counted by equation 4 and using collected microscopic data. Vehicles with greater headway than estimated desired time distance are assumed as free vehicles.

4.2 Queuing

Since the vehicles headway at the capacity level has been considered, and there is remarkable discrepancy between drivers' behavior before and after queuing, the data related to queue have to be eliminated. In order to recognizing and eliminating the relevant data, the speed-flow diagram is used. Indeed, flow drop i.e. breakdown, and its relevant speed is the best factor for the queue recognition [Chung, Rudjanakanoknad and Cassidy, 2007; Dowling et al., 1997]. The speed-flow curves have been suggested for the sake of finding the speed corresponded to breakdown [Maerivoet and De Moor, 2008].

4.3 Mathematical Modelling

The hypothesis of statistical modelling is based on accidental input data with no trend [Luttinen 1996]. Three tests suggested for determining data trend are: 1) Weighted sign test, 2) Kendall rank correlation test and 3) exponential ordered scores test. In order to trend data, correlation tests are utilized. The pair data are compared in this test [Stuart and Ord 1991]. Because of the complicated calculations, statistical software is used. It has to be considered that, generally, traffic data has a specific trend [Luttinen, 1996].

The model parameters are estimated after investigating the data trend. The model parameters estimation is the most vital step in statistical modelling. These parameters can be estimated by several methods. The most well- known methods of model parameters analysis are Moments method, Maximum likelihood and Chi-

square. The most common method of parameter estimation is Maximum likelihood [Luttinen, 1996].

5. Modeling Procedure

The headway data modeling is conducted in two steps which are: preparing and filtering the input data and statistical modelling. In first step, the input data for the follower vehicles before the queuing have filtered using the following time criteria and related speed of flow drop. In second step, the statistical models, related to vehicles' headway for each freeways lane have been estimated.

5.1 First Step: Preparing Input Data Filtering Free Vehicles

Drivers' reaction time in Iran has been investigated by Sohrabi et al. in Hamedan Medical University. This study has been carried out on 46 individuals (including 10 females and 36 males) and has been complemented on each volunteer using a simulator machine for one hour. The sample contains includes drivers with driving license who drive both professionally and unprofessionally. Also, includes variety of age, gender and driving experience. The drivers' reaction has been recorded has been analyzed by SPSS software in next step. Results have demonstrated that the mean reaction time of decelerating is 1320 milliseconds. Also, the time of releasing the accelerator has been estimated 559 mil-

liseconds [Sohrabi et al., 2013].

If the driver's reaction time is considered as the sum of perception, decision and action time [Elefteriadou 2014], the sum of releasing accelerator pedal and pushing on the decelerating pedal will be equal to the driver's reaction time. Consequently, the driver's reaction time in Iran has been determined 1879 milliseconds.

The equation 5 is used to estimate the driver's sensitivity coefficient. v_n^t , v_{n+1}^t and a_{n+1}^t have been collected and analyzed as microscopic data which is discussed before. In next step, the sensitivity coefficient has been estimated using regression model. Modelling process has been done using SPSS. The linear regression model with zero constant value has been applied and result is shown in equation 6 and Figure 1.

$$v_n^t - v_{n+1}^t = 1.646 \times a_{n+1}^t$$
 (6)

According to outputs, the corrected R2 has been determined 0.933. Also, as is presented is Table 1, related P-values of model and independent variable are zero, which means, the model and independent variable are significant with 95% confidence.

As it was mentioned, the desired time distance is sum of the reaction time and driver's sensitivity coefficient. Therefore, this value is determined as 3.525, which is meant the vehicles that have time headway less than 3.525 are follower vehicles and the others are free ones. Free vehicles have been eliminated for data base.

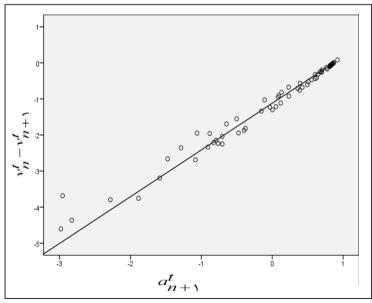


Figure 1. The Equation of Estimated Regression in order to Determining the Driver's Sensitivity Coefficient

Table 1. Results of model statistical tests

N	Todel	Independent Variable	Independent	
Corrected R ²	P-Value Model (F)	P-Value	Variable Coefficient	
0.933	0.00	0.00	1.646	

Filtering queue data

Speed and flow data of the study section are fitted for 5- minute periods as shown in Figure 2. The data is collected for one year period of study section (almost 100,000 observations) and each point represents an observation.

As it is depicted, flow drop have been occurred in flow speed 68 kilometers per hour, where the data density is reduced. In consequence, the data with mean speed more than 68 kilometers per hour have been utilized in this study.

5.2 Second Step: Modelling

In this step, the filtered data have been extracted according to maximum vehicles' headway and minimum vehicles' mean speed, and modelling has been conducted by statistical software, Minitab16. The trendless feature of data has been examined using correlation method as mentioned before. Finally, modelling has been carried out for each lane and the entire section and model's parameters have estimated. (Table 2 to 5) Eight well-

known distribution functions in traffic studies are utilized to find the best fitted model of observed headway.

6. Evaluation Models

In order to find the best distribution function, models have been evaluated by three criteria and the best model has been presented.

6.1 Applicability

The applicable model should have not only simple structure and definition, but also specific statistical attitudes. In addition, this model can be reviewed and the model parameters can be estimated. Therefore, the well-known statistical models in this study are considered and experimental models are not investigated.

According to the conducted studies up to now, in this study 4 models are investigated. These 4 models are:

- 1- Normal distribution function and exponential distribution function
- 2- Lognormal distribution function and 3-parameter

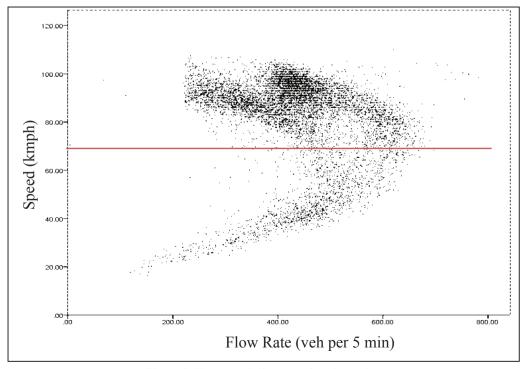


Figure 2. Flow-speed diagram of study section

Table 2. Parameters of the headway model for first lane data

Model	Parameters					
_	Location	Shape	Scale	Threshold		
Gamma	NA	11.067	0.146	NA		
3- parameter Gamma	NA	1.881	0.368	0.928		
Normal	1.620	NA	0.549	NA		
Exponential	NA	NA	1.620	NA		
Lognormal	NA	0.437	0.289	NA		
3- parameter Lognormal	NA	0.049	0.316	0.217		
Weibull	NA	2.928	1.810	NA		
3- parameter Weibull	NA	1.354	0.752	0.935		

NA = Not available.

Table 3. Parameters of the headway model for second lane data

Model		Paramete	ers	
_	Location	Shape	Scale	Threshold
Gamma	NA	10.236	0.193	NA
3- parameter Gamma	NA	3.184	0.360	0.835
Normal	1.979	NA	0.670	NA
Exponential	NA	NA	1.979	NA
Lognormal	NA	0.633	0.307	NA
3- parameter Lognormal	NA	0.229	0.392	0.461
Weibull	NA	2.997	2.212	NA
3- parameter Weibull	NA	1.754	1.245	0.875

NA = Not available.

Table 4. Parameters of the headway model for third lane data

Model -	Parameters						
Model	Location	Shape	Scale	Threshold			
Gamma	NA	1196315	0.228	NA			
3- parameter Gamma	NA	181.804	0.056	-7.450			
Normal	2.732	NA	0.744	NA			
Exponential	NA	NA	2.732	NA			
Lognormal	NA	0.963	0.302	NA			
3- parameter Lognormal	NA	0.478	0.575	0.697			
Weibull	NA	4.373	3.012	NA			
3- parameter Weibull	NA	9.493	6.132	-3.077			

NA = Not available.

Table 5. Parameters of the Headway Model for the EntireSection

Model	Parameters					
-	Location	Shape	Scale	Threshold		
Gamma	NA	9.644	0.187	NA		
3- parameter Gamma	NA	2.368	0.391	0.874		
Normal	1.800	NA	0.638	NA		
Exponential	NA	NA	1.800	NA		
Lognormal	NA	0.535	0.314	NA		
3- parameter Lognormal	NA	-0.122	0.342	0.352		
Weibull	NA	2.859	2.016	NA		
3- parameter Weibull	NA	1.556	1.033	0.877		

 $\overline{NA} = Not available.$

lognormal distribution function

- 3- Weibull distribution function and 3-parameter weibull distribution function
- 4- Gamma distribution function and 3-parameter gamma distribution function

6.2 Validity

In modelling, validating models is the most prominent step. In this study, the models' goodness of fit is investigated and the best model will be determined. The goodness of fit tests can be applied in order to investigating the hypothesis of equality in observed data distribution with specific distribution. The zero hypothesis of this test (H0) depicts the equality of fitted distribution function on observed data and the reputed distribution function. The contrast hypothesis (HA) illustrates the difference of the models.

In order to testing the goodness of fit, several tests have been proposed: Chi- Square test, Kolmogorov- Smirnov and Anderson & Darling tests are widely used in statistical studies. Although all the methods have equal hypotheses, it has been demonstrated that Anderson & Darling test (A-D) will have better results concerning to headway data essence. In this method, the maximum value will be estimated as the difference of model and observed data distribution function. This discrepancy is belonged to middle part of function and the discrepancies are slight in beginning and ending part of the function. These discrepancies are weighted according to the location in function which each point has specific weight [Luttinen 1996]. In this study, the A-D method will be applied in order to calculating the model goodness of fit. Table 6, presented the goodness of fit of estimated models.

Table 6. The Goodness of Fit of Headway Models

Model	Firs	t Lane	Secon	nd Lane	thire	d Lane	Entire	Section
-	A-D	P Indicator	A-D	P Indicator	A-D	P Indicator	A-D	P Indicator
Gamma	49.780	0.005<	42.457	0.005<	29.915	0.005<	49.780	0.005<
3- parameter Gamma	12.640	0.000	10.662	0.000	17.554	0.000	12.640	0.000
Normal	109.603	0.005<	76.219	0.005<	57.733	0.005<	109.603	0.005<
Exponential	617.820	0.003<	336.036	0.003<	319.189	0.003<	617.820	0.003<
Lognormal	28.258	0.005<	28.550	0.005<	20.236	0.005<	28.258	0.005<
3- parameter Lognormal	5.156	0.000	3.731	0.000	13.963	0.000	5.156	0.000
Weibull	107.168	0.010<	77.992	0.010<	57.236	0.010<	107.168	0.010<
3- parameter Weibull	26.916	0.005<	16.931	0.005<	25.938	0.005<	26.916	0.005<

As it is obvious, the 3-parameter gamma and 3-parameter lognormal present acceptable results with 95% confidence level. Indeed, the zero hypotheses is not rejected in these models. Comparing the A-D value of these two models demonstrates that the 3-parameter lognormal can present more compatible results than other data.

6.3 Reasonability

Concerning to statistical data characteristics and traffic flow theory, the obtained model should be compatible to users' behavior. The last step of headway modelling is investigating the models with the purpose of compatibility to drivers' behavior. As discussed before, both log normal and gamma distribution have been used in previous traffic studies which satisfies the reasonability criteria.

7. Conclusion

In this study, using headway method, capacity of the basic freeway section was determined. Therefore, headway distribution functions estimated for each freeway lanes. The essential data of the Iran's freeways were collected and the filtered according to 2 assumptions: 1) the following vehicles, and 2) before queuing. Finally, modelling for function determination was done by mathematics methods and the models were analyzed on basis of 3 factors: 1) Applicability, 2) Validity, and 3) Reasonability. At last, the lognormal model is introduced as the best distribution function of the headway data for first, second, third lanes and the entire section. According to the relation of headway and flow rate, the capacity values have been estimated for each lane and the entire section for a 3-lane freeway basic section as. On the other hand, the capacity proposed value of HCM under basic section condition and considering the ratio of buses, is depicted in Table 7.

Comparing the estimated values and HCM proposed

values that are the bases of designing in Iran, the suggested capacity for basic freeway section is 12% less than HCM's and is estimated as 6099 vehicle per hour for 3 lanes. In agreement with the drivers' behavior as well as vehicles' low performance in Iran this difference seems to be rational. The contrast also reported from field observations of bottlenecks of constructed roads that were not predicted in designing procedure. In addition, there are remarkable discrepancies in the estimated capacity among different freeways' lanes, where, there is 22% and 43% variance in capacity of lane 2 and 3 compare with HCM. This would be result of the fact that buses are just allowed to use second and third lane, and consequently, vehicles with lower technical performance cross through these lanes in Iran. The introduced lognormal model for drivers' headway could be used in several safety and behavioral studies by representing the driver behavior in congested flow in Iran freeways'. For example, comparing the drivers' headway and minimum allowable distance would results finding the risky drivers in further studies. Besides, learning of the discrepancy in practical and theoretical freeway capacity, further studies are needed to find the capacity estimation manual for Iran freeways in order to optimize the construction costs.

8. References

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Table 7. Capacity of Each Lane and the Entire Section for a 3-Lane Freeway basic Section

Lane	Headway Model (veh per hr)	HCM (veh per hr)	Capacity Value Variance From HCM (percent)
3	2268	2320	2
2	1820	2320	22
1	1320	2320	43
Entire Section	6099	6960	12

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