Optimization of Delay Time at Signalized Intersections Using Direction-Wise Dynamic PCE Value

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Abstract

The study worked on direction-wise dynamic Passenger Car Equivalent (PCE) model to optimize delay at signalized intersection under heterogeneous traffic condition. Static PCE values are usually practiced to transform heterogeneous traffic into homogeneous stream. However, the PCE values are dynamic due to variation in vehicle composition. Video sensors were used to count direction-wise classified vehicle at various signalized intersections for this research. The adaptive PCE values were compared with the PCE values used in the manual of Roads and Highway Department (RHD), Bangladesh. Furthermore, Synchronous regression method was performed to estimate saturation flow. Field total delay was estimated from queuing diagram considering residual queue, arrival and saturation flow rate. After that, signal timing was adjusted by optimization of total delay. RHD manual based PCE was observed to be underestimated comparing to direction-wise dynamic PCE in case of field delay and optimized delay by 6.79% and 27% respectively. Turning vehicles occupy more space and time and influence operation of signalized intersection. Hence, conventional PCE estimation failed to comply with actual scenario, and consequently, could optimize signal timing inaccurately. This study can be used as a framework to calibrate practiced PCE values in the road capacity manual and design traffic signal.

Keywords: Direction-wise dynamic PCE, Right-turning vehicle, Heterogeneous traffic, isolated signal optimization, delay.

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1. Introduction

Improving mobility and reducing traffic delay through implementing efficient transportation strategy is the crucial challenge in urban areas. Smooth operation and control of traffic stream depends largely on appropriate management of signalized intersection. Increasing travel time and reducing vehicle speed are the primary consequence of traffic delays at intersections. Precise estimation of vehicle delays at signalized intersections is the key component in planning, designing and analyzing traffic signal control. Estimation of delays at isolated signalized intersection relies on signal parameters such as cycle time, green time and traffic characteristics such as saturation flow rate, arrival rate and residual queue. Traffic parameters are relying on classified vehicle counting. In heterogeneous traffic stream, because of variety of maneuvering in terms of time and space among different vehicle classes, their influence on signalized intersection varies widely. To overcome aforementioned effect, Highway Capacity Manual provides value of Passenger Car Equivalent [Adams et al. 2015]. PCE value is usually adopted to provide a mechanism for converting all diverse types of vehicles into reference vehicle i.e. car. It is based on the concept that number of passenger cars that are displaced (in terms of time and space) by a single type of vehicle under existing traffic, roadway and control scenario [Akcelik, 1980]. Because of static and dynamic characteristics of different vehicle categories, PCE values largely depend on turning movement and vehicle composition of prevailing traffic stream. Hence, PCE values vary over time and space and it is preferred to adopt direction-wise vehicle composition based on dynamic PCE model rather than fixed PCE values for all scenarios. Several studies, such as, [Lam,1994], [Cuddon and Ogden,1992] revealed

that headways of same type of vehicle are varying for different types of movement i.e. through, left turning and right turning during saturation time. [Shao et al. 2011] studied that PCE of particular type of vehicle for through traffic is smaller than that of right turning traffic. Therefore, separate PCE are required to adopt for different movements to quantify effects of vehicle composition on the delay estimation of signalized intersection. In this study, we utilized PCE model to incorporate dynamic direction-wise PCE to estimate field total delay as well as optimized signal timing.Turning movement at intersections affects and controls the capacity of a signalized intersection through its volume and maneuvering. Hence, estimating volume of turning vehicle accurately is a major concern for traffic planners and engineers over a long time. Vehicle headways for turning vehicles are greater than those of through traffic because of turning radius. Therefore, saturation flow rate in a turning lane is less and PCE value is more, correspondingly roadway capacity becomes less. The PCE value of different types of vehicle for the large cities of developing countries are quite different than developed one. The reasons are— traffic are heterogeneous with various sized motorized vehicle, significant presence of non-motor vehicle, vehicle follow poor lane discipline and proportion of turning vehicle are high. PCE value accounts vehicle classification with different static and dynamic characteristics, which cause a variety of lateral and longitudinal gap among vehicles on the roadway. Moreover, heavy traffic is usually dominant and control the signalized system. As a result, delay estimation for homogeneous traffic, may lead to a biased estimation in case of heterogeneous traffic condition. Our investigation tried to overcome these challenges and included direction-wise dynamic PCE value for through and right turning

vehicles separately in traffic delay estimation at signalized intersection.PCE can be estimated from several features of traffic stream, such as, headway, volume (traffic count) and speed along Hence, transportation development authorities in various countries developed different standards on regional basis. PCE values vary for different cities as per geometric pattern, traffic system, travel pattern and so on. PCE values used for designing and analyzing signalized intersections in Dhaka on the basis of Roads and Highways Department (RHD) manual, Bangladesh [Geometric Design Standards of RHD, 2001]. It should be noted that RHD adopted asynchronous regression technique to estimate PCE values for heterogeneous traffic condition. The road intersections of Dhaka city are becoming more congested day by day with proportioning increment of right-turning vehicle. It is to mention that Dhaka city follow right hand driving rule, hence right turning vehicles are conflicting traffic stream with through traffic in a signalized intersection. Therefore, static PCE value provided by RHD of Bangladesh are not appropriate. Several researches have been performed to estimate PCE for various types of vehicle in Dhaka. [Saha et al. 2007] estimated the PCE values for through traffic at the intersections of Dhaka City. Later, the similar research was extended by [Mamun et al. 2012] for right turning vehicle of signalized intersections. Both of the above researchers used headway method for PCE estimation. In this paper, the researchers compared two methods of PCE estimation to compute field delay and optimize signal timing. One is conventional static PCE values based on RHD manual of Bangladesh, where direction-wise traffic at intersections are not considered; PCE values are based on lane group width and percentage of non-motorized vehicle (NMV). The other alternative method utilizes

with roadway and vehicle wise characteristics. Inaccurate estimation of traffic flow parameter under various traffic and roadway occurs due to use of fixed set of PCE value.

synchronous regression method to estimate dynamic PCE value for through and right turning vehicle separately. The regression method is flexible and more suitable for the traffic condition where traffic does not follow strict lane discipline, in addition, where heterogeneity of traffic stream is significant and right turning vehicle are high. PCE values estimated by both methods are used to calculate saturation flow rate, arrival flow rate and residual queue. By utilizing geometric shape of queuing diagram—similarly mentioned by [Christofa and Skabardonis, 2011]—total field delay were computed for signal cycles using PCE values estimated by both methods. After that, signal optimization was performed for the improvement of real time traffic operation. Assuming minimum green time for each lane group and fixed cycle time, Generalized Reduce Gradient (GRG) nonlinear technique was used to minimize the objective function which consisted of summation of the delay in the cycles. The proposed alternative method is more comprehensive and precise to estimate delay and optimized signal timing at urban intersections than the conventional one and can be utilized for other countries as well.

Our Developed PCE model can be a framework to measure PCE accurately which can be helpful for capacity and Delay Estimation. The paper is structured as following—the first section of our paper provides introduction of the study. The second section presents literature review. The third and fourth section show methodology for field survey and PCE estimation respectively. The fifth represents saturation flow and the next section includes delay estimation procedure. The

last section discusses with the concluding remarks.

2. Literature Review

Delay is a measure of effectiveness, evaluates performances under different demand, control and operating conditions of a signalized intersection. Delay at signalized intersection depends on arrival flow rate, saturation flow, under and over saturated traffic to measure delay [Akçelik, 1980]. Delay model proposed by [Teply et al. 1984] and [Akçelik, 1980] were practiced by the Canadian capacity guide and Australian road research board respectively. [Fambro and Rouphail, 1997] improved accuracy of time dependent model. HCM 2000 proposed time dependent delay model, which is capable to estimate delays for over saturated traffic. It is widely practiced in USA. [Benekohal and El-Zohairy, 2001] considered various arrival patterns and increased accuracy for delay estimation at signalized intersection. [Zhang and Tong, 2008] estimated delay for queue spillback and left-turn blocking conditions at signalized intersection. [Qiao et al. 2002] adopted artificial intelligence technique to improve accuracy of oversaturated condition, however, no specific delay estimation formula was considered. Simulation based TRANSYT model [Li and Gan, 1999] can estimate delay at signalized intersection and optimize signal timing as well. Nevertheless, measuring field delay is much more complex than delay estimation through theoretical and simulation based model. The challenges increase many folds, in case of heterogeneous traffic and presences of significant proportion of turning vehicle at signalized intersection. Several researches have been conducted on heterogeneous traffic to measure delay at signalized intersections. [Arasan and Jagadeesh, 1995] used probabilistic approach on heterogeneous traffic to measure saturation flow and field delay at signalized intersections in

signal timing and so on. There are several theoretical models available for delay estimation. [Webster, 1958], [Akçelik, 1980] and HCM 2000 estimated delays based on queuing theory. Webster's delay model considered Poisson arrival, steady state flow, however also, overestimated delays at higher degree of saturation. Akçelik developed stochastic model arrival and departure pattern considering both

Chennai city. [Ko et al. 2007] utilized GPS to collect delay data of signalized intersections. They studied speed profiles, acceleration and deceleration to estimate control delay. However, the proposed technique was not suitable for closely spaced intersections and over-capacity condition. [Kumar and Dhinakaran, 2012] estimated PCE value of mixed traffic based on speed ratio and space occupancy ratio and later calculated saturation flow and delay at signalized intersections in Tamil Nadu, India. Their study found that delay estimated by HCM 2000 varies significantly than field delay and less suitable for mixed traffic due to variation in size, maneuvering, controlling, static and dynamic characteristics. [Maini and Khan, 2000] mentioned significant lateral movement of smaller sized vehicles at the queue of signalized intersection, vis-à-vis, affected saturation flow and delays. [Minh et al. 2010] adopted modified Webster's formula on heterogeneous traffic for measuring delay at pre-timed signalized intersections. They used multiple regression to calculate PCE values for different vehicle classes. [Verma et al. 2018] used modified Webster's delay model considering various types of vehicle categories which shared same right-of-way at signalized intersection. Later, they optimized the traffic signal timing using micro-simulation in Bangalore city. The study of PCE at signalized intersections for homogeneous and heterogeneous traffic include different methods, such as, headway method, delay based method, queue discharge flow based method, travel time

based method, multiple regression technique and simulation technique. [Saha et al. 2009] used headway ratio for the estimation of PCE for various types of through traffic at intersections of Dhaka city. [Bhattacharya and Mandal, 1980] established PCE model for intersections in Calcutta and [Sarraj and Jadili, 2012] measured PCE at signalized intersections in Gaza utilizing headway method. However, Suitability of headway method is limited to steady state traffic stream and it is laborious, time consuming to collect headway data of classified vehicle in [Radhakrishnan and Mathew, 2011] developed queue discharge flow method and [Mohan and Chandra, 2017] introduced queue discharge rate for PCE measurement at signalized intersections. This method is inappropriate where lane discipline is compromised by higher maneuverability of smaller vehicle in the queue and smaller vehicle occupy front row by squeezing gap available on road space in the mixed traffic. [Mahidadiya and Juremalani, 2016]; [Kumar and Dhinakaran,2013]; [Bhatt and Patel, 2017] utilized travel time ratio and static area of vehicle, which was introduced by [Chandra et al. 1995] to estimate PCE at signalized intersection under mixed traffic condition. Chandra's method also has some criticisms, such as, vehicle in congested traffic condition have similar speed and influenced area of a vehicle in field is often larger than the projected area. [Kellar and Saklas, 1984] estimated PCE for heavy vehicle using TRANSYT simulation model under different signal settings. The study was limited to homogeneous traffic condition. [Alex and Isaac, 2015] calculated dynamic PCE for signalized intersections by TRAFFICSIM model under varying traffic composition, like speed, volume and approach width.[Asithambi et al. 2017] utilized simulation model to estimate PCE using headway ratio method traffic which do not follow any lane discipline at signalized intersections in

heterogeneous traffic accurately. [Zhao, 1998]; [Benekohal and Zhao, 2000] introduced delay based PCE measurement at signalized intersections. [Cao et al. 2009] proposed motorcycle equivalent unit (MCU) using delay for urban signalized intersections. [Rahman et al. 2003] calculated PCE for large vehicles based on delay at signalized intersections. Delay based method assumed that queue is generated due to speed reduction of faster vehicles by slower vehicles. Hence, this method is appropriate only for high volume homogeneous traffic.

Chennai, India. Later, they developed regression model of PCE based on simulation results obtained for various vehicle compositions, traffic volume, levels, presence of heavy vehicle and road widths. However, simulation methods have similar drawbacks, such as, those cannot represent actual field scenario accurately. Regression based classified vehicle count are widely used for PCE measurement at signalized intersections under mix traffic condition. [Patel and Dhamaniya, 2003] conducted regression-based stream equivalent technique to estimate PCE and thus calculated saturation flow of signalized intersections. [Kimber et al. 1986] compared synchronous and asynchronous regression method of classified vehicle count for PCE measurement at signalized intersection. The study revealed that results obtained by synchronous regression complies with headway ratio method closely. [Hadiuzzaman et al. 2008] utilized synchronous regression model to estimate PCE factor for traffic without lane discipline at signalized intersections in Dhaka city. Saturated green time was considered as dependent variable and classified vehicle passing during this time was taken as independent variable of the regression model. [Parvathy et al. 2013] evaluated PCE in the signalized intersection in Kerala. They conducted linear regression among saturated green time and number of different types of vehicle crossing

during saturated green time. [Minh et al. 2010] performed multiple linear regression to estimate PCE of four categories of vehicle under mixed traffic condition and calculated delay at signalized intersections. [Adams et al. 2015] investigated variation of on saturation flow rate due to presence of motorcycle at signalized intersection in Ghana. They assumed saturated green time as a function of vehicle passing stop line during green time to calculate PCE. [Simha, 2017] performed regression analysis with respect to green time in various signalized intersections in Mumbai city. PCE value of different types of vehicle were ratio as coefficient of respective vehicle and coefficient of passenger car. Turning vehicle at an approach of signalized intersection have different maneuverings (in terms of time and space occupied in the road) than those of through traffic. [Pretty, 1966] revealed that the movement of right turning vehicle can reduce saturation flow in a signalized intersection. Similarly, [Webster, 1967] investigated right turning traffic as capacity reducer of a signalized intersection by obstructing the movement of through traffic. [Bhattacharya and Mandal, 2017] proposed right turning factors for several types of vehicle at controlled intersections in Calcutta, India.[Cuddon and Ogden, 1992] used regression model to relate left and right turning cars with the turning radius in terms of through car equivalent. [Lam, 1994] studied on saturation flows at traffic signals in Hong Kong and calculated PCE factors for through and turning traffic separately. PCE factor for turning vehicles adopted in analysis and design at signalized intersections in Dhaka city are usually taken from the Geometric design standards provided by Roads and Highways Department, Bangladesh, which was established almost two decades ago. On contrary, volume of right turning vehicle and their percent composition is raising regularly with the increasing population and high growth rate of motorized vehicle in Dhaka city. Therefore, PCE

factor provided may not be appropriate for Dhaka city.[Mamun et al. 2012] utilized headway ratio to estimate PCE for several types of right turning vehicle at signalized intersections in Dhaka, Bangladesh. Unlike developed countries, traffic stream in Dhaka have weak lane discipline, have wide range of vehicle with various sizes including significant NMV, drivers taken up any position of road width —which cause queuing and discharging of vehicle at signals at a disorder pattern. However, PCE measuring through headway ratio method required lane-by-lane approximation of headways for different types of vehicle. Hence, the aforementioned technique is difficult to measure accurately and not suitable for the traffic condition of Dhaka city.

[Hadiuzzaman et al. 2008] studied PCE and saturation flow at signalized intersections in Dhaka city, where there was lack of lane discipline and vehicle form queue with no clear pattern. There were lateral movement of vehicle for having tendency to occupy place in front of the queue at the intersections. There may be one follower vehicle have more than one leader vehicle and vis-à-vis, one leader vehicle may have more than one follower vehicle. Moreover, narrower vehicle, such as, motorcycles occupy front of the queue and discharge in a group during the subsequent discharge. It is difficult to record individual headway of vehicle accurately in the stop line as described in [Mamun et al. 2009]. Therefore, vehicle count was more appropriate method as adopted in [Hadiuzzaman et al. 2008]. Similarly, [Parvathy et al. 2013], [Minh et al. 2010], [Adams et al. 2015] and [Simha, 2017] estimated PCE at various signalized urban intersections, where classified vehicle counts were regressed against green time and ratio of coefficient of each type of vehicle with passenger car were used as corresponding PCE.

Measuring field delay at signalized intersection is challenging. [Rouphail et al. 2013] calculated the total delay from the triangle area encircled by the

additive arrival and departure curve. In this research, field total delay was measured through the method suggested by [Christofa and Skabardonis, 2011] which is focused on direct observation of queue forming vehicle at the intersection. The method directly measures total delay faced by the vehicle. Number of residual vehicle in queue was measured by counting number of vehicles in queue after the end of green time for each phase. Arrival rate was measured in mid-block and both arrival rate and saturation flow was estimated using direction-wise PCE value proposed in this research. The details of cumulative curve and queuing diagram were described by [May, 2010].

No prior studies have been conducted to estimate the direction-wise dynamic PCE value in context of Dhaka city. Moreover, we have adopted regression method to estimate PCE value, which is more appropriate for heterogeneous traffic with poor lane discipline. In this study, we have incorporated direction-wise PCE to optimize delay at signalized intersections, which is noble in this point of view.

To obtain maximum utilization of traffic signal time , both accurate estimation of direction-wise PCE Values and effective combination of green time are required which ensure Optimum delay time.[Mania et al. 2016 ;Pajecki et al. 2019 ;Saw et al. 2015 ; Zargari et al. 2014]

3. Field Survey

Traffic survey data of eight selected intersections of Dhaka City were used for the study, which is shown in Fig. 1. After that, PCE model was developed using regression method and delay at I-7 was estimated. The traffic was heterogeneous in nature with lack of lane discipline. Vehicles comprised bus, car, microbus and auto-rickshaw having dominant amount of car and motor cycle. Data from study area were collected using Digital video camera during the period of November 2018 to December 2018.The intersections were selected considering large amount of vehicle and availability of high-rise building near the intersection. The video camera was set at the roof top of the building located near the intersections and was focused on covering the one leg of the intersection. The video was recorded carefully so that it would cover full queue formed on the study approach. The recording was done for about 120 minutes to 180 minutes for each approach. Signal timing Data i. e. cycle time, phase number, phase length were collected at the pre-timed signals. To extract the desired information, the recorded video was played in several times on a large screen TV monitor.
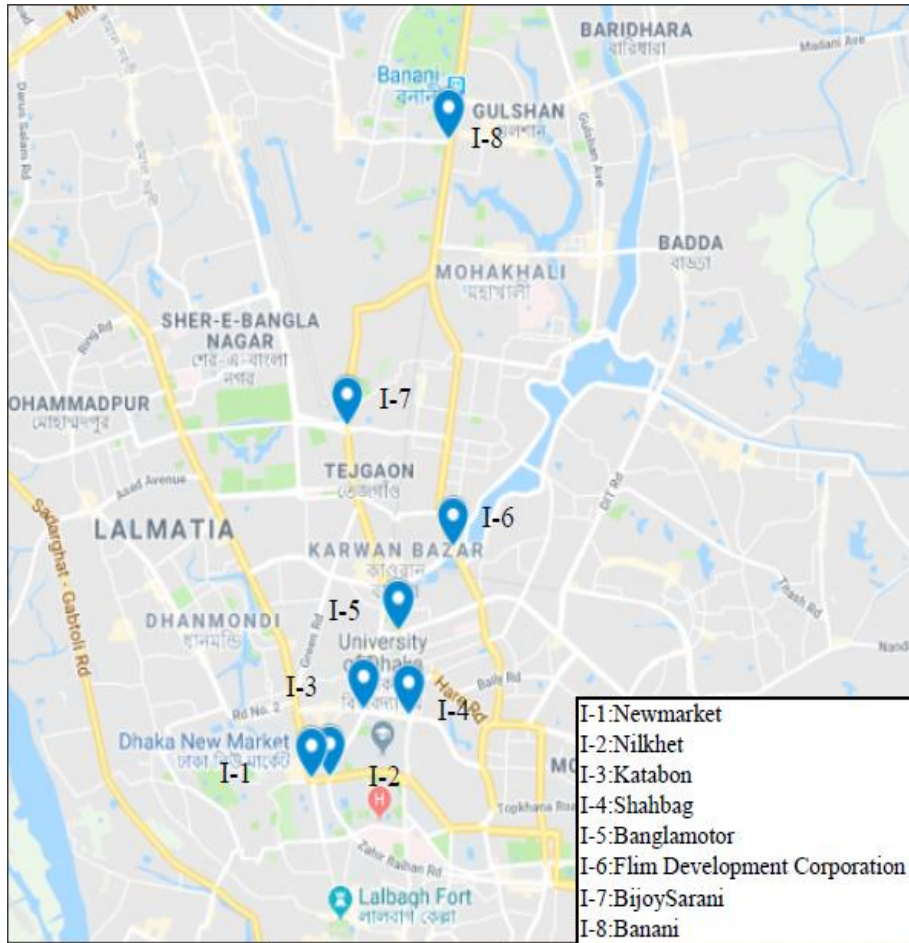
Figure 1: Google map showing studied eight intersections in Dhaka city.

4. Estimation of PCE

To incorporate the dynamic effect of directional movement of mix vehicle at various composition, proposed PCE model can be a framework for more accurate estimation rather than static PCE Value.

4.1 Direction-wise PCE

Due to variable Vehicle composition and turning effect PCE is considered as dynamic parameter .Synchronous regression estimated approximately more accurate PCE compared to other methods.

4.1.1 Synchronous Regression

Number of researchers, such as, [Branston and Vanzuylen, 1978], [Branston and Gipps, 1981], [Martin and Voorhees, 1981] have measured lost times and saturation flows using 'synchronous' regression. In this method, departed vehicle of every category is counted during time periods starting and finishing with the moment of departure of a vehicle (excluding first few second for start up loss time). Although it is possible to record individual headways of vehicles passing through the stop line, nonetheless the headway method cannot be applied because the vehicles are not moving in a line and one vehicle may have

more than one leader. Consequently, the headway ratio method cannot be applied for this study purpose. Because of the difficulties in measuring headways of vehicles in non-lane based traffic conditions and due to the fact that total approach width is adopted for analysis of saturation flows in the current study, the option of counting is more appropriate in this respect. Although the and PCE values simultaneously and is therefore attractive. However, investigation by [Kimber, 1986] revealed that the synchronous regression method produced a good approximation of both PCE factors and saturation flow which were underestimated by the asynchronous multiple regression technique. Hence, the synchronous counting method is more attractive in terms of data collection and analysis.

In this research, determination of PCE values have been performed by synchronous regression technique. A mathematical model based on regression analysis is developed to show variation of Saturation flow with traffic stream. The proposed regression equation is as given,

$$T = a_o + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5 + a_6x_6 \qquad (1)$$

Where,

T = saturated green time (sec), a_o = y-intercept, a_1, a_2, a_3, a_4, a_5, a_6 = coefficient for bus, microbus, car, auto rickshaw, motor cycle and non-motorized vehicle (NMV) respectively, x_1, x_2, x_3, x_4, x_5, x_6 = proportion of vehicles of each category passing through the stop line in time T.

Then, Passenger car equivalent of vehicle type i,

Road Note 34 method requires data in simple classified vehicle count format, it does not allow one to calculate PCE values which are essential for saturation flow (in PCE) measurement. Due to this limitation, it is also not considered here as an alternative method. The option left is the regression method which gives saturation flow

$$PCE_i = a_i / a_3 \qquad (2)$$

Where,

a_i = regression coefficient for particular type of vehicle

Firstly, PCE values of various vehicle classes are calculated by Eq. (1) and (2) for different approaches of eight intersections in Dhaka city. The obtained PCE values are biased to specific vehicle composition. Hence, those are regressed over corresponding various vehicle compositions to develop generalized PCE model shown in Eq. (3). Then unbiased PCE values are calculated using the corresponding coefficient of PCE model and %vehicle composition at I-7 as represented in Table 2 and Table 1 respectively.

4.1.2 Vehicle composition

Passenger car equivalent (PCE) values of a specific vehicle class are dynamic for the various approaches and those are changed over time due to variation in vehicle composition. Therefore, fixed passenger car equivalent values for different vehicles is not accurate for the non-lane based heterogeneous traffic stream. Vehicle composition of traffic of four approaches at I-7 are presented in the following Table 1.

Table 1. %Vehicle composition of direction-wise traffic at various approaches of I-7.

Approaches	Direction	BUS	Micro Bus	CAR	Auto Rickshaw	Motorcycle
Southbound	Through	2.39%	12.60%	24.93%	18.57%	41.51%
	Right	2.50%	38.34%	14.25%	12.87%	32.04%
Eastbound	Through	0.00%	3.51%	15.17%	20.28%	61.04%
	Right	2.79%	41.80%	29.10%	13.93%	12.38%
Westbound	Through	0.14%	7.42%	15.99%	13.54%	62.90%
	Right	0.00%	19.29%	7.07%	41.58%	32.07%
Northbound	Through	9.17%	7.09%	15.62%	17.84%	50.29%
	Right	0.14%	20.86%	13.38%	24.89%	40.72%

4.1.3 PCE Model

PCE value was dynamic for varying vehicle composition at different time of the day .To study the complete variation in PCE due to different traffic composition, regression technique based mathematical model was developed. PCE model enables accurate measurement of PCE values for the various type of vehicles in a heterogeneous traffic condition .The suggested regression model is shown in the Eq. (3). The Direction-wise dynamic PCE model obtained by regression of PCE value against vehicle composition. PCE value is given by,

$$PCE_i = a_o + a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 + a_5 x_5 + a_6 x_6 \quad (3)$$

Where, $a_1, a_2, a_3, a_4, a_5, a_6$ = coefficient for Bus, Micro bus, Car, Auto-rickshaw, Motorcycle, NMV and $x_1, x_2, x_3, x_4, x_5, x_6$ = Composition of Bus, Micro bus, Car, Auto-rickshaw, Motorcycle, NMV respectively.

Table 2. represents the regression analysis coefficients for different type of vehicle and corresponding R^2 value for each PCE model.

4.2 RHD Manual Based PCE

Table 3 depicts RHD manual based PCE values for different road width having 0% of NMV.

Table 2. Regression Coefficient for various types of vehicle

Vehicle	Movement	Intercept	Coefficients						R^2
		a_0	Bus, a_1	Micro Bus, a_2	Car, a_3	Auto Rickshaw, a_4	Motorcycle, a_5	NMV, a_6	
Bus	Through	1.137	0.012	0.011	0.012	0.006	0.01	0	1.00
	Right	2.008	0.006	0	0	0.009	0	0	1.00
Micro bus	Through	2.102	-0.005	-0.007	-0.017	-0.001	-0.009	-0.008	0.56
	Right	1.351	-0.0122	-0.001	-0.001	-0.0003	0	0.003	0.63
Auto-rickshaw	Through	-0.643	0.009	0.011	0.017	0.011	0.013	0.012	0.86
	Right	0.699	0.027	0	0.001	-0.001	0.004	0.001	0.63
Motor cycle	Through	1.608	-0.009	-0.022	-0.021	-0.01	0	-0.014	0.79
	Right	-0.84	0	0.01	0.014	0.015	0.014	0.011	0.74
NMV	Through	1.396	-0.04	-0.012	-0.008	-0.009	-0.009	-0.01	0.17
	Right	19.376	0	-0.143	-0.187	-0.19	-0.198	-0.19	0.61

Table 3. PCE values obtained from RHD Manual

Approaches	Approach width (m)	Motorcycle	Car	Microbus	Bus	CNG
Southbound	10.5	0.03	1.0	1.53	1.99	0.41
Eastbound	14.0	0.01	1.0	1.4	2.09	0.44
Westbound	7.0	0.06	1.0	1.45	1.95	0.36
Northbound	10.5	0.03	1.0	1.53	1.99	0.41

5. Estimation of Saturation Flow (in PCE/hr)

Maximum number of vehicle passing during green time was calculated as Saturation flow rate. It was estimated in PCE/hr using both the direction-wise and RHD manual based PCE for different vehicle categories. To obtain time headway, saturated green time (T sec) was divided by passenger car equivalent of various types of vehicle. In following equation (4), Inverse of time headway represents the saturation flow for particular approach. Saturation Flow was calculated using Multiple Liner Regression Methods.

$$S = \frac{(PCE_{bus} \, x_1 + PCE_{micro \, bus} \, x_2 + PCE_{car} \, x_3 + PCE_{CNG} \, x_4 + PCE_{motor \, cycle} \, x_5 + PCE_{NMV} \, x_6) * 3600}{T} \quad (4)$$

Table 4 shows that saturation flow estimated based on direction-wise is higher than RHD manual based PCE in all approaches of the studied intersection.

Table 4. Saturation flow estimated by direction-wise PCE method

Intersection	Approach	Saturation Flow based on Direction-wise PCE (PCE/hr)	Saturation Flow based on RHD based PCE (PCE/hr)
	Northbound	5566.3	4549.13
I-7	Southbound	7706.75	6311.51
	Westbound	5179.55	2907
	Eastbound	5769.35	3629.85

6. Estimation of Delays (in PCE-hr)

In case of delay measurement, Residual queue ,flow rate, combination of green time are the most influencing parameter which include the effect of mix vehicle .

6.1 Formulation of total field delay

In oversaturated conditions, Total Delay depends on the number of vehicles in the residual queues for particular lane groups. New Residual queue of lane group j given by old residual queue plus arriving vehicle minus the leaving vehicle, $N_{j,T}$, can be determined by the following equation,

$$N_{j,T} = N_{j,T-1} + q_{j,T} \sum_{i=Ij-1}^{P} G_{i,T} + q_{j.T} G_{Ij,T} - s_{j.T} G_{Ij,T} \quad (5)$$

Where,

$N_{j,T-1}$ = residual queue expressed as number of vehicle in lane group j during cycle T-1 (PCE),

$q_{j,T}$ =average flow rate arriving towards lane group j during cycle T (PCE/hr),

I_j = ending phase in lane group j during a cycle,

$G_{i,T}$ = portion of green time length in lane group j during cycle T (hr), and

$s_{j,T}$ = saturated flow rate leaving lane group j (PCE/hr).

Fig.2 represents additive number of traffic present in lane group j at an intersection during cycles $T-1$ and T .Considering the intersection as 4 phases signal, Portion of green time distributed for lane group j will be $G_j = G_4$. Area under the geometry shows the total delay of traffic stream in lane group j (denoted by $D_{j,T}$) which are present from end of green time for particular lane group j during previous cycle T-1 until end of green time for particular lane group j during design cycle T . Similarly, queuing diagrams for total delay can be obtained for rest of lane groups to estimate of the delay for all vehicles at the intersection during design Cycle time which can be expressed as in the form of following equation .

$$D_T = \sum_{j=1}^{J} D_{j,T} \quad (6)$$

Where,

J = Number of total lane groups in the intersection

D_T = Total delay of all vehicle in the intersection during design cycle Time T

For oversaturated conditions, total delay of

traffic stream in lane group j during cycle Time T can be calculated as,

$$D_{j,T} = \frac{1}{2}\left(N_{j,T-1} + q_j \sum_{i=Ij-1}^{P} G_{i,T}\right) \sum_{i=Ij-1}^{P} G_{i,T} + \frac{1}{2}\left(2N_{j,T-1} + 2q_j \sum_{i=Ij-1}^{P} G_{i,T}\right)G_{Ij,T} + \frac{1}{2}\left(q_{j.T}G_{Ij,T} - s_{j.T}G_{Ij,T}\right) \quad (7)$$
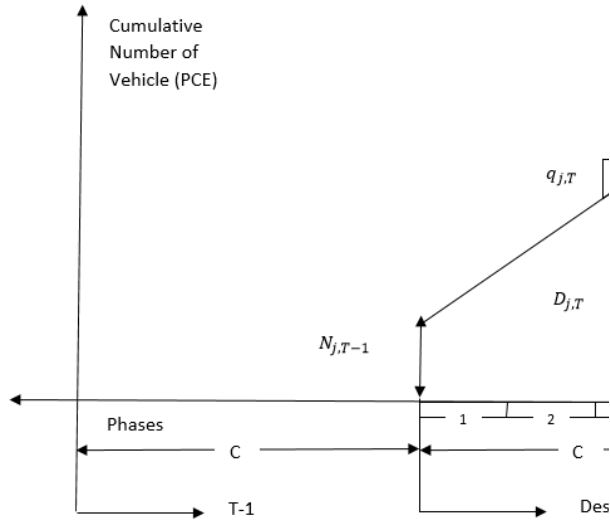

Fig. 2. Graphical representation of total delay in queuing diagram

Field delay was measured for the approaches of I-7. The average delay for each class of vehicle was calculated by the difference between departure and arrival time. Counting interval for classified vehicle was 10 seconds. The counted classified vehicle was then multiplied by the average delay per vehicle which can be expressed as vehicle seconds of delay for each classified vehicle on the approach during the counting period. The vehicle seconds of delay for each classified was then multiplied by the PCE value to get total delay in PCE-hr. Counting period was about 15-20 minutes to estimate delay. Table 5 represents total field delay estimated in various approaches of I-7 by direction-wise and RHD manual based PCE.

Control delay has been calculated for determination of LOS in intersection .Total delay is based on s/veh.

6.2 Measurement of field delay

Table 5. Total field delay measured by direction-wise PCE and RHD Manual based PCE*

Approaches	Total delay by direction-wise PCE (PCE-hr)	Total delay by RHD based PCE (PCE-hr)
Southbound	222.49	230.32
Northbound	164.27	160.30
Westbound	86.79	68.77
Eastbound	106.68	81.58
Total delay	580.24	540.97

*Note: Total field delays are measured in Vehicle-hr and then converted to corresponding PCE-hr in each method

Table 6. Level of Service based on Control Delay (HCM 2000)

Intersection	Approach	Number of Lane	Width of lane	Cycle Time(Sec)	Saturated green time(Sec)	Arrival Rate(PCU/Hr)	Saturation flow(PCU/Hr)	Capacity	PCU	Approach wise Control Delay(Sec/PCU)	Control Delay(Sec/PCU)	LOS
	SB	3	3.86		41	1174	3476	713		233		F
	EB	3	3.97		74	2411	3948	1460		131		
I-1	WB	3	3.97		43	2270	3877	834		254		
	NB	3	3.86	200	37	1727	3300	611		268	886	
	SB	3	3.92		33	759	2564	465		141		F
	EB	3	3.97		49	874	2655	715		99		
I-2	WB	3	3.97		75	1039	2770	1141		66		
	NB	3	3.92	182	25	522	2280	313		150	456	
	SB	2	3.11		25	301	1184	282		61		F
	EB	2	3.33		25	329	1292	308		60		
	WB	2	3.33		25	406	1622	386		57		
I-3	NB	2	3.11	105	25	297	1036	247		67	245	
	SB	3	4.11		41	574	3125	641		93		F
	EB	3	4.16		74	1128	3948	1461		55		
I-4	WB	3	4.16		43	953	3877	834		106		
	NB	3	4.11	200	37	507	3300	611		93	347	
	SB	3	4.11		74	459	2938	402		295		F
	EB	3	4.09		194	900	4457	1598		141		
I-5	WB	3	4.09		194	903	5208	1868		136		
	NB	3	4.11	541	74	453	2963	405		290	862	
	SB	2	4.01		145	378	1317	455		140		F
	EB	2	3.97		145	361	1183	408		149		
I-6	WB	2	3.97	420	125	320	1022	304		190	479	
	SB	3	3.5		147	5893	7707	1818		945		F
	EB	4	3.5		158	3155	5769	1463		600		
I-7	WB	2	3.5		132	3488	5180	1097		939		
	NB	3	3.5	623	181	4494	5566	1617		776	3260	
	SB	3	3.89		75	911	1883	706		100		F
	EB	2	3.78		75	884	1768	663		104		
I-8	NB	3		200	45	524	1636	368		133	337	

6.3 Delay Optimization

For minimization of signalized intersection total delay, a pre-timed signal control is selected. Basic assumption for this formulation is to consider fixed cycle time with pre-defined phase sequence. Another assumption made for arrivals and service times as Deterministic for all vehicles vehicle in the intersection and real time for transit vehicle. After running mathematical program for every cycle, Minimized total delay can be obtained by changing combination of green time of different approaches assuming constant minimum green time for each lane group j, $G_{j\ min}$. The optimization of mathematical program in following equation can be operated for any design Cycle time T.

Objective function, z = minimize $\sum_{i \in I_j} D_T$ (8)

Subject to, $\sum_{i=1}^{P} G_i = C$

$$\sum_{i \in I_j} G_i \geq G_{j\ min}$$

Table 6 shows optimized total delay calculated by direction-wise PCE .The Values given in the parenthesis are calculated by the RHD manual based PCE values.

Delay Optimization was done by optimizing Eq. (8) in Microsoft excel solving using Generalized Reduce Gradient (GRG) nonlinear technique .The Generalized Reduce Gradient method is a precise and accurate method for solving nonlinear programming (NPL) problems.

Table 7. Optimization of Total Delay

Parameter		Northbound		Westbound		Southbound		Eastbound
Final Residual queue (PCE)	$N_{1,T}$	895.747 (674.165)	$N_{2,T}$	620.167 (338.695)	$N_{3,T}$	1105.195 (473.761)	$N_{4,T}$	666.486 (682.589)
Initial Residual queue (PCE)	$N_{1,T-1}$	320.502 (353.29)	$N_{2,T-1}$	245.766 (81.21)	$N_{3,T-1}$	411.646 (391.77)	$N_{4,T-1}$	284.4 (265.32)
Saturation flow (PCE/hr)	$s_{1,T}$	5566.3 (4549.13)	$s_{2,T}$	5179.55 (2907)	$s_{3,T}$	7706.75 (6311.51)	$s_{4,T}$	5769.35 (3629.85)
Arrival rate (PCE/hr)	$q_{1,T}$	4494.29 (3190)	$q_{2,T}$	3488.84 (1865.2)	$q_{3,T}$	5892.96 (4177.2)	$q_{4,T}$	3154.76 (2301.8)
Queue forming time (hr)	$G_{2,T} + G_{3,T} + G_{4,T}$	0.14079 (0.1286)	$G_{1,T} + G_{3,T} + G_{4,T}$	0.15244 (0.1778)	$G_{1,T} + G_{2,T} + G_{4,T}$	0.135755 (0.13652)	$G_{1,T} + G_{2,T} + G_{3,T}$	0.154347 (0.1404)
Delay in approach (PCE-hr)	$D_{1,T}$	104.1481 (88.595)	$D_{2,T}$	85.64827 (44.574)	$D_{3,T}$	133.6067 (89.384)	$D_{4,T}$	88.64814 (78.254)
Optimum green time (s)	$G_{1,T}$	193.1485 (236.9347)	$G_{2,T}$	151.2176 (60)	$G_{3,T}$	211.2815 (208.5222)	$G_{4,T}$	144.35 (194.5429)
Optimum total delay (PCE-hr)	D_T	412.051 (300.81)						

6.4 Comparison of Total Field Delay and Optimized Delay

Total field delay estimated by direction-wise PCE is higher than RHD manual based PCE. Direction-wise PCE considered separate PCE values for right turning traffic, which were higher than through traffic. Similarly, optimized delay calculated utilizing RHD manual based PCE

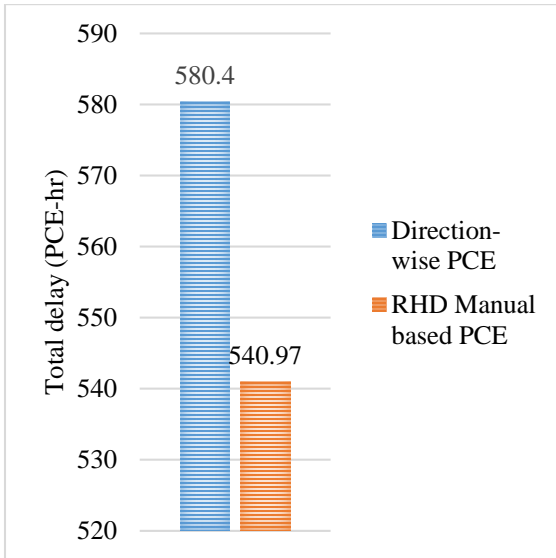showed lower result comparing to direction-wise PCE.

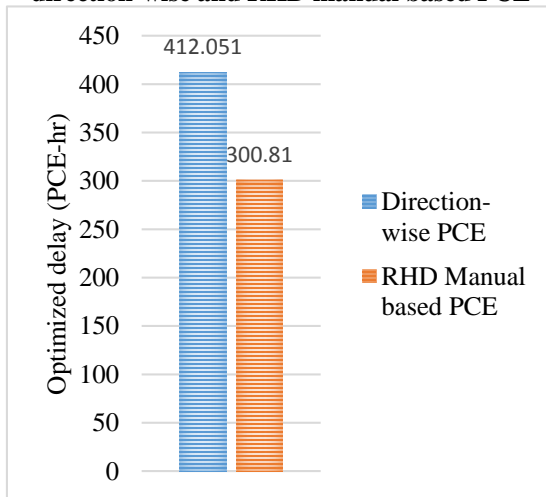**Figure 3. Comparison of Field delays estimated by direction-wise and RHD manual based PCE**

Figure 4. Comparison of Optimized delays estimated estimated by direction-wise and RHD manual based PCE

7. Concluding Remarks

The PCE values of right turning vehicle have important role on the estimation of saturation flow; thus influence the measurement of delay and signal optimization. The proposed direction-wise PCE value model depended on classified vehicle counts and saturated green time at each approach, as well as, those considered through and right turning traffic separately. Whereas, RHD manual based PCE values varied with the

percentage of NMV, width of each lane group and there was no separate PCE values for turning vehicle. Right turning vehicles took more space and time while maneuvering at a signalized intersection, hence, those had different PCE values compared to through traffic. Right turning traffic were of significant proportion in total traffic at the studied I-7. Those comprised 60.56%, 33.24%, 23.44% and 20.96% of total traffic for Southbound, Northbound, Eastbound and Westbound respectively. As a result, RHD manual based PCE was inaccurate in PCE values estimation, as it considered through and right turning vehicle together. In addition, RHD manual based PCE underestimated saturation flow rate compared to direction-wise PCE values by 18.1% (Southbound), 18.3% (Northbound), 37.1% (Eastbound) and 43.87% (Westbound). Similarly, total delay and optimized delay estimated by RHD manual based PCE underestimate direction-wise PCE by 6.79% and 27% respectively. Hence, the mathematical model of direction-wise PCE values can be applied as a framework for designing and analyzing signalized intersections in Dhaka city, as well as, other metropolitan of developing countries, where traffic conditions are heterogeneous. Appropriate estimation of PCE values can improve the accuracy of delay calculation. Several researches such as, [Teply, 1989], [Kumar and Dhinakaran, 2012] and [Minh et al. 2010] adopted theoretical delay models to estimate delays and found some correlation with measured field delays. However, delay at signalized intersection cannot be precisely measured through theoretical delay models. In this research, field delay at approaches of signalized intersection was measured by the method adopted by [Christofa and Skabardonis, 2011], which can calculate delay from queuing diagram directly and precisely. To encounter the effects of heterogeneous traffic, classified vehicle counts per hour obtained for arrival rate and queue length were converted into proposed

direction-wise dynamic and RHD manual based PCE. The above mentioned first method provides better estimation of delay at signalized intersection. The study found that the analysis for non-lane based heterogeneous traffic condition depends on vehicle composition and the PCE values of same type of vehicle vary at different approaches of a signalized intersection. In this research, synchronous regression model was developed to measure direction-wise dynamic PCE values for five types of classified vehicles at a signalized intersection. Afterward, saturation flow, total delay and optimized delay were estimated for better intersection design. The regression model for PCE doesn't account the effect of light in vision and time. The effect of light in vision (day and night) and time (peak and off-peak) can be introduced to this model. In addition, saturation flow depends on various factors which affects the PCE value. In the study all approaches of the intersection were selected having almost flat surface. Saturation flow also gets affected by parking facility near intersection

and slope of level terrain. All these factors can be incorporated into the study and developed new. Moreover, the recommended direction-wise PCE method for delay optimization may be verified for large number of intersections of other cities as well. The real-time optimization of total delay in any intersection can be done using direction-wise PCE with predicted value of vehicle composition, arrival rate and residual queue. Overall, the proposed method can be used as framework for the analysis and design of pre-timed signalized intersection under heterogeneous traffic condition.

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9. References

Transportation Engineering, ASCE, Volume 121, No.4. pp. 397- 404.

- Asaithambi, G., Sekhar Mourie, H. and Sivanandan, R. (2017) "Passenger Car Unit Estimation at Signalized Intersection for Non-lane Based Mixed Traffic Using Microscopic Simulation Model", Periodica Polytechnica Transportation Engineering, Vol. 45, No. 1, pp. 12-20.

- Benekohal, R. F., & El-Zohairy, Y. M. (2001) "Multi-regime arrival rate uniform delay models for signalized intersections", Transportation Research Part A: Policy and Practice, Vol. 35, No. 7, pp. 625–667.

- Benekohal, R. F., and Zhao, W. (2000) "Delay-based passenger car equivalents for heavy vehicles at signalized intersections", Transportation Research Part A: Policy Practice. Vol. 34, No. 6, pp. 437–457.

- Bhatt, M., and Patel, P. (2017) "Determination of dynamic PCU in Ahmedabad city", International Journal of Advance Research

- Adams, C. A., Zambang, M.A.M., and Boahen, R. O. (2015) "Effects of motorcycles on saturation flow rates of mixed traffic at signalized intersections in Ghana", International Journal of Traffic and Transportation Engineering, Vol. 4, No. 3, pp. 94–101.

- Akçelik, R. (1980) "Time-dependent expressions for delay, stop rate and queue length at traffic signals", Australian Road Research Board, Report No. AIR367-1.

- Alex, S., and Isaac. K. P. (2015) "Dynamic PCU values at signalized intersections in India for mixed traffic", International Journal of Traffic and Transportation Engineering, Vol. 5, No. 2, pp. 197–209.

- Arasan, T. V. and Jagadish, K. (1995) "Effect of Heterogeneity of Traffic on Delay at Signalized Intersections", Journal of

and Innovative Ideas in Education , Vol. 3, No. 2, pp. 1179–1184.

- Bhattacharya, P. G., and Mandal, A.G. (1980) "Investigation of passenger car equivalents at controlled intersection in Calcutta." In Proce, Highway Research Bulletin, Indian Roads Congress, Vol. 14, pp. 41–64. New Delhi, India: Indian Roads Congress.

- Branston, D., & Gipps, p. (1981) "Some experience with a multiple linear regression method of estimating parameters of the traffic signal departure process", Transportation Research, Vol. 6A, pp. 445-458.

- Branston, D., & Vanzuylen, J. (1978) "The estimation of saturation flow, effective green time and passenger car equivalents at traffic signals by multiple linear regression", Transportation Research, Vol. 12, No. 1, pp. 47 – 33.

- Cao, N. Y., Sano, K., Minh, C. C. and Matsumoto, S. (2009) "Motorcycle equivalent units of bus based on delay at signalized intersections in urban road", In Vol. 7 of Proc., Eastern Asia Society for Transportation Studies, pp. 326–326. Tokyo: Eastern Asia Society for Transportation Studies.

- Chandra, S., V. Kumar, and Sikdar, P. K. (1995) "Dynamic PCU and estimation of capacity of urban roads", Indian Highways, Vol. 23, No. 4, pp. 17–28.

- Christofa, E. and Skabardonis, A. (2011) "Traffic signal optimization with application of transit signal priority to an isolated intersection", Transportation Research Record, Vol. 2259, No. 1, pp.192-201.

- Cuddon, A.P. and Ogden, K.W. (1992) "The effect of heavy vehicles on saturation flows at signalised intersections", Proceedings of the 16th ARRB Conference, Perth, Australia, November 9-13, Vol.16 (5), pp.1-18.

- Fambro, D. B., & Rouphail, N. M. (1997) "Generalized delay model for signalized intersections and arterial streets", Transportation Research Record: Journal of the Transportation Research Board, Vol. 1572, No. 1, pp. 112–121.

- Hadiuzzaman, M., Rahman, M.M. and Karim, M.A. (2008) "Saturation flow model at signalized intersection for non-lane based traffic", Canadian journal of transportation, Vol. 2, No. 1, pp. 78-90.

- Highway Research Board. (1965.) "Highway Capacity Manual, Special Report 209, Chapter 9: Signalized intersections, Third edition", National Research Council, Washington, DC. U.S.A.

- Keller, E. L. and Saklas, J. G. (1984) "Passenger car equivalents from network simulation", Journal of Transportation Engineering, Vol. 110, No. 4, pp. 397–411.

- Kimber, R. M. (1986) "The prediction of saturation flow for road junction controlled by traffic signal", Transport and Road Research Laboratory Report 67.

- Kimber, R. M., M. McDonald and Hounsell, N. (1986) "The prediction of saturation flow for road junction controlled by traffic signal", Transport and Road Research Laboratory, Report 67.

- Ko J, Hunter M. and Guensler R. (2007) "Measuring Control Delay Using second-by-Second GPS Speed Data", In Transportation Research Board 86th Annual Meeting, TRB, National Research Council, Washington, DC.

- Kumar, R. P., and Dhinakaran, G. (2013) "Estimation of delay at signalized intersections for mixed traffic conditions of a developing country", International Journal of Civil Engineering. Vol. 11, No. 1, pp. 53–59.

- Kumar, R.P. and Dhinakaran, G. (2012) "Estimation of delay at signalized intersections for mixed traffic conditions of a developing country", International Journal of Civil Engineering, Vol. 11, No. 1.

- Lam, W.H.K. (1994) "Saturation flows and passenger car equivalents at signalized Intersections in Hong Kong", Proceedings of the 2nd International Symposium on Highway Capacity, Vol. 2, U.S.A., pp. 385-394.

- Li, M.T. and Gan, A.C. (1999) "Signal timing optimization for oversaturated networks using TRANSYT-7F", Transportation Research Record, Vol. 1683, No. 1, pp.118-126.

- Mahidadiya, A. N. and Juremalani, J. (2016) "Estimation of passenger car unit value at signalized intersection", International Journal of Scientific Research in Science, Engineering and Technology, Vol. 2, No. 2, pp. 1317–1324.

- Maini P. and Khan S. (2000) "Discharge characteristics of heterogeneous traffic at signalized intersections", Proceedings of fourth International Symposium on Highway Capacity 2000, pp. 258–270.

- Mamun, A. A., Mahmud, H. M. I., Afroz, M., Hossain, Q. S., Kaiser, S. M. S. U. (2012) "Estimation of Passenger Car Equivalent (PCE) of right-turning vehicles at signalized intersection in Dhaka Metropolitan City, Bangladesh", Advances in Transportation Studies an international Journal, Section B 26, pp. 47-56.

- Mania, E.V., Forde, A. and Robinson, R.M. (2016) "Impact of optimally minimizing delay times on safety at signalized intersections in urban areas case study: the city of Virginia Beach", International journal of transportation engineering, Vol. 3, No. 4, pp. 277-288.

- Martin and Voorhees Associates (1981) "Saturation flows and lost times at traffic signal junctions flow for road junction controlled by traffic signal", Working Paper TSN77, unpublished.

- May, A.D. (1990), "Traffic flow fundamentals", Chapter 12, Queuing analysis.

- Minh, C. C., Binh, T. H., Mai, T.T. and Sano, K. (2010) "The delay estimation under heterogeneous traffic conditions", Journal of the Eastern Asia Society for Transportation Studies Vol. 8, pp. 1583–1595.

- Minh, C.C., Binh, T.H., Mai, T.T. and Sano, K. (2010) "The delay estimation under heterogeneous traffic conditions", Journal of the Eastern Asia Society for Transportation Studies, Vol. 8, pp.1583-1595.

- Ministry of Communication.(2001). "Geometric design standards of RHD (Roads and Highways Department)", Government of the People's Republic of Bangladesh, Bangladesh.

- Mohan, M. and Chandra, S. (2017) "Queue clearance rate method for estimating passenger car equivalents at signalized intersections", Journal of Traffic Transportational Engineering, Vol. 4, No. 5, pp. 487–495.

- Pajecki, R., Ahmed, F., Qu, X., Zheng, X., Yang, Y. and Easa, S. M. (2019) "Estimating passenger car equivalent of heavy vehicles at roundabout entry using micro-traffic simulation", Frontiers in Built Environment, Vol. 5, No. 77.

- Parvathy, R., T. Sreelatha, and Z. K. Reebu. (2013) "Development of new PCU values and effect of length of passenger cars on PCU", International Journal of Innovative Research in Science, Engineering and Technology, Vol. 2, No. 1, pp. 344–351.

- Patel, P. and Dhamaniya, A. (2019) "Stream equivalency factor for mixed traffic at urban signalized intersections", Transportation Research Procedia, Vol. 37, pp.362-368.

- Pretty, R.L. (1966) "The effects of right-turning vehicles on saturation flow through signalized intersections", Proceeding of 3rd

ARRB Conference, Vol. 3, No. 1, pp. 460-470.

- Qiao, F., Yi, P., Yang, H., & Devarakonda, S. (2002) "Fuzzy logic based intersection delay estimation", Mathematical and Computer Modelling, Vol. 36, No. 11, pp. 1425–1434.

- Radhakrishnan, P., and Mathew, T.V. (2011) "Passenger car units and saturation flow models for highly heterogeneous traffic at urban signalized intersections", Transportmetrica, Vol. 7, No. 2, pp. 141–162.

- Rahman, M. M., Okura, I. and Nakamura, F. (2003) "Measuring passenger car equivalents (PCE) for large vehicles at signalized intersections", Journal of the Eastern Asia Society for Transportation Studies. Vol. 5, pp. 1223–1233.

- Rouphail, N., Tarko, A., & Li, J. (1999) "Traffic flow at signalized intersections", Transportation Research.

- Saha, P., Hossain, Q. S., Mahmud, H. M. I. and Islam, M. Z. (2009) "Passenger car equivalent (PCE) of through vehicles at signalized intersections in Dhaka Metropolitan City, Bangladesh", Journal of International Association of Traffic and Safety Sciences (IATSS), Vol.33, No.2, pp.99-104.

- Sarraj, Y., and Jadili, I. (2012) "Estimating passenger car unit factors for buses and animal driven carts in Gaza City, Palestine", IUG Journal of Natural Studies Vol. 20, No. 2, pp. 99–116.

- Saw, K, Katti, B. K. and Joshi, G. (2015) "Literature review of traffic assignment: static and dynamic," International Journal of Transportation Engineering, Vol. 2, No. 4, pp. 339–347.

- Shao, C.Q., Rong, J. and Liu, X.M. (2011) "Study on the saturation flow rate and its influence factors at signalized intersections in China", Procedia-Social and Behavioral Sciences, Vol. 16, pp.504-514.

- Simha, G. H. S. V. (2017) "Estimation of saturation flow at various signalized intersections", Open Access International Journal of Science and Engineering Vol. 2, No. 11, pp. 42–48.

- Teply, S. (1989) "Accuracy of delay surveys at signalized intersections", Transportation Research Record TRB, National Research Council, Washington, D.C., Vol. 1225, pp. 24-32.

- Teply, S., Allingham, D. I., Richardson, D. B. and Stephenson, B. W. (1984) "Canadian capacity guide for signalized intersections", Canada: Transportation Association of Canada

- TRB (Transportation Research Board) (2000) "Highway Capacity Manual, Chapter 16: Signalized intersection, National Research Council", Washington, DC. U.S.A.

- Verma, A., Nagaraja, G., Anusha, C.S. and Mayakuntla, S.K. (2018) "Traffic Signal Timing Optimization for Heterogeneous Traffic Conditions Using Modified Webster's Delay Model", Transportation in Developing Economies, Vol. 4, No. 2, pp.13.

- Webster, F. V., (1967) "Effect of right-turning vehicle at traffic signals", Road Research Laboratory, Laboratory Report 86, Crowthorne, U.K.

- Webster, F. V. (1958) "Traffic Signal Settings", Road Research Technical Paper, Vol. 39.

- Zargari, S. A., Dehghani, N. and Moraveji, G. (2014) "Optimal signal control in urban road networks with high priority congested centers", International Journal of Transportation Engineering, Vol. 2, No. 1, pp. 1-12.

- Zhang, Y. and Tong, J. (2008) "Modeling left-turn blockage and capacity at signalized intersection with short left-turn bay", Transportation Research Record: Journal of the Transportation Research Board, Vol. 2071, No. 1, pp. 71–76.

- Zhao, W. (1998) "Delay-based passenger car equivalents for heavy vehicles at signalized intersections", Procedia - Traffic and Transportation Studies, pp. 691–700. Reston, VA: ASCE, 1998.