A Novel Approach for Optimization of Transportation Problem in Chaos Environment

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Abstract

Nature is characterized by its chaotic behavior. Mathematics is considered one of the appropriate tools to achieve the best definition of possible its chaos variables and process. Classical mathematics deals with the numbers as static and meaningless, but chaos mathematics deals with it as dynamic evolutionary, and value- added. This paper attempts to introduce the transportation problem representation in chaos environment and also the necessity of the model is investigated. An approach for determining the chaos best solution is proposed briefly. The advantage of the proposed approach is accomplished with the associated ordinary number and the number of iterations arriving to the best solution is reduced. A numerical example is given to illustrate the utility, effectiveness and applicability of the approach for the problem.

Keywords: Transportation problem; Chaos numbers; Initial basic feasible solution; North-west corner method; Stepping –Stone; Chaos best solution.

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1. Introduction

Transportation problem (TP) is an operational research for its wide application in real life. This a special kind of optimization problem which concerned with finding the minimum cost of transporting for a single commodity from a given number of sources to given number of destinations. Transportation or shipping problem (TP) involves determining the amount of goods or items to be transported from a number of sources to a number of destinations. TP is a specific case of linear programming (LP) problems and a special algorithm has been developed to solve it.

The data of the TP problem include the following:

- Level of supply at each source and the amount of demand at each destination.
- Transportation cost unit of the commodity from each source to each destination.

The basic TP was developed by [Hitchcock, 1941]. Efficient methods to find the solutions have developed by [Dantzig, 1951; Charnes et al., 1953; Kasana& Kumar,2005; Pandian& Natarajan, 2010, and Shenoy et al.,1991]. Some of the reputed method for finding an initial basic feasible solution of TP have discussed as: North West Corner Method [Hamdy, 2007], Row Minimum Method [Anam et al. 2012], Column Minimum Method [Anam et al. 2012], Least Cost Method [Hamdy, 2007],Vogel's Approximation Method [Pandian& Natarajan, 2010], Extremum Difference Method [Kasana& Kumar,2005], Highest Cost Difference Method[Khan, 2011], Average Cost Method [Rashid et al. 2013] , etc,.

In many scientific areas, such as systems analysis and operations research, a model has to be set makes this possible. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers. [Dubois and Prade, 1980] extended the use of algebraic operations on real numbers to fuzzy numbers by the use of a fuzzification principle. A enormous authors tested the TP in various fuzzy environment as integer fuzzy [Tada and Ishii, 1996], type-2 fuzzy [Liu et al. 2014, Kundu et al. 2014; Gupta and Kumari, 2017], [Omar and Samir, 2003], and [Chanas and Kuchta, 1996] discussed the solution algorithm or solving the TP. [Mahewari and Ganesan, 2018] proposed a simple approach for solving TP having pentagonal fuzzy numbers. [Gabrel, Lacroix, Murat, and Remli, 2014] presented an extensive computation analysis of the 2- stage robust location TP. [Roy, 2016] analyzed the multi-choice stochastic TP where the cost of the objective and the demand follow multi- choice parameters, and the supply follow logistic distribution. [Daman, 2017] developed a model of the uncertain entropy multi- item solid transportation problem (STP) in which the supply capacities, demands, conveyances and transportation capacities are represented as uncertain variables. [Guo, Wang, and Zhou, 2015] investigated a TP under uncertain and random environment. They proposed a conceptual uncertain model for the problem, where the supplies are considered as random variables, and the cost and the demands are uncertain variables. [Vidgya and Ganesan, 2018] introduced methodology for solving fuzzy multi-objective TP problems with triangular fuzzy numbers in all of costs, demands and supplies. [Li and Lai, 2000] applied fuzzy programming approach to multi-objective TP. [Maity et al., 2016] studied a multi-objective TP under uncertain environment, where they considered the supply and demand as uncertain variables. Also, fuzzy multi- choice goal programming is used to select the proper goal to the objectives of the proposed multi- objective TP. [Zhang, Peng, Li, and Chen, 2016] studied fixed charge STP applying uncertainty theory to the crisp STP. [Kaur, Rakshit, and Singh, 2018] proposed a simple approach to obtain best compromise of multi- objective TP. [Bit, Biswal and Alam, 1993] applied fuzzy programming for multi-objective STP. The interval number TP are converted into the ordinary multi- objective problems based on [Das, Goswami, and Alam, 1999]. [Ammar and Khalifa, 2014] investigated the efficient solution and stability of fully fuzzy multi-objective STP. [Ahmed et al.2016] proposed a new approach for finding an initial basic feasible solution to the TP problems. [Kumar et al. 2019] introduced a new computing procedure for solving the fuzzy Pythagorean TP.

Nature forces and processes are infinite. They interact with other in a chaotic manner. They follow a spontaneous order that is chaos. Chaos is the ordered disorder of nature. The conventional definition and perception of order is short- term oriented since it does not allow change or evaluation. However, nature is in continuous change, progress, and evaluation because it is governed by infinity of dimensions including time, space, and state [Ketata, Satish, and Islam, 2006].

Nature knowledge requires the development of new tools to understand better its variables and process and their diverse interaction [Gutzwiller, 1991; Ott, 2002; Strogatz, 2001; Peitgen, Jurgens, and Saupe ,2004].

Mathematics is the study of relationships using numbers, shapes, and quantities. It employs signs, symbols, and proofs. It includes arithmetic, algebra, calculus, geometry, and trigonometry. In classical mathematics, a number is static and meaningless. However, in chaos mathematics, a number is dynamic, evolutionary, and value- added.

In this paper, a mathematical model of TP is represented in chaos environment. The necessity of the model is discussed. Also, a novel approach for solving the problem is proposed.

The outlay of the paper is constructed as follows: Section 2 offers some preliminaries needed in the paper. Section3 introduces some of notation and index needed in the problem formulation. In section 4, mathematical model for TP with chaos numbers cost is formulated. Section 5 introduces a novel approach for solving the chaos TP. In section 6, a numerical example is given for illustration. Finally some concluding remarks are reported in section7.

2. Preliminaries

In order to discuss our problem conveniently, basic concepts and results related to chaos numbers are recalled [Ketata et al., 2006].

Definition 1 (Chaos number). A chaos number x_a is neither static nor rigid. It changes over time and space.

Definition 2. Let x_a and y_b be two chaos numbers. The arithmetic operations on x_a and y_b are:

1. $x_a + y_b = (x + y)_{a+b}$,
2. $x_a - y_b = (x - y)_{a-b}$,
3. $x_a \times y_b = (x \times y)_{bx+ay+ab}$,
4. $x_a \div y_b = \left(\frac{x}{y}\right)_u$, where $u = \frac{(x+a)\times y - x\times\times(y+b)}{(y+b)\times y}$,
5. $0_a \times x_b = 0_{ax+ab}$,
6. $0_a \div x_b = 0_{\left(\frac{a}{x+b}\right)}$,
7. $x_a \div 0_b = (b \times x)_{ab}$.

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3. Index and Notation

3.1 Index

In this transportation problem, the following indices are made:

i: Source index for all $i = 1, 2, \ldots, m$.

j: Destination index for all $j = 1, 2, \ldots, n$.

3.2 Notation

In this transportation problem, the following notation can be used:

x_{ij}: Number of units of the product transported from ith source to jth destination.

C_{ij}^{ch} : Chaos cost of one unit quantity transported from ith source to jth destination.

x_i: Total availability of the product at the source i.

x_j: Total demand of the product at the destination j.

$\hat{Z}$: Cost function with ordinary number.

4. Problem Definition

Consider transportation problem with m sources and n destinations in which the decision maker is uncertain about the precise values of transportation cost from ith source to jth destination, but there is no uncertainty about the supply and demand of the product. The mathematical model of the problem is

$$\min Z^{ch} = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^{ch}\, x_{ij}$$

Subject to

(1)

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, \ldots, m,$$
$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, \ldots, n,$$
$$x_{ij} \geq 0; \ \forall\, i, j.$$

5. Procedure for Proposed Novel Approach Based On Chaos Number

5.1 Assumptions of the Proposed Novel Approach

- **Requirement:** The supply units entering from each source must be distributed to destinations.
- **Feasible solution:** The transportation problem with chaos numbers have a feasible solution if and only if
  $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$.
- **Cost:** The cost depends only on the number of units transported and unit of transportation cost not on distance and conveyance.
- **Input:** All the parameters represented by either crisp or chaos numbers.

5.2 Chaos Initial Basic Feasible Solution for Problem (1)

The steps of the procedure to determine the chaos initial basic feasible solution can be u

Step1: Consider the TP with chaos number of cost C_{ij}^{ch}.

Step2: Apply the North West corner method to obtain basic feasible solution.

5.3 Chaos Optimal Solution

Here, the stepping- stone method is applied for obtaining the best solution of the chaotic TP problem.

6. Numerical Examples

6.1 Example 1

Consider the TP with three plants $A, B,$ and C manufacture a product with a very large market. $A, B,$ and C have respectively 100, 120, and 120 metric tons available to deliver to five warehouses, numbers 1, 2, 3, 4, and 5.Warehouse 1 demands 40 units, warehouse 2 demands 50 units, warehouse 3 demands 70 units, warehouse 4 demands 90 units, and warehouse 5 demands 90 units. The transportation cost between plants and warehouses is given in the following table

Tabl11. Data input for chaos transportation problem

Warehoused \ Plants	1	2	3	4	5	Supply
(1) A	$4_{0.25}$	$1_{0.5}$	$2_{0.25}$	$5_{0.25}$	29_{0}	100
(2) B	4_{0}	$4_{0.25}$	$1_{0.5}$	$4_{0.5}$	$6_{0.75}$	120
(3) C	$4_{0.5}$	$2_{0.5}$	6_{0}	$5_{0.75}$	$7_{0.5}$	120
Demand	40	50	70	90	90	

Starting from the North- West Corner, the basic solution is given in table2

Tabl12. First chaos stepping- Stone solution

Warehoused \ Plants	1	2	3	4	5	Supply
(1) A	40	50	10	0	0	100
(2) B	0	0	60	60	0	120
(3) C	0	0	0	30	90	120
Demand	40	50	70	90	90	

The best solution is $x_{11} = 40_0$, $x_{12} = 50_0$, $x_{13} = 10_0, x_{23} = 60_0, x_{24} = 60_0$, $x_{34} = 30_0, x_{35} = 90_0$, and the corresponding chaos cost is

$Z^{ch} = 40_0 \times 4_{0.25} + 50_0 \times 1_{0.5} + 10_0 \times 2_{0.25} + 60_0 \times 1_{0.5} + 60_0 \times 4_{0.5} + 30_0 \times 5_{0.75} + 90_0 \times 7_{0.5} = 1310_{240}$.

The corresponding cost expressed in terms of associated number is $\hat{Z} = 1550$.

Various stepping- stone method for searching a best optimal solution and we continue this process with the improved solution to find a better solution. The process is illustrated as in the tables 3 and 4

Table 13. Second best solution for example 1

Warehoused \ Plants	1	2	3	4	5	Supply
(1) A	0	30	70	0	0	100
(2) B	0	0	0	30	90	120
(3) C	40	20	0	60	0	120
Demand	40	50	70	90	90	

The best solution is $x_{12} = 30_0$, $x_{13} = 70_0$, $x_{24} = 30_0, x_{25} = 90_0, x_{31} = 40_0$, $x_{32} = 20_0, x_{34} = 60_0$, and the corresponding chaos cost is

$Z^{ch} = 30_0 \times 1_{0.5} + 70_0 \times 2_{0.25} + 30_0 \times 4_{0.5} + 90_0 \times 6_{0.75} + 40_0 \times 4_{0.5} + 20_0 \times 2_{0.5} + 60_0 \times 5_{0.75} = 1330_{190}$ and $\hat{Z} = 1520$.

Table 14. Final best solution for example 1

Warehoused \ Plants	1	2	3	4	5	Supply
(1) A	0	50	50	0	0	100
(2) B	0	0	20	10	90	120
(3) C	40	0	0	80	0	120
Demand	40	50	70	90	90	

The best solution is $x_{12} = 50_0$, $x_{13} = 50_0$, $x_{23} = 20_0, x_{24} = 10_0, x_{25} = 90_0$, $x_{31} = 40_0, x_{34} = 80_0$, and the corresponding chaos cost is

$Z^{ch} = 50_0 \times 1_{0.5} + 50_0 \times 2_{0.25} + 20_0 \times 1_{0.5} + 10_0 \times 4_{0.5} + 90_0 \times 6_{0.75} + 40_0 \times$

$4_{0.5} + 80_0 \times 5_{0.75} \quad = 1310_{200}$ and $\hat{Z} = 1510$.

6.1 Example 2

Consider the following TP problem

Table 15. Data input for chaos transportation problem

Warehoused / Plants	1	2	3	4	Supply
(1) Q_1	11_0	$18_{0.5}$	33_1	38_2	26
(2) Q_2	4_1	15_1	$36_{0.5}$	24_1	24
(3) Q_3	18_1	6_1	14_1	$23_{0.5}$	30
Demand	17	23	28	12	

Starting from the North- West Corner, the basic solution is given in table6 Tabl16. Final chaos stepping- Stone solution (Example 2)

Warehoused / Plants	1	2	3	4	Supply
(1) Q_1	17	9	0	0	26
(2) Q_2	0	14	10	0	24
(3) Q_3	0	0	18	12	30
Demand	17	23	28	12	

The best solution is

$x_{11} = 17_0,\ x_{12} = 9_0, x_{22} = 14_0, x_{23} = 10_0, x_{31} = 18_0, x_{34} = 12_0$, and the corresponding chaos cost is

$Z^{ch} = 17_0 \times 11_{0.5} + 9_0 \times 18_{0.5} + 14_0 \times 15_1 + 10_0 \times 36_{0.5} + 18_0 \times 14_1 + 12_0 \times 23_{0.5} = 1447_{56}$, and is $\hat{Z} = 1503$.

7. Concluding Remarks

In this paper, the transportation problem involving data represented in the chaos numbers has studied and also the necessity of the model has investigated. An approach for determining the chaos best solution has proposed briefly. Also, we shown how to chaotic the classical stepping- stone.

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Conflicts of Interest

The author declares no conflicts of interest.

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